*IMVI* *ISSN (p) 2303-4882, ISSN 1840-4383*
*Open Mathematical Education Notes*
*Vol. 7(2017), 29-42*
www.imvibl.org / *JOURNALS / IMVI OMEN*
DOI: 10.7251/OMEN1701029R

# Prospective B&H Elementary School Teachers' Understanding of Processes with Basic Geometric Concepts

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**Abstract.** This study describes the mathematical proficiency of pre-service elementary school teachers in geometry within the paradigm: mathematical and methodical types of knowledge are necessary for teachers understanding of teaching and students' learning of the basic geometric concepts. In order to define the parameters that allow us to consider students' skills is mentioned paradigm, we tested 63 students of the third and fourth years of studies at two pedagogical faculties for teacher education in Bosnia and Herzegovina on a few questions about the relationship between the basic geometric objects - points, lines and planes. We analyzed students' reflections on these questions using quantitative and qualitative methods. Although the tested students have solid intuitive understanding of basic geometric objects (the 'level 0' by van Hiele's classification), their understanding of the process with these geometric objects is much lower. This paper is written to formulate the hypothesis that the difficulties that have been observed in the tested population were not the result of students' intellectual deficiency but, to a significantly greater extent, of insignificantly built components of geometric thinking during the previous school education.

**Key words and phrases:** *Pre-Service Teachers, Geometric thinking, basic geometric concepts - points, line and planes, and their interrelationships*

**Subject mathematics classification (2010): 97A40, 97B50, 97G10, 97E30**
**Didactic subject classification (2010): A40, B50**

## INTRODUCTION

The territory of the former Yugoslavia was socially and politically transformed during the civil war of 1990-1995. Bosnia and Herzegovina [B&H] was the geographically central area of the former Yugoslavia. Those transformations involve many changes in education system in B&H. Many segments of this system have been significantly changed. We estimate that the vast majority of these changes strongly increased the entropy of this system. We are deeply convinced that there is a deep gap between official information about the success of school systems in B&H offered by the social and academic communities on the one hand, and the real success of education systems, on the other. We formed this belief by working over 35 years as a lecturer in several mathematical courses at three different universities in B&H. For almost fifteen years, the author served as a lecturer in several courses in the field of mathematics teaching methodology for elementary and high school education. This includes the research in mathematical education in our school systems - the system of Pre-primary, Elementary (Primary and Middle), High school Education and teacher education for the mentioned school systems.

In B&H, there is no external method for assessing success in the school systems. Sometimes, under the pressure of independent educational researchers, the social community organizes a partial assessment of the success of pupils in mother tongue and mathematics. The results are extremely alarming. They are generally not available to the general public. One reliable possibility of gaining insights into the quality of mathematics education in the school systems in B&H is entrance examinations in mathematics, if they are organized at all. Another possibility is that a researcher as a university teacher independently undertakes the investigation of this phenomenon.

In order for the reader of this paper to gain at least a partial insight into the unusualness of school systems in B&H, we are providing two examples:

- It is a common practice that the management of school institutions, on a personal basis, constructs a significantly higher success in the mathematical literacy of the tested population than it is in reality.
- It is not uncommon for the management of particular school institutions in B&H to foster a commission assessment of students' success in mathematical literacy without the involvement and consent of responsible teachers. This process is generally an accepted practice in elementary and high schools and takes place with the knowledge of the social, academic and general public. A similar practice is not rare at some of the faculties of universities.

In this socio-political environment, there is a very small number of scientific texts related to research in mathematics education. It is not uncommon for researchers to feign the results in their texts with the intention not to disturb public opinion, fearful of the reflections of the socio-political community.

