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An Improved Analytical Model for Predicting Waterflood Injectivity in Niger-Delta Marginal Oilfield Reservoirs

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Abstract The first problem encountered by the reservoir engineer in predicting or interpreting fluid displacement behavior during secondary and enhanced oil recovery project of a stratified reservoir is that of understanding the behaviour or performance of each layer of the stratified reservoir. Accurate prediction of the rate at which fluid can be injected into each layer of stratified reservoir is a key economic variable that must be considered when evaluating a stratified reservoir waterflood project. Each layer's position and velocity of flood front, time of breakthrough, fluid production rate, waterflood project's life and consequently, the economic benefits will be directly affected by the rate at which fluid can be injected into each layer of the reservoir. This paper aim at developing a new analytical injectivity model for predicting injection rate into each layer a stratified reservoir introducing the new concept of two-phase (fractional) flow behind the displacement front coupled with physical properties, fluid mobilities ahead and behind the flood front for each layer of the stratified system, Bottom-hole injection pressure, producing well pressure, and average reservoir pressure at the start of injection and flood front advancement in successive layers of the reservoir at a real point in time. The developed model was validated by its application to a stratified reservoir from a particular Niger-Delta Marginal Oilfield and the result was compared against that of 17 years real waterflood injection history performance (data) of the field and that of two existing analytical models. Analysis showed that, the new injection rate model prediction result actually fit with that of the field performance whereas the two existing models with single phase assumption over-predicted the total injectivity of the field which actually confirm the effectiveness of this new analytical injectivity model. Therefore, for accurate prediction of water flood oil recovery from stratified reservoir in Niger-delta oil fields, this new analytical model should be used for accurate estimation of time of water breakthrough, fluid production rate, waterflood project's life and the economic benefits, this analytical model should be employed to accurately predict each layer's performance and thereby estimating the total composite reservoir injectivity performance in either homogenous or stratified reservoirs and also valid for performance prediction of Water Injection rate during Pressure Maintenance Operations of Niger-delta oil fields.

Keywords Stratified Reservoir, Marginal Oilfield, Layer injection rate, Enhanced oil recovery, Two-phase Flow

1. Introduction

It has been proposed that most reservoirs are laid down in a body of water by a long-term process, spanning a variety of depositional environments, in both time and space. As a result of subsequent physical and chemical re-organization, such as compaction, solution, dolomitization and cementation, the reservoir characteristics are further changed. The main geologic characteristic of all the physical rock properties that have a bearing on reservoir behavior when producing oil and gas is the extreme variability in such properties within the reservoir itself, both laterally and vertically, and within short distances. It is important to recognize that there are no homogeneous reservoirs, only varying degrees of heterogeneity. Statistical as well as geological criteria[1,2] usually are used to divide the pay zone between adjacent wells into a number of horizontal layers each with his own properties, these properties may include permeability, porosity, thickness, saturation, faults and fractures, rock facies and rock characteristics. Such reservoirs usually are called 'stratified' reservoirs. This variation in properties affect the performance of stratified oil reservoirs during primary and secondary recovery process. One of the key elements influencing recovery performance during water flooding of a stratified system is the water injection rate or fluid injectivity.

Injection rate is a key economic variable that must be considered when evaluating a water flooding project for a stratified reservoir. Each layer's position and velocity of flood front, time of breakthrough, fluid production rate, water flood project's life and consequently, the economic benefits will be directly affected by the rate at which fluid can be injected into each layer of the reservoir. Estimating the injection rate is also important for the proper sizing of injection equipment and pumps. Although injectivity can be best determined from small-scale pilot floods, there are several empirical methods for estimating water injection rate for regular pattern floods into homogenous and single layer reservoir between a producer and an injector, although none of them has been able to accurately account for injection rate into each layer of stratified reservoir in which the composite will give the total injectivity of the stratified reservoir.

