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Research Article

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Development of Predictive Model to Monitor Groundwater Flow Rate in Aquifer

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Abstract The study presented in this research assessment is to investigate the flow rate through aquifer for an unconfined aquifer of two wells in Obagi Community spanning 2.85km. The Laplace equations of one-dimension flow were solved, considering recharge as functional parameter and model for estimation of flow rate. The flow rate model was evaluated and simulated using concept of finite element method and flow rate at different nodes were determined. Results revealed that as groundwater flow rate moves from zero distance to considered distance through the aquifer, there is reduction in magnitude flow rate by FEM as a results of wide spread of flow rate within the aquifer zone.

Keywords flow rate, aquifer, finite element method, developed model, theoretical

1. Introduction

The assessment of subsurface flow is essential since about 30% of the world's fresh water resources exist in the form of groundwater. Further, the subsurface water forms decisive input for sustenance of life and vegetation in arid zones. Due to its significant source of water supply, various aspects of groundwater dealing with the exploration, development and utilization have been extensively studied by researcher from different disciplines, such as geology, geophysics, geochemistry, agricultural engineering, fluid mechanics and civil engineering, Water in the soil mantle is called subsurface water and is considered in two zones such as saturated zone and aeration zone. All earth materials, from soils to rocks have pore spaces. Although these pores are completely saturated with water below the water table, from the groundwater utilization aspect only such material through which water moves easily and hence can be extracted with ease are significant [1-18].

The study in this article is to evaluate groundwater flow rate through aquifer of two wells in Obagi Community in Ogba/ Egbema/ Ndoni local government area of Rivers State, Nigeria.

2. Methodology

2.1. Study Area

The study area is made up of 17 Communities of Egi Clan in Ogba Kingdom, lies between GPS coordinate elevation 20m, N05^{O}14.522[1] and E06O40.157[1].For proper mapping out of water table elevations for the study area, two wells were observed in Obagi Community (Well A-Well B spanning 2.85 km) carefully for months to generate elevations at different node of interest-because the water table map is a series of contours of equal or different elevations that resemble land topographic contours. Precipitation is the main source of groundwater recharge, flow rate intrusion of river water is also considered as form of recharge and unconfined case of aquifer is considered based field observations.

Figure 1: Location map of study area

2.2. Model Concepts: Unconfined Aquifer

Figure 2 shows the pattern of groundwater flow between two rivers with different water levels.

If the flow within water bearing aquifer zone is assumed to be one dimensional and steady state with a hydraulic conductivity (k). Then the Laplace equation is,

