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ON DOUBLE WEIGHTED MEAN STATISTICAL CONVERGENCE

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Abstract

In this paper, we have established some new theorems on double weighted mean statistical convergence of double sequences, which gives some new results and generalizes the some previous known results of Karakaya.

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1. Introduction

Let $K \subset IN \times IN$ (the set of positive integers) be a two dimensional set of positive integers and let $K(m,n)$ be the number of (j,k) in K such that $j \le m$ and $k \le n$. If the sequence $K(m,n)$ has a limit and limit is exist, then K has a double natural density and defined as (MURSALEN etc. [4])

$$\delta_2(K) = m,n \xrightarrow{\lim} \infty \frac{|K(m,n)|}{mn}$$

where $|K(m,n)|$ denotes the cardinality of $K(m,n)$.

A double sequence of real numbers $(x_{jk}, j,k = 0,1,2...)$ is said to be statistically convergent to some number L, if for $\varepsilon > 0$, the set (FREEDMAN etc. [2].

$$K(\varepsilon) = \{(j,k),\ j \le m, k \le n : |x_{jk} - L| \ge \varepsilon\} \tag{1.1}$$

has double natural density zero. In this case we write,

$$ST - \lim x_{jk} = L \tag{1.2}$$

Let $\{p_n\}$ and $\{q_n\}$ be two sequences of the positive real constants, such that

$$P_m = \sum_{j=0}^{m} p_j \to \infty, \text{ as } m \to \infty, \ (p_0 > 0, p_m \neq 0) \tag{1.3}$$

$$Q_n = \sum_{k=0}^{n} q_k \to \infty \text{ as, } n \to \infty, \ (q_0 > 0, q_n \neq 0) \tag{1.4}$$

The double weighted means of a given double sequence $\{x_{jk}\}$ are denoted by the $(\bar{N}, p_m, q_n)$ means and the sequence-to-sequence transformation defined as (FEKETE [1])

$$t_{m,n} = \frac{1}{P_m Q_n} \sum_{j=0}^{m} \sum_{k=0}^{n} p_j q_k x_{jk}, \quad m,n = 0,1,2.... \tag{1.5}$$

The sequence $\{x_{jk}\}$ is said to be double weighted means or $(\bar{N}, p_m, q_n)$ summable to L. If

$$t_{m,n} \to L, \quad \text{as } m,n \to \infty \tag{1.6}$$

and we may write

$$x_{jk} \to L(\overline{N}, p_m, q_n)$$

If p_m=1 and $q_n = 1$, then

$$t_{m,n} = \frac{1}{m} \cdot \frac{1}{n} \sum_{j=0}^{m} \sum_{k=0}^{n} x_{jk} \tag{1.7}$$

this is denoted by (C,1,1) and called the double Cesáro Summability. (RHOADES etc. [5])
If,

$$m,n \xrightarrow{\lim} \infty \frac{1}{P_m Q_n} \sum_{j=0}^{m} \sum_{k=0}^{n} p_j q_k \left| x_{jk} - L \right| = 0 \tag{1.8}$$

the sequence $X = \{x_{jk}\}$ is said to be strong $(\overline{N}, p_m, q_n)$ summable to L and it is denoted by

$$\left| \overline{N}, p_m, q_n \right| = \left\{ X = \{x_{jk}\} : m,n \xrightarrow{\lim} \infty \sum_{j=0}^{m} \sum_{k=0}^{n} p_m q_n \left| x_{jk} - L \right| = 0, \text{ for some } L \right\} \tag{1.9}$$

The matrix $A = (a_{nk})$ for $(\overline{N}, p_m, q_n)$ – summability is given by

$$a_{nk} = \begin{cases} \dfrac{p_j q_k}{P_m Q_n}, & if \ j \leq m, \quad k \leq n \\ O, & if \ j > m, \quad k > n \end{cases} \tag{1.10}$$

Before, we state the main result of this paper, let us give the some more definitions.
A sequence $X = \{x_{jk}\}$ is said to be double weighted statistical convergent, if for given $\varepsilon > 0$,

$$m,n \xrightarrow{\lim} \infty \frac{1}{P_m Q_n} \left| \left\{ j \leq m, k \leq n : p_j q_k \left| x_{jk} - L \right| \geq \varepsilon \right\} \right| = 0 \tag{1.11}$$

The set of double weighted statistical convergence sequence is denoted by $ST_{\overline{N}}$ as follows.

$$ST_{\overline{N}} = \left\{ X = \{x_{jk}\} : m,n \underrightarrow{\lim} \infty \frac{1}{P_m Q_n} \left| j \leq m, k \leq n : p_j q_k \left| x_{jk} - L \right| \geq \varepsilon \right| = 0, \text{ for some L} \right\} \tag{1.12}$$

If the sequence $X = \{x_{jk}\}$ is $ST_{\overline{N}}$ is convergent, then we also use the notation $x_{jk} \to \mathrm{L}(ST_{\overline{N}})$.