Deppe [3] and Muskat [4] developed simple mathematical formulas which relate injection rate and injection pressure for a number of regular well patterns. Studies by Muskat [4] of steady state pressure distributions in various well patterns with unit mobility ratio show that most of the pressure changes between injection and producing wells occurs in areas near the wells where flow is essentially radial. Even for the complex nine-spot pattern, radial flow occurs in the vicinity of injection and producing wells. Recognizing that radial flow occurs near injection and producing wells, the largest changes in injectivity occur in these radial flow regions, it was concluded by Deppe [3] that the injection rates in any pattern can be approximated by dividing the pattern into regions where radial and linear flow predominate. As a result, Deppe [3] developed a simple equation that could be used to compute injection rate for a variety of geometrical configurations including both regular and irregular patterns. Prats, et al [10] developed an analytical method whereby injection rates can be calculated for an enclosed five-spot well pattern where oil, gas, and water saturations are present. This is one of the few methods which has attempted to quantify the effect of an initial gas saturation. Caudle and Witte [6] used the results of their investigation to develop a mathematical expression that correlates the fluid injectivity with the mobility ratio and areal sweep efficiency for five-spot patterns, the authors presented their correlation in terms of the conductance ratio *i.e.*, which is defined as the ratio of the fluid injectivity at any stage of the flood to the initial (base) injectivity. Craig [7,8] developed another method for predicting injection performance which can be applied to stratified systems with or without free gas present. This method, which uses the correlations of Caudle and Witte [6] to predict injection rate as a function of mobility ratio and areal sweep efficiency.

Most of these authors assume unit mobility ratio (M) situation in their derivation, where fluid mobilities in the water zone and oil zone portions of the reservoir are equal, i.e., $M = 1$, meaning that, fluid injectivity does not change as the flood front advances after gas fill-up, the models were derived using a single homogenous layer with the assumption of piston-displacement mechanism, meaning injectivity for a particular well pattern is independent of the size of the area swept by water but is directly proportional to the fluid mobility involved, and time is not also explicitly related to prediction. Furthermore, none of these methods consider variation in injection rate into different layers of reservoir as the displacement process progresses. The determination of injectivity under these conditions reduces to a geometrical problem which results in simple analytical relationships. The aim of this work is to developed an improved analytical model for predicting injection performance which can be applied to stratified systems with or without free gas present and can be applied under either favourable or unfavourable fluid mobility ratio for five spot flood pattern. This correlation will be develop taking into account; two-phase (fractional) flow behind the flood front, physical properties of each layer of the stratified system, fluid mobilities ahead and behind the flood front, bottom-hole injection pressure, producing well pressure, and average reservoir pressure at the start of injection and flood front advancement in successive layers of the reservoir at a real point in time.

2. The Model Development

The rate at which fluid can be injected per unit pressure difference between injection and producing wells, depends upon the following factors: (1) Physical properties of the reservoir rock and fluids, such as: k_o, k_{ro}, k_{rw}, μ_w, μ_o, and h. (2) Area swept by the injected water and oil bank. (3) Fluid mobilities in the water zone and oil bank.(4) Well geometry, pattern, spacing. (5) Position of flood front. (6) bottom-hole injection pressure, producing well pressure, and average reservoir pressure at the start of injection. The new injectivity model will be developed taking into account all the above factors using the following simplifying assumptions and a physical model as shown below:

Assumptions:

1. The system is linear and horizontal and flow is incompressible, isothermal, and obeys Darcy's law.
2. There is fractional flow displacement type i.e. two-phase (fractional) flow behind the flood front
3. Each layer has different relative permeability characteristics.
4. Each layer is different in thickness, porosity, permeability, and fluid saturation. These properties highly varied between different layers.
5. The displacement is at constant pressure drop between the injector and the producer.