Our intention with this paper is to describe the situation in mathematical education in B&H at the time in which our research was carried out and exposed - from 1995 until 2017 - for the documentation of the relationship between the social and academic community relating to their school systems and especially in mathematics education. In order to get a closer look at our convictions about the very low mathematical literacy of a significant number of upper high school students in B&H, we have aligned specific events, or the lack of characteristic phenomena in mathematics education in the last twenty years. This includes:

- Analyzing the results of the mathematical literacy of the tested candidates when enrolling at the universities;
- Assessment of mathematical knowledge of pre-service kindergarten teachers;
- Assessment of mathematical knowledge, mathematical abilities and math skills of pre-service primary school teachers;
- Assessment of mathematical knowledge, mathematical abilities and math skills of pre-service middle school teachers;
- Assessment of mathematical knowledge, mathematical abilities and math skills of pre-service high school teachers.

The state of mathematics education in B&H can be described in the first approximation by the following observations:

- Mathematical education of children in the pre-school system does not exist or it is inadequate.
- The mathematical education of pupils in the Primary school system is not completely inadequate.

- Positive achievements of students' mathematical education in the Middle School system can be registered with a small percentage of the population.
- The satisfactory success in mathematical literacy of pupils in the high school system can be registered in a satisfactory way for an extremely small percentage of the population. An exception to this claim is a group of students from some classical gymnasiums.

Given that researchers have consistently found inadequacies in elementary pre-service teachers' geometric understanding and spatial ability, the purposes of this study were:

(1) to assess elementary pre-service teachers' geometry content knowledge;
(2) to access elementary geometry pedagogical content knowledge; and
(3) their spatial visualization skills in terms of the first three van Hiele's levels of geometric thought;
(4) to access elementary geometry pedagogical content knowledge to identify misconceptions held by pre-service teachers with regards to geometric content.

Within the context of this study, it was assumed that not all pre-service teachers were functioning at the Informal Deduction level ('level 2') of the van Hiele model [24]. Also, we assumed that the tested population has no further knowledge of why the students of lower grades of elementary schools are taught these basic geometric concepts. In addition, our intention by this testing was to determine whether there is a significant link between the students' real understanding of the teaching process with these basic geometric concepts, on the one hand, and the expected outcomes of this teaching, on the other.

What is the problem with this text? - It's a natural question that is being asked. In the author deepest belief, the most important problem in designing texts with such intentions is the principled-philosophical orientation of the author in relation to the issues that he analyzes in the article. How does the author take the right attitude about the phenomena he/she analyzes in the offered text?

The researcher in Mathematical Education of the school system in their academic community is primarily responsible for the academic integrity. Also, he/she should be responsible for the principle of correct and complete informing of the academic public about the conclusions deduced in the research based on the collected data.

## BACKGROUND

In the teaching of mathematics in the lower grades of elementary schools we try to develop a geometric view within 'level 1' (according to van Hiele's classification, [24] among students. It is natural to assume that their teachers should have the ability to understand geometric contents at a minimum 'level 2'. Of course, we fully agree with the belief that there are mathematical contents in lower grades whose interpretation requires a higher level of understanding of geometry than 'level 2'. This includes, among other things, the concept of 'the right angle' and the concept of 'mutually perpendicular lines', the concept of 'line perpendicular to the plane', and many others.

Geometry deals with mental entities constructed within the boundaries of common sense through the use of geometrical representations using points, lines, planes, angles and shapes. These are not simply representations of actual objects experienced in the world. Geometric concepts are the result of human ability of abstraction. According to Fischbein [8], a geometric representation encompasses both figural and conceptual characters. Figural characters depict properties that represent a certain geometric concept and can be classified as external (embodied materially on paper or other supported entity) or iconical (focused on visual images). According to Mesquita [14], a geometric concept can also be determined in terms of ‘finiteness’ (referring to specific forms) and ‘objectivity of the assumed design’ with no reference made to specify its forms. On the other hand, conceptual characters are concept image – the collective mental pictures in our collective mind, their corresponding properties and processes that are associated with the concept. Such an image represents an abstracted entity, bound by its formal concept definition - a form of words used to specify that concept and its development through the process of visualizing.