2. Main Results

Concerning double weighted mean statistical convergence of double sequences, we have proved the following theorems.

Theorem 2.1: If the sequence $\{x_{jk}\}$ is $\left|\overline{N}, p_m, q_n\right|$-summable to L, then the sequence $\{x_{jk}\}$ is $ST_{\overline{N}}$ convergent and the inclusion, $\left|\overline{N}, p_m, q_n\right| \subset ST_{\overline{N}}$ is proper.

Theorem 2.2: Let $P_m \to \infty,\ Q_n \to \infty$ and $p_j q_k \left|x_{jk} - L\right| \le M$, for all $j,k \in IN$. If $x_{jk} \to L(ST_{\overline{N}})$, then $x_{jk} \to L(\overline{N}, p_m, q_n)$

Theorem 2.3: Let $Let\left(\frac{P_m}{m}, \frac{Q_n}{n}\right) > 1$ or $\left\{\left(\frac{P_m}{m}\right) > 1 \text{ and } \left(\frac{Q_n}{n}\right) > 1\right\}$, for all . $m,n \in IN$. If $x_{jk} \to L(ST)$, then $x_{jk} \to L(ST_{\overline{N}})$ and inclusion is proper.

Theorem 2.4: If the sequence $\{p_m\}$ and $\{q_n\}$ are bounded sequences, such that $\lim\sup\left(\frac{P_m}{m}, \frac{Q_n}{n}\right) < \infty$ or $\left\{\lim\sup\left(\frac{P_m}{m}\right) < \infty \text{ and } \lim\sup\left(\frac{Q_n}{n}\right) < \infty\right\}$, the $ST_{\overline{N}}$ is equivalent to ST

3. Proof of the Theorems

The proof of the our theorems are as follows

Poof of the Theorem 2.1:

Let the sequence $\{x_{jk}\}$ be $\left|\overline{N}, p_m, q_n\right|$-summable to L and $K_\varepsilon = \{j \le m, k \le n : p_j q_k \left|x_{jk} - L\right| \ge \varepsilon\}$. Then for a given $\varepsilon > 0$ we have

$$\begin{aligned}\frac{1}{P_m Q_n}\sum_{j=0}^{m}\sum_{k=0}^{n} p_j q_k \left|x_{jk} - L\right| &= \frac{1}{P_m Q_n}\sum_{\substack{j=0\\ j\varepsilon k_\varepsilon}}^{m}\sum_{\substack{k=0\\ k\varepsilon K_\varepsilon}}^{n} p_j q_k \left|x_{jk} - L\right| \\ &+ \frac{1}{P_m Q_n}\sum_{\substack{j\notin K_\varepsilon\\ j=0}}^{m}\sum_{\substack{k=0\\ k\notin K_\varepsilon}}^{n} p_i q_k \left|x_{jk} - L\right| \\ &\ge \frac{1}{P_m Q_n}\left|\{j \le m, k \le n : p_j q_k \left|x_{jk} - L\right| \ge \varepsilon\}\right|\end{aligned}$$

Hence, we obtain that the sequence $\{x_{jk}\}$ is $ST_{\overline{N}}$ convergent to L.

Now by the following example, it is shown that the inclusion is proper. Let us define the sequence $X = \{x_{jk}\}$ as follows.

$$x_{jk} = \begin{cases} \sqrt{jk}, & if\ k = n^2 \ \& \quad j = m^2 \\ O, & if\ k \ne n^2 \ \& \quad j \ne m^2 \end{cases}$$

Let $p_n = 1,2,3....$ and $q_n = 1,2,3....$

$$\frac{1}{P_m Q_n}\left|\{j \le m, k \le n : p_j q_k \left|x_{jk} - 0\right| \ge \varepsilon\}\right|$$
$$= \frac{\sqrt{m}\ \sqrt{n}}{P_m\ Q_n} \to 0, \text{ as } n \to \infty$$

and

$$\frac{1}{P_m Q_n}\sum_{j=0}^{m}\sum_{k=0}^{n} p_j q_k \left|x_{jk} - 0\right| = \frac{1}{P_m Q_n}\sum_{j=1}^{m}\sum_{k=1}^{n} p_{j^2}, q_{k^2},\ x_{j^2 k^2} \to \infty, \text{ as } n \to \infty$$

Hence, the inclusion $\left|\overline{N}, p_m, q_n\right| \subset ST_{\overline{N}}$ is proper.