In order to derive a comprehensive model for predicting injection rate during water flooding of a stratified reservoir, there is a need to derive or introduce new correlation for; two-phase average total fluid mobility ahead and behind the flood front, two phase mobility ratio, position of flood front in any layer and consequently the new water injection rate model will be developed.

a) Development of The Two-Phase Average Total Fluid Mobility Model

In order to describe water-oil flow behaviour in a stratified system represented by physical model (fig. 1) above, consider it first at the time when water has advanced a distance X_1 in the most permeable layer: this is illustrated by fig.2 below:

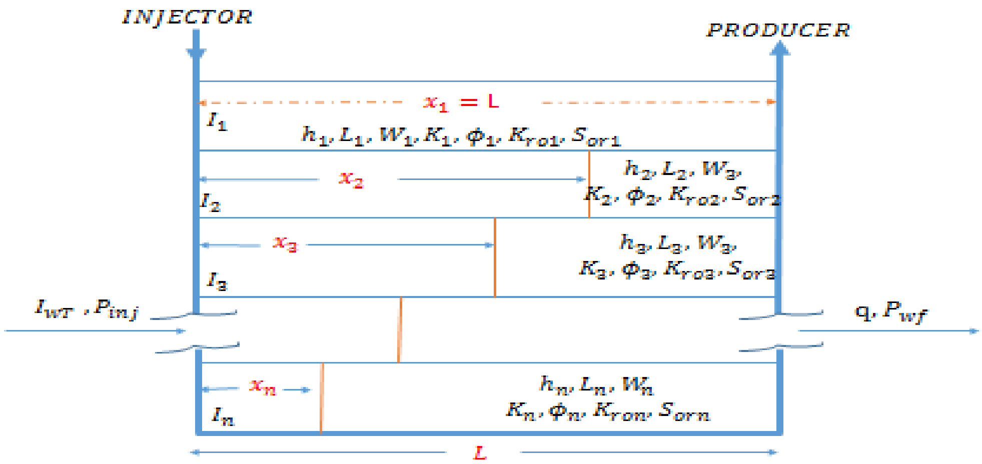
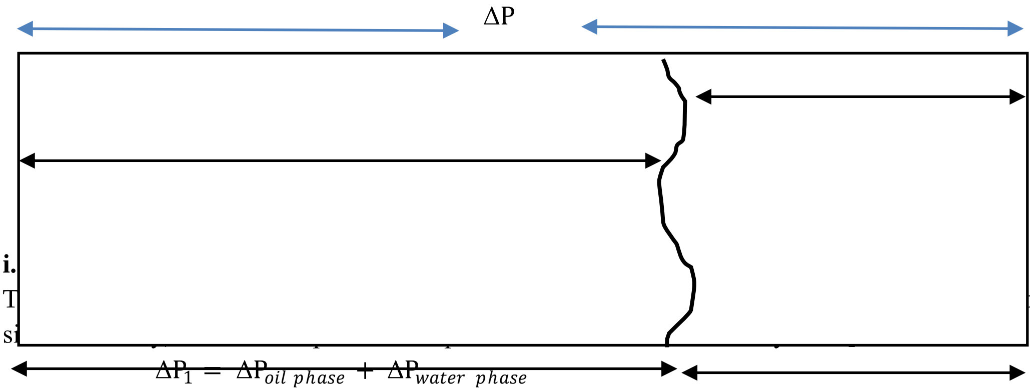


Figure 1: Physical Model Showing Linear Flow in a Stratified Reservoir



i.

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$$\Delta P_1 = \Delta P_{oil\ phase} + \Delta P_{water\ phase} \quad (1)$$

In terms of Darcy linear flow equation for steady state incompressible flow

$$\Delta P = \frac{i_w L\mu}{KA_1} = \frac{i_w L}{\left|\frac{\bar{K}}{\mu}\right|_1 A_1} \quad (2)$$

$$\Delta P_1 = \frac{i_w \mu_w X_1}{K_w A_1} + \frac{i_o \mu_o X_1}{K_o A_1} \quad (3)$$

$$\frac{i_w X_1}{\left|\frac{\bar{K}}{\mu}\right|_1 A_1} = \frac{i_w \mu_w X_1}{K_w A_1} + \frac{i_o \mu_o X_1}{K_o A_1} \quad (4)$$