A large number of researchers in mathematics education devoted a long-lasting attention to the assessment of the knowledge of geometric contents of pre-service teacher of Primary grades in elementary schools. For example, a reader can look at the texts [1], [2], [5], [6], [10], [12], [13], [16], [17], [18], [22] and [23].

In the past few years, in the field of mathematics education, everything is more distinct paradigms (for example, see [2] and [3]):

(1) School mathematics / Collective knowledge of mathematical content;

(2) Mathematical knowledge required for mathematical teachers / Specialized knowledge of mathematical content;

(3) Methodological knowledge required for mathematical teachers / Special knowledge which linking mathematical contents to students, and

(4) Understanding the process of teachers' teaching and students' learning in mathematics classroom / Specific skills that link the contents of mathematics and teaching skills.

More recently, to this characterization were added two types of knowledge:

(5) Knowledge of mathematics curriculum, and

(6) Knowledge related to the mathematical horizon \ the frontiers of mathematics knowledge.

This author also published (independently and in collaboration) several texts in which he analyzed specific phenomena of interest with the research. See for example the following texts: [7], [11], [15], [19-21].

In the text [7] we identified students’ problems with the perception of triangles: (a) students do not distinguish between the border of a triangle and the space inside this border; (b) students, in most cases, are not able to determine this geometric figure; (c) most of the tested students do not know if the triangle is basic or the concept of a triangle is introduced by definition. In article [8], we described our vision of several worry some phenomena in mathematical education in B&H from the socio-political point of view.

## METHODOLOGY

### Participants

The sample for this study was comprised of N = 63 (± 1) students of the third and the fourth years of studies at the two pedagogical faculties for teacher education in B&H. All these students had previously attended and passed the examinations of two one-semester courses in mathematics. In the third and fourth year of college, these students are taught the knowledge and skills of teaching pupils of lower grades of elementary schools of mathematics through two one-semester courses. In the first course, they hear lectures on theories of mathematical education (Traditional approach, Theory of didactic situations, Theory of realistic mathematical education, etc.), about mathematical thinking (arithmetic, early-algebraic, algebraic, geometric, logical, etc.) and so on.

The test was conducted on April 18, 2011.

### Instrumentation

Students were tested on a set of questions related to basic geometric concepts and their interrelationships. This set consists of three series of questions:

The first series: assessment of the level of cognitive knowledge of the tested students on basic geometric concepts of points, lines and planes and their interrelations;

The second series: evaluation of students didactical knowledge about levels of development of geometrical thinking during the teaching of primary grades' students of these concepts;

The third series: identifying logical tools using which primary grades students are taught in these classroom's activities.

### The Tasks

In accordance with the goals of mathematics teaching setting in the Bloom's taxonomy, the teacher should stimulate student success in the cognitive domain (knowledge, understanding and acceptance of processes with basic geometric concepts) and encourage the development of components in the affective domain (logical and geometric thinking and socio-mathematical norms).

**A.** *The first series of questions*

Q1. Describe the relationship between a point and a line.
Q2. Describe the relationships between two lines.
Q3. Describe the relationship between a point and a plane.
Q4. Describe the relationships between a line and a plane.
Q5. Describe the relationships between two planes.

At 'the intuitive level' of understanding geometry, we assume that students accept the existence of the following classes of geometric concepts: points [$\mathcal{T}$], straight lines (= lines) [$\mathcal{L}$], and planes [$\mathcal{P}$]. To determine the relationship between the elements within each of these entities, as well as the relationships between the elements of different clusters, we use the term '*incidence*'. In the teaching of mathematics in the Primary School, this term is interpreted through several different words. A successful teacher of mathematics in the Primary School should enable their students to understand and accept the specificity of the relationship between the basic geometric concepts - points, lines and planes. Also, teachers should teach students how to use specific language categories in these classroom activities.