Proof of the Theorem 2.2:

Let $x_{jk} \to L(ST_{\overline{N}})$ and $K_\varepsilon = \{j \le m, k \le n : p_j q_k \left|x_{jk} - L\right| \ge \varepsilon\}$

Since $P_m \to \infty, Q_n \to \infty$ and $p_j q_k \left|x_{jk} - L\right| \le M$

For all $j, k \in IN$ and for a given $\varepsilon > 0$ we have,

$$\frac{1}{P_m Q_n}\sum_{j=0}^{m}\sum_{k=0}^{n} p_j q_k \left|x_{jk} - L\right| = \frac{1}{P_m Q_n}\sum_{j=0}^{m}\sum_{k=0}^{n} p_j q_k \left|x_{jk} - L\right|,\ j,k \in K_\varepsilon$$
$$+ \frac{1}{P_m Q_n}\sum_{j=0}^{m}\sum_{k=0}^{n} p_j q_k \left|x_{jk} - L\right|,\ j,k \notin K_\varepsilon$$
$$\le \frac{M}{P_m Q_n}\left|\left\{j \le m, k \le n : p_j q_k \left|x_{jk} - 0\right| \ge \varepsilon\right\}\right| + \varepsilon$$

Since, $\in$ is arbitrary, we have that $x_{jk} \to L\left|\overline{N}, p_m, q_n\right|$

Hence, completes the proof of theorem.

Proof of the Theorem 2.3:

For $\varepsilon > 0$, we have,

$$\frac{1}{m}.\frac{1}{n}\left|\{i \le m, k \le n : \left|x_{jk} - L\right| \ge \varepsilon\}\right|$$
$$\le \frac{1}{mn}\left|\{j \le m, k \le n, p_j q_k \left|x_{jk} - L\right| \ge \varepsilon\}\right|$$
$$= \left(\frac{P_m Q_n}{mn}\right)\frac{1}{P_m Q_n}\left|\{j \le m, k \le n : p_j q_k \left|x_{jk} - L\right| \ge \varepsilon\}\right|$$
$$\ge \frac{1}{P_m Q_n}\left|\{j \le m, k \le n : p_j q_k \left|x_{jk} - L\right| \ge \varepsilon\}\right|$$

Since, $\left(\frac{P_m}{m} > 1\right)$ and $\left(\frac{Q_n}{n} > 1\right)$, hence $x_{jk} \to L(ST_{\overline{N}})$

Hence, completes, the proof of theorem.

Proof of the Theorem 2.4:

For given $\varepsilon > 0$, we have,

$$\frac{1}{m}.\frac{1}{n}\left|\{i \le m, k \le n : \left|x_{jk} - L\right| \ge \varepsilon\}\right|$$

$$= \frac{1}{mn}\left|\{ j \le m, k \le n : p_j q_k \left|x_{jk} - L\right| \ge \varepsilon\}\right|$$
$$\le \left(\frac{P_m}{m}\right)\left(\frac{P_n}{n}\right)\frac{1}{P_m Q_n}\left|\{ j \le m, k \le n : p_j q_k \left|x_{jk} - L\right| \ge \varepsilon\}\right|$$
$$\le \frac{1}{P_m Q_n}\left|\{ j \le m, k \le n : p_j q_k \left|x_{jk} - L\right| \ge \varepsilon\}\right|$$

Since, $\limsup\left(\frac{P_m}{m}\right) < \infty$ and $\limsup\left(\frac{Q_n}{n}\right) < \infty$,

we have,

$$x_{jk} \to L(ST_{\overline{N}}) \Rightarrow x_{jk} \to L(ST)$$

Hence, completes the proof of theorem.

4. Corollary

Our theorems have the following results as a corollary.

Corollary 4.1

By replacing double sequence $\{x_{j,k}\}$ by the sequence $\{x_k\}$ and putting $q_n=1$, our theorem reduces to the theorems of KARAKAYA [3].

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