$$\frac{d^2h}{dx^2} = 0 \quad (1)$$

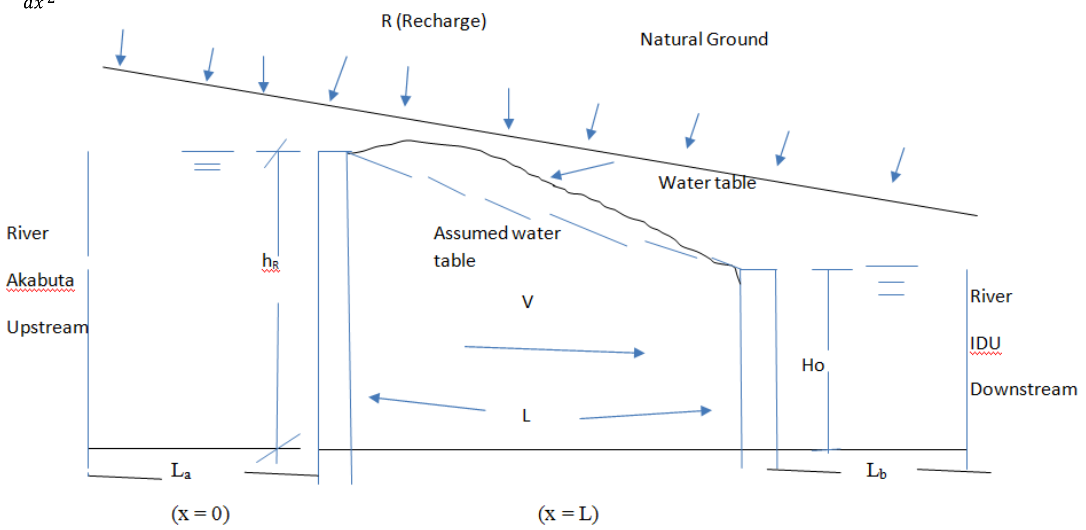

Figure 2: Unconfined aquifer

Integrating equation (1)

$$h^2 = ax + b \quad (2)$$

Boundary condition 1: h = h_R at x = 0

Therefore, equation (2) becomes

$$b = h_R^2 \tag{3}$$

Differentiating equation (3)

$$\frac{2hdh}{dx} = a \tag{4}$$

Application of Darcy's equation

$$q = -kh\frac{dh}{dx} \tag{5}$$

Equation (2) gives

$$h^2 = 2h\frac{dh}{dx}x + h_R^2 = h_R^2 - \frac{2qx}{k} \tag{6}$$

Considering boundary condition 2: h = h_0 at x = L

$$h_0^2 = h_R^2 - \frac{2qL}{k} \tag{7}$$

Therefore,

$$q = \frac{k}{2L}(h_R^2 - h_0^2)$$ Equation (5) yields

$$h^2 = h_R^2 - \frac{x}{l}(h_R^2 - h_0^2) \tag{8}$$

Equation (8) gives the variation in height of the water table. And the above equations hold only in the event of no recharge. Recharge is the proportion of rainfall that eventually finds its way in to the aquifer and raises the water level. If recharge is R, then

$$\frac{dq}{dx} = R \tag{9}$$

Calling Darcy's law

$$q = -kh\frac{dh}{dx}$$

Therefore,

$$\frac{dq}{dx} = -kh\frac{dh}{dx} = R \tag{10}$$

Integrating equation (9) twice

$$h^2 = \frac{Rx^2}{k} + ax + b$$

As in the non- recharge case, boundary conditions are the same, giving

$$b = h_R^2$$

And $a = \frac{(h_0^2 - h_R^2)}{l} + \frac{Rl}{K}$

Putting and re-arranging, we arrive at

$$h = h_R^2 - \frac{x}{l}(h_R^2 - h_0^2) + \frac{Rx}{k}(l - x) \tag{11}$$

This equation determines the shape of the water table line and parabolic. The flow rate through the aquifer can now be determined from the following.

Differentiating equation (9)

$$2h\frac{dh}{dx} = \frac{h_R^2 - h_0^2}{l} + \frac{R}{k}(l - 2x) \tag{12}$$

Equation (11) becomes

$$\frac{-2q}{k} = \frac{h_R^2 - h_0^2}{l} + \frac{R}{k}(l - 2x)$$

$$q = \frac{k}{2L}(h_R^2 - h_0^2) - \frac{R}{2}(l - 2x) \tag{13}$$

It is observed that flow rate varies with respect x. Therefore, differentiating Equation (13) with respect to x gives,

$$\frac{dq}{dx} = \text{R} \tag{14}$$

Functional parameter of the model equation, taking R = 1x10^{-4}m/d and L = 2850m

2.3 Application of finite element method

Stage-1: Discretization and selection of Approximation function

One – dimensional stretch were assumed and three elements with six nodes and the flow rate through the aquifer of each node were evaluated.

Stage – 2: deviation of element equations. Applying Galerkins Weighted Residuals Method GWRM to the governing one – flow rate equation (14) is expressed as:

$$\int_o^l N^T[\frac{dq}{dx} - R]\,dx = 0 \qquad (15)$$

Stage 3: Assembling of individual evaluated terms of Equation (15)

In Equation (14) one-dimensional stretch were assumed in this investigation in order to establish the assembling equation which will generate the flow rate at each node of interest.