**B.** *The second series of question*

What level of understanding of geometry is induced to students by teaching them links between these basic geometric concepts?

In teaching interrelations of the basic geometric concepts, teachers should use models to enable pupils to gain insight into these interactions. In these processes, the teachers should use specific language categories in explaining each of the special cases of the relationship between the basic geometric concepts. For example:

('Level 0' – Visualization) A point is incidental with the line or is not incidental with the line (based on the logical 'principle of Tercium non datur' - TND) and there can be no possibility that a point to be incidental and not to be incidental with the line at the same time (based on the logical 'principle of non-contradiction' - nonK). To indicate the first situation, we use the wording "a point is *on* the line". In the second case, the teacher describes the situation by the words: "the point *is not on* the line".

(‘Level 1’ – Analysis) The teacher must make sure that the students accept the validity of the following statements: (a) the line consists of points; (b) there are plenty of points on the line; (c) by observing the points on the lines from left to right, we note that the points are in a relationship with one another - this relationship is described by the words *before* (and *after*).

Teaching school students the concept of line in the sense that the line consists of infinitely many points, that the line is not bounded either on the left or the right, is very complex, cognitively demanding and far exceeds ‘level l’.

**C.** *The third series of questions*
What are the tools of logical thinking that our students understand and accept when we teach them the relationship between these basic geometric concepts?

Many people from political and academic life in our social community often talk about the development of logical thinking among pupils in the Primary grades of Elementary schools. Unfortunately, in neither document related to mathematical teaching in these grades there are no concrete instructions about which logic tools teachers should teach to their students. We draw attention of the reader to the fact that mathematical curricula for Primary grades in Elementary schools in our educational environment are written with only two to three pages of text.

Which and what kind of material and methodical instructions can be offered to teachers trough these two or three pages of text? - This is a question that every teacher should ask himself during methodical preparation for teaching mathematics.

In interpreting the relationship between the point and the line, the teacher has the opportunity to emphasize the following principle to their pupils: *a point is on the line or the point is not on the line.* Also, the teacher can point out the principle: *it cannot be that a point can and cannot belong be the line at the same time*. It is quite acceptable that the teacher omitted a deeper explanation of these principles. However, in interaction with the students, each time the teacher should emphasize that these two principles are always valid. In addition, the teacher should point out the difference between these principles and the properties of the geometric concepts to which they apply.

## DATA COLLECTED AND THEIR ANALYSIS

### Data collection of the first series of questions

In the first set of tasks, the first question was: **Q1**. *Determine the relationship between the point and the line.*

Table 1 shows the distribution of student responses to question Q1 (N = 63):

| quality of response | ∅ | -1 | 0 | 1 | 2 | Σ |
|---|---|---|---|---|---|---|
| number | 19 | 7 | 9 | 15 | 13 | **63** |
| percentage | 30.16 | 11.11 | 14.29 | 23.81 | 20.63 | |

**Table 1**: Distribution of students respond on the question 1.1 (N = 63)

**Codes**: ∅ - the student did not offer any answers; -1 - the student offered a response that cannot be classified; 0 - unacceptable answer; 1 – acceptable answer; 2 – correct answer.

**Analysis:** Unfortunately, for over half of the tested population it was not possible to register quality knowledge about the relationship between the point and the line. Only 44.44% (15+13) of the tested students expressed their thoughts on the relationship between the geometric concepts of the point and the line in an acceptable way; i.e., using appropriate geometric categories. The author has hard time to accept that such high percentage of tested students are not able to, with acceptable terms, describe their perception of the geometric concepts of points, lines and their mutual relations. It appears that these students have never been asked in their previous school education to demonstrate their understanding of the relationship between the points and the lines. In addition, the author found that the tested candidates have significant difficulties with understanding the concept of (potential) infinity. They often described the infinity of the line by the words "the line can be unlimitely extended". In doing so, they always omitted explanation of terms used in the offered description. This deficiency, detected in the tested population of future teachers, if not eliminated, will create deep misunderstandings among their students during their practice for many years to come.