Solving for average fluid mobility, $\left|\frac{K}{\mu}\right|$ behind the front

$$\frac{i_w X_1}{A_1}\frac{1}{\left|\frac{\bar{K}}{\mu}\right|_1} = \frac{i_w X_1}{A_1}\left[\frac{\mu_w}{K_w}\right] + \frac{i_w X_1}{A_1}\left[\frac{\mu_o}{K_o}\right] \quad (5)$$

Therefore

$$\left|\frac{\bar{K}}{\mu}\right| = \left[\frac{\mu_w}{K_w} + \frac{\mu_o}{K_o}\right]^{-1} \quad (6)$$

But $K_w = K_{rw} K \; and \; K_o = K_{ro} K$

$$\left|\frac{\bar{K}}{\mu}\right| = K_1\left[\frac{\mu_w}{K_{rw}} + \frac{\mu_o}{K_{ro}}\right]^{-1} = \bar{\lambda}_t \quad (7)$$

Where $\bar{\lambda}_t$ is the average total two phase mobility behind the front. Equation (7) gives the average total fluid mobility behind the front at distance X_1

ii. Average Fluid Mobility Ahead of the Displacement Front.

Consider the pressure drop across the distance ($L - X_1$) ahead of the front in fig.1

$$\Delta P_2 = \Delta P_{oil\ phase} + \Delta P_{water\ phase} \quad (8)$$

Using Darcy's equation

$$\Delta P_2 = \frac{i_w (L - X_1)}{\left|\frac{\bar{K}}{\mu}\right| A_1} \quad (9)$$

$$\Delta P_{oil\ phase} = \frac{i_o \mu_o (L - X_1)}{|K_o| A_1} \quad (10)$$

$$\Delta P_{water\ phase} = \frac{i_w \mu_w (L - X_1)}{|K_w| A_1} \quad (11)$$

$$\frac{i_w (L - X_1)}{\left|\frac{\bar{K}}{\mu}\right| A_1} = \frac{i_w \mu_w (L - X_1)}{K_w A_1} + \frac{i_o \mu_o}{K_o A_1}(L - X_1) \quad (12)$$

$$\left|\frac{\bar{K}}{\mu}\right| = \left[\frac{\mu_w}{K_w} + \frac{\mu_o}{K_o}\right]^{-1} \quad (13)$$

But $K_w = K_{rw} K \; and \; K_o = K_{ro} K$

$$\left|\frac{\bar{K}}{\mu}\right| = K_1\left[\frac{\mu_w}{K_{rw}} + \frac{\mu_o}{K_{ro}}\right]^{-1} \quad (14)$$

Since it is assumed that no water is flowing ahead of the front, therefore $K_{rw} = 0$

$$\left|\frac{\bar{K}}{\mu}\right| = K_1\left[\frac{\mu_o}{K_{ro}}\right]^{-1} \quad (15)$$

Equation (15) gives the average Total fluid mobility ahead of the displacement front at distance ($L - X_1$) as shown in fig 2.

b) Development of Two Phase Mobility Ratio

Mobility ratio can be defined mathematically as

$$m = \frac{\lambda_{displacing\ \ fluid\ flowing\ \ behind\ the\ front}}{\lambda_{displacing\ \ fluid\ flowing\ \ ahead\ the\ front}} \quad (16)$$

But it was assumed that oil and water flow simultaneously behind the front and oil only ahead of the front, then we have Two-phase mobility ratio m_{tp}

$$m_{tp} = \frac{\bar{\lambda}_t}{\lambda_o} = \frac{\overline{\lambda_o + \lambda_w}}{\lambda_o} \tag{17}$$

Where

$\bar{\lambda}_t$ = average total two phase mobility behind the front evaluated at average water saturation behind the front

$$m_{tp} = \frac{K_1 \left[\frac{\mu_w}{K_{rw}} + \frac{\mu_o}{K_{ro}}\right]^{-1}}{K_1 \left[\frac{\mu_o}{K_{ro}}\right]^{-1}} \tag{18}$$