3. Results

3.1. Approach of Linear Shape Approximation Function of Finite Element

The individual terms of Equation (15) can be calculated by the application of the linear shape functions or finite element method as follows:

Applying the finite element method of obtaining a solution to equation (15) and the domains are discretized into elements. A linear shape function approximation was adopted for this research work as given:

Linear element approach,

$$q\,(x) = N_i^e q_i + N_{i+1}^e q_{i+1} = [N][q] \qquad (16)$$

Where,

$$N_i^e = 1 - \frac{x}{l}$$

And

$$N_{i+1}^e = \frac{x}{l}$$

Evaluation of first term of Equation (15) gives,

$$\int_o^l N^T \frac{dq}{dx}\,dx = \int_o^l N^T \frac{d}{dx}[N][q]dx$$

$$= \frac{1}{2}\begin{vmatrix} -1 & 1 \\ -1 & 1 \end{vmatrix}\begin{vmatrix} q_1 \\ q_2 \end{vmatrix} \qquad (16)$$

Evaluation of second term of Equation (15) gives,

$$\int_o^l N^T R dx = \frac{Rl}{2}\begin{vmatrix} 1 \\ 1 \end{vmatrix} \qquad (17)$$

Putting Equation (17) and (18) into Equation (15) gives,

$$\frac{1}{2}\begin{vmatrix} -1 & 1 \\ -1 & 1 \end{vmatrix}\begin{vmatrix} q_1 \\ q_2 \end{vmatrix} - \frac{Rl}{2}\begin{vmatrix} 1 \\ 1 \end{vmatrix} = 0 \qquad (18)$$

Substituting the values of R, L and q_1 into Equation (18) and simulating flow rate through the aquifer gives,

Table 1: Simulated flow rate through aquifer by finite element method

Distance (m)	*Flow rate through aquifer* ($\frac{m^3}{d}$)
0	0.56
570	0.44
1140	0.34
1710	0.22
2280	0.12
2850	0.01

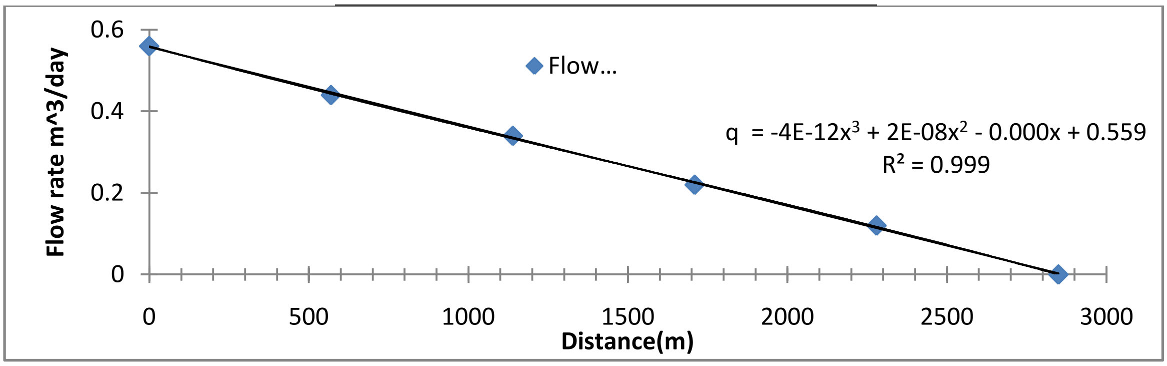

Figure 3: Plot of flow rate verses distance of investigation

Figure 3 and Table 1 illustrate the relationship between flow rate and distance of investigation in this research. For Obagi Community within the two wells of research assessment, a flow rate modeled equation is generated as, $q = -4E - 12x^3 + 2E - 8x^2 + 0.559$ and coefficient of determination of $R^2 = 0.999$. The developed modeled equation of flow rate is found useful in predicting and monitoring flow rate through aquifer spanning 2.85km. Through the developed modeled equation the flow rate at any given distances along the investigated length can be established or calculated.