Some of the students' answers to the first question are shown below for illustration purposes.

1.1 Acceptable answers

- A point can be on a line or it can be out of the line.

1.2 Unacceptable answers

- A point is out of a line.
- A point and a line are in such a relationship that the point is on the line and that several points in the series constitute one straight line.
- A line can go through the point, or the line can be at the point.
- Each point lies on a line, and one line is marked by two points.
- The point can be on the line. Two points on the line determine one segment, or if they are outside or below it. A line can pass through a point. A point and a line can be missed.
- A line passes through one point, or the point and the line pass each other.
- The point $A$ is on the line $p$, or the point $A$ is parallel to the line $p$.
- A point and a line determine one plane.
- A point and a line can lie in the same plane; a point can be on a line.

1.3 Unclassified answers

- A line is limited in relation to a point.
- A line can be far from the point.
- A point and a line can be on the line, and that it is next to it.
- The relation between the point and the right is that the law passes through certain points.
- With two different lines, there is one and only one line that is related to them.

The next task was: Q2. *Determine the possible relationships between the two lines.*

Table 2 shows the distribution of student responses to question Q2 (N = 63):

| quality of response | ∅ | -1 | 0 | 1 | 2 | Σ |
|---|---|---|---|---|---|---|
| number | 16 | 5 | 8 | 16 | 18 | **63** |
| percentage | 25.4 | 7.94 | 12.7 | 25.4 | 28.57 | |

**Table 2**: Distribution of students respond on the question 1.2 (N = 63)

**Codes**: ∅ - the student did not offer any answers; -1 - the student offered a response that cannot be classified; 0 - unacceptable answer; 1 – acceptable answer; 2 – correct answer.

**Analysis:** The two lines lie on the same plane, or they do not lie on the same plane. The case where the two lines do not lie on the same plane, we describe by the words: lines are *not encountered*. If the two lines lie on the same plane, then the following options are possible: (1) they are *parallel* - they do not have any common point; (2) they have a single common point - they *intersect*; (3) all the points of two lines are common - in this case we say that these lines *coincide*.

45 (= 16+5+8+16) tested candidates, or 71.43% of the total number, did not mention the lines that do not lie on the same plane. The concept of 'two lines that do not lie on the same plane' is not cognitively demanding - this relationship between the two real lines is easily visualized using the perceptible models in each school classroom. On the other hand, 66.67% of the total number of tested candidates tried to express their thoughts about the interaction of the two lines that lie at the same plane. Only 8 of them gave unacceptable answers. 53.97% of the tested students offered acceptable thoughts on the mentioned problem. What is the problem with 46.03% of the tested population in relation to this issue? - This is the question the author asked himself when he collected feedback on the second question. How is it possible that adults cannot imagine the interaction of two lines in the same plane? If we accept that some of them have no inborn ability to eloquently express their thoughts about the issue, it is entirely natural to expect that most of them would not be able to design a model of relations between two lines in the same plane. The registration of missing skills in some of the tested population strongly suggests that there were gaps in their previous mathematics education. The following conclusions seem to be truly acceptable: Mathematics teachers in their previous education did not at all, or not often, talk about the mentioned relationships among these basic geometric concepts. Also, they did not necessarily require from their students to demonstrate solid knowledge about these geometric issues.

In order to gain insight into the ways of student reflection, here are some of the answers:

2.2 Unacceptable answers

- Two lines have one common point.
- Two lines can be perpendicular and parallel.
- Two straight lines are lines that are parallel. They coincide.
- The point and the lines have no common intersection.
- The relationship between the two lines can be that they intersect and that they are parallel.

2.3 Unclassified answers

- The two lines $p$ and $q$ pass by if and only if they lie on the same plane.
- Two straight lines are two parallel lines that are limited.

The third question of the first cluster was: Q3. *Determine the relationship between a point and a plane.*

Table 3 shows the distribution of student responses to question Q3 (N = 64):

| quality of response | ∅ | -1 | 0 | 1 | 2 | Σ |
|---|---|---|---|---|---|---|
| number | 17 | 15 | 9 | 16 | 7 | **64** |
| percentage | 26.56 | 23.44 | 14.06 | 25.0 | 10.94 | |

**Table 3**: Distribution of students respond on the question 1.3 (N = 64)

**Codes**: ∅ - the student did not offer any answers; -1 - the student offered a response that cannot be classified; 0 - unacceptable answer; 1 – acceptable answer; 2 – correct answer.

**Analysis**: As in the first glance on student responses may indicate, the third answer is identified with its specificity. It seems that this kind of thinking is not difficult to reconstruct. The student imagines the plane in the form of a part of the plane limited by rhomboid. If we accept this reconstruction, then student's visualization of the point-to-plane relation is quite understandable.

According to the collected reflections, 41 (or 64.06%) responses are not good. Because of the case visualization of the point-and-plane relationship, consideration of this relationship and its acceptance, it was expected that most of the tested population offered acceptable respons. How are adults who have heard more courses in mathematics in the previous education have not completely consolidated their knowledge of this interaction? Would the conclusion: 'The student does not know the answer to this question' – be acceptable at all? The most agreeable conclusion we could apply is: The students did not listen to lectures about the mentioned interaction, or they did not listen often enough in order to remember. It is also possible to conclude: 'Students did not consolidate their knowledge of point-plane interactions during their previous education.'

Some of the students' reflections on this issue are exposed to below.

3.3 Unclassified answers

- The point may be next to the plane, and it can be in plane.
- The mutual relationship between a point and a plane is the points on a plane can be many.
- The point in relation to a plane can be in front of the plane, behind the plane or on the side of the plane.
- A plane can be far from the point.
- The plane can have several points.

The fourth question in the first cycle of tasks was Q4. *Determine the relationships between a line and a plane.*

Table 4 shows the distribution of student responses to question Q4 (N = 64):

| quality of response | ∅ | -1 | 0 | 1 | 2 | Σ |
|---|---|---|---|---|---|---|
| number | 19 | 15 | 13 | 13 | 4 | **64** |
| percentage | 29.69 | 23.44 | 20.31 | 20.31 | 6.25 | |

**Table 4**: Distribution of students respond on the question 1.4 (N = 64)

**Codes**: ∅ - the student did not offer any answers; -1 - the student offered a response that cannot be classified; 0 - unacceptable answer; 1 – acceptable answer; 2 – correct answer.

**Analysis**: Relationship between a line and a plane can be: (a) the line does not belong to the plane - The line and a plane do not have common points. To describe this situation, we use the wording – 'the line and the plane *are parallel*'. (b) The line can lie in the plane. In this case, we say all points of the line lie in the plane. (c) A line can have the *intersection* with a plane - The line and the plane have only one common point. The interaction between a line and a plane could be derived by student trough the following thinking. First step: a line and a plane can be in the relation of incidence, or the line and the plane are not in the relation of incidence (by the logical principles of TND). The second step: if the line and the plane are not in the relation of incidence, i.e. if they have no common points we say that they are *parallel*. The third step: if they have only one common point, we say that the line *perforate* the plane.

Students routinely visualize the plane as rhomboid. For this reason it is not at all unusual for them to have relationships: the line is ahead or behind the plane, or the line is from the side in relation to the plane. What is common in 47 tested students (or 73.44% of the total) is that they did not offer an acceptable answer. Unfortunately, only 4 of the tested students showed that they fully understand the relationship between the line and the plane. Let's ask ourselves: ‘How is it possible that only a quarter of the tested population has a consolidated knowledge of the relationship between the line and the plane?’ On the basis of the collected data, we estimate that these students do not have the feeling for the boundless two-dimensional space. How is that possible?