Table 2: Theoretical determined flow rate and finite element method determined flow rate through aquifer

T. Flow rate through aquifer ($\frac{m^3}{d}$)	*FEM. Flow rate through aquifer* ($\frac{m^3}{d}$)
0.56	0.56
0.57	0.44
0.58	0.34
0.59	0.22
0.62	0.12
0.63	0.01

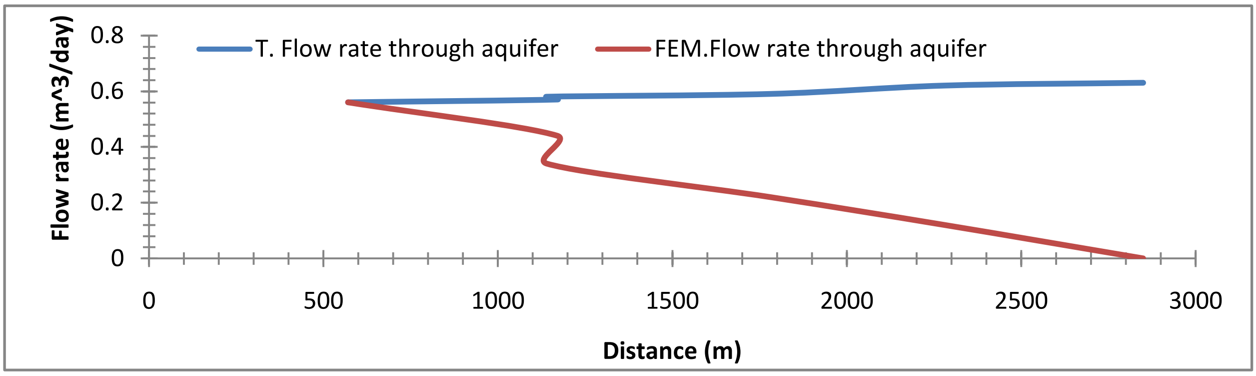

Figure 4: Plot of theoretical &finite element method obtained flow rate verses distance of investigation

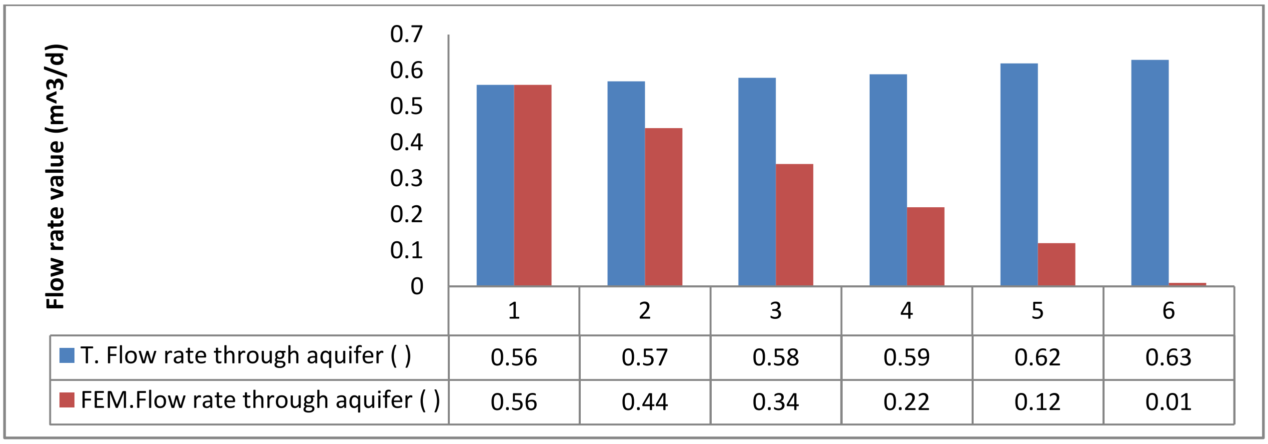

Figure 5: Plot of finite element method flow rate and theoretical flow rate

Table 2, Figure 4 and Figure 5 present the results of flow rate obtained at different nodes of interest by method of finite element method and theoretical flow rate. The flow rate through the aquifer in x-direction determined by approach finite element method shows that at the initial node, a flow rate of $0.56m^3/day$were simulated and results revealed as flow rate dispersed through aquifer there is reduction in magnitude of flow rate at the nodes and along direction flow while theoretical obtained flow rate results gives an increased in flow rate along direction of flow, accounting for influence of other in filtration of water into the aquifer as given flow rate of $0.56m^3/day$ is been simulated through the aquifer.

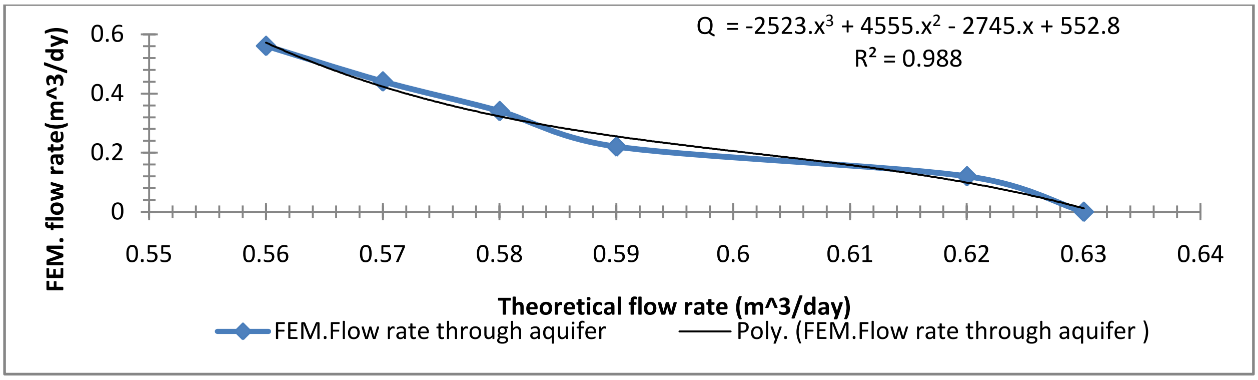

Figure 6: Plot of finite element method obtained flow rate verses theoretical flow rate

Figure 5 demonstrate the relationship between flow rates evaluated using concept of finite element method (FEM) and theoretical method. The FEM. Flow rates and the theoretical flow rates were used to validate the developed monitoring and simulating modeled equation flow rate and an acceptable coefficient of determination of $R^2 = 0.988$ was established indicating the trustworthiness of the model developed within span length of 2.85km in simulating and monitoring flow rates through the water bearing aquifer zone.

4. Conclusions

The following conclusions are drawn from this research

[a]. The flow rate through the aquifer in x-direction determined using the tool of finite element method [FEM] shows that at the initial node, a flow rate of $0.56m^3/day$that were simulated revealed that as flow rate dispersed through aquifer reduction in magnitude of flow rates at the nodes and along the direction of flow while theoretical obtained flow rates results gives an increased in flow rate along the direction of flow, accounting for influence of other infiltration of water into the aquifer for given flow rate of $0.56m^3/day$ simulated through the aquifer.

[b]. The finite element method [FEM] obtained flow rate and the theoretical determined flow rate were used to validate the developed monitoring and simulating modeled equation of flow rate and a satisfactory coefficient of determination of $R^2 = 0.988$ was established indicating the dependability of the model developed within span length of 2.85km in simulating and monitoring flow rates through the water bearing aquifer zone of Obagi Community.

[c]. Evaluation in this research work established flow rate model of, $q = -4E - 12x^3 + 2E - 8x^2 + 0.559$ and coefficient of determination of $R^2 = 0.999$. This developed equation is considered fit in predicting flow rate through the aquifer along the considered distance.

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