Some of the student responses are shown below.

4.2 Unacceptable answers

- A line and a plane can have a common intersection, they can be coincidental or they can be missed.
- A line and a plane can be coinciding or they can be parallel.
- A line may belong to a plane, or it may be out of the plane. The line can cut the plane; the line and the plane can be parallel; they can be coinciding; they can be divergent.

4.3 Unclassified answers

- A line and a plane can fit in.
- A line and a plane have such a relationship that the line can cut the plane, that they are parallel or that the radar lies next to the one.

The last question in the first round of tasks was: Q5**.** *Determine the relationships between two planes*.

Table 5 shows the distribution of student responses to question Q5 (N = 62):

| quality of response | ∅ | -1 | 0 | 1 | 2 | Σ |
|---|---|---|---|---|---|---|
| number | 18 | 14 | 21 | 6 | 3 | **62** |
| percentage | 29.03 | 22.58 | 33.87 | 9.68 | 4.84 | |

**Table 5**: Distribution of students respond on the question 1.5 (N = 62)

**Codes**: ∅ - the student did not offer any answers; -1 - the student offered a response that cannot be classified; 0 - unacceptable answer; 1 – acceptable answer; 2 – correct answer.

**Analysis**: In accordance with the logical TND principle, it is clear that the two planes can be in the relation of incidence or not in the relation of incidence. If two planes are not in the relation of incidence, i.e., if they do not have common points, then we say that they are *parallel*. If two planes have at least two different common points then they have a common line - this claim is valid in geometry as an axiom. So, the teacher should encourage his students to accept the significance of this claim based on the visualization of this relationship of two planes. In any classroom environment there is at least one model for visualizing the observed relationship. Of course, teachers can design this model by themselves. The third option of relation between the two planes is their coincidence.

How is it possible that over 4/5 of the tested population (more precisely: 18+14+21 or 85.48%) did not offer acceptable consideration on this issue? Suppose they had a lecture on these phenomena in geometry during their previous mathematics education. What conclusions can we deduce from the previous two statements? Why do most of the tested students have no solid knowledge of these basic geometry phenomena?

Some of the students' reflections, classified into the clusters of unacceptable and unclassified answerers, are presented below.

5.2 Unacceptable answers

- Two planes *p* and *q* are cut if and only if they have no common points.
- Two planes have a common intersection.
- The relationship between the two planes can be: they are coinciding; they can be cut; they can be diverging.
- The two planes can be parallel to each other, to cut, to coincide, to have a common point or the line.
- Two planes can be cut at one point; they can be parallel or passable.
- The two planes are parallel to each other.

5.3 Unclassified answers

- If all the planes have one common point and do not coincide, then their intersection is a plane.

  Then we say that these planes are cut along one line.
- The two planes determine the two lines that are cut.
- Two planes are passable if they do not lie in the same plane.
- The relation between two planes is that they are infinite.

**Student responses on the second circle**

The distribution of students' recognition of the cognitive levels of the geometric thinking developed by solving the problems of Question 1 is shown in the following table:

| quality of response | ∅ | -1 | 0 | 1:0 Level 0 | 1:0-1 Level 1 | 2 Level 2 | Σ |
|---|---|---|---|---|---|---|---|
| number | 10 | 2 | 7 | 44 | 17 | 5 | **63** |
| percentage | 15.87 | 3.17 | 11.11 | 69.84 | 26.98 | 7.94 | |

**Table 6**: Distribution of students respond on the question 2 (N = 63)

In this table, displaying numbers is not the same as in the previous tables. Numbers 17 (column 1: 0-1, Level 1) and number 5 (column 2, Level 2) are contained in number 44 (column 1: 0, Level 0) and number 5 is contained in number 17.

**Codes**: ∅ - the student did not offer any answers; -1 - the student offered a response that cannot be classified; 0 - unacceptable answer; 1:0 – acceptable answer - student recognizes level 0; 1:0-1 – acceptable answer - student recognizes levels 0 and 1; 2 – correct answer - student recognizes levels 0, 1 and 2.

**Analysis**: The first thing to notice when analyzing these data is that only slightly less than 16% of the tested students did not take part in thinking about this task. Only 7 of them offered unacceptable considerations. Almost 70% (precisely: 69.84%) of the tested population recognizes ‘level 0’ in analyzing the relationship between basic geometric entities. So, we can conclude that most of the tested students have the ability to visualize the relationship between basic geometric concepts as they have knowledge of what is implied at level 0. The author estimates that this conclusion is due to the fact that these students just finished the course 'Methods of Teaching Mathematics'. Only 1/4 of the tested candidates recognize the characteristics of ‘level 1’ applied in this specific case. The author believes that this population of prospective teachers should continually improve their mathematical and methodical competences. The responsibility for these obligations should be taken by the appropriate academic institutions of our social community.

### Student responses on the third circle

Students’ identification of logical tools [TND] and non-contradiction [nonK] is presented in the following table:

| quality of response | ∅ | -1 | 0 | 1 | 2 | Σ |
|---|---|---|---|---|---|---|
| number | 19 | 22 | 7 | 13 | 2 | **63** |
| percentage | 30.16 | 34.92 | 11.11 | 20.63 | 3.17 | |

**Table 7**: Distribution of students respond on the question 3 (N = 63)

**Codes**: ∅ - the student did not offer any answers; -1 - the student offered a response that cannot be classified; 0 - unacceptable answer; 1 – acceptable answer - student recognizes TND; 2 – correct answer - student recognizes TND and nonK.

**Analysis**: The collected data of student reflections according to the requirements to express their understanding of the basic logic tools that can be perceived in the process of teaching geometry, suggests a very worry some conclusion: Future teachers have very poor knowledge of logical tools that appear in mathematics classes of Primary grades in Elementary school. 65.08% of the total number of tested population does not know about logical tools in the Primary school mathematics. We allow the academic part of this community to assess what knowledge about logical tools is necessary for the quality mathematics education in Primary grades of Elementary school.

## PERSONAL OBSERVATION

Results of this evaluation and identification indicated that the tested pre-service teachers are functioning primarily at ‘level 0’ (visualization) in van Hiele’s classification. On the basis of the collected reflections, we estimate that the tested candidates with significant difficulties exhibit their understanding of the geometric concepts and processes of basic geometric concepts within ‘level 1’ (analysis). Their geometric considerations are based on the ability to recognize and accept basic geometric concepts - points, lines and planes. With tremendous difficulties, the tested candidates constructed an acceptable description of the relationships among these basic geometric concepts.

The results of this investigation provide evidence that prospective teachers should be given the opportunity to learn about van Hiele's levels of geometric thought and to be able to independently evaluate their own level of geometric thinking. Students of Teachers Education faculties should also be enabled to acquire experiences that include the aforementioned basic geometric concepts in all their interrelations. They should also be able to acquire the necessary methodical knowledge of the elements of geometric and logical thinking associated with activities including points, lines and planes. They also need to understand and accept the knowledge of why these pedagogical issues are important and that this knowledge will be useful their future work. Geometric experiences must go beyond memorization of geometric descriptions in order for pre-service teachers to develop an ability to logically analyze the properties of the basic geometric concepts. Until pre-service elementary teachers can successfully demonstrate knowledge and understanding of the geometry content that they would be expected to teach, they will be unable to fully comprehend the methodology of mathematics. Elementary school teacher educators (for example: The Republic Pedagogic Institute of the Republic of Srpska, the department for mathematics didactic at universities) should take into account pre-service teachers' existing geometric misconceptions and purposefully address those issues during the discussion of geometry content in Primary grades of Elementary schools.

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Received by editor: 02.11.2017; in revised version 07.12.2017.
Available online: 11.12.2017.