Therefore,

$$m_{tp} = \frac{\left[\frac{\mu_w}{K_{rw}} + \frac{\mu_o}{K_{ro}}\right]^{-1}_{@\overline{sw}}}{\left[\frac{\mu_o}{K_{ro}}\right]^{-1}_{@s_{wi}}} \tag{19}$$

Equation (19) gives the two-phase mobility ratio.

c) Modelling of Position of Flood Front in any Layer after Breakthrough in Layer 1 (most permeable layer)

Consider figure 2, the total pressure drop across this layer is:

$$\Delta P = P_{iwf} - P_{wf} = \Delta P_1 + \Delta P_2 \tag{21}$$

In terms of Darcy's linear flow

$$\Delta P_1 = \frac{i_w \mu_w X_1}{K_w A_1} + \frac{i_o \mu_o X_1}{K_o A_1} \tag{22}$$

$$\Delta P_2 = \frac{i_w \mu_o (L - X_1)}{K_o A_1} \tag{23}$$

$$\Delta P = \frac{i_w L}{\left|\frac{\bar{K}}{\mu}\right| A_1} \tag{24}$$

Substituting Equation (22 -24) into equation into equation (21) and solving for average mobility in the layer

$$\left|\frac{\bar{K}}{\mu}\right| = K_1 L \left[\frac{\mu_w X_1}{K_{rw}} + \frac{\mu_o X_1}{K_{ro}} + \frac{\mu_o (L - X_1)}{K_{ro}}\right]^{-1} \tag{25}$$

Therefore, the average injection flux is:

$$U_1 = \frac{i_w}{A_1} = \frac{\left|\frac{\bar{K}}{\mu}\right|_1 \Delta P}{L} \tag{26}$$

$$U_1 = K_1 \Delta P \left[\frac{\mu_w X_1}{K_{rw1}} + \frac{\mu_o}{K_{ro1}} X_1 + \frac{\mu_O}{K_{ro1}} (L - X_1)\right]^{-1} \tag{27}$$

The actual velocity of the flood front is given by the expression

$$V_1 = \frac{dx_1}{dt} = \frac{U_1}{\phi_1 \Delta S_{w1}} \tag{28}$$

Where ΔS_{w1} represent the change in water saturation across the front. Therefore

$$\frac{dx_1}{dt} = \frac{K_1 \Delta P}{\phi_1 \Delta S_{w1}} \left[\frac{\mu_w X_1}{K_{rw1}} + \frac{\mu_o}{K_{ro1}} X_1 + \frac{\mu_O}{K_{ro1}} (L - X_1)\right]^{-1} \tag{29}$$

And

$$\Delta P = constant = \frac{\phi_1 \Delta S_{w1} \frac{dx_1}{dt}}{K_1} \left[\frac{\mu_w X_1}{K_{rw1}} + \frac{\mu_o}{K_{ro1}} X_1 + \frac{\mu_O}{K_{ro1}} (L - X_1)\right]$$

Similarly, for second layer,

$$V_2 = \frac{K_2 \Delta P}{\phi_2 \Delta S_{w2}} \left[\frac{\mu_w X_2}{K_{rw2}} + \frac{\mu_o}{K_{ro2}} X_2 + \frac{\mu_O}{K_{ro2}} (L - X_2) \right]^{-1}$$

$$\Delta P = constant = \frac{\phi_2 \Delta S_{w2} \frac{dx_2}{dt}}{K_2} \left[\frac{\mu_w X_2}{K_{rw2}} + \frac{\mu_o}{K_{ro2}} X_2 + \frac{\mu_O}{K_{ro2}} (L - X_2) \right] \quad (30)$$

Equating equation (28) and (30)

$$\frac{\phi_1 \Delta S_{w1} \frac{dx_1}{dt}}{K_1} \left[\frac{\mu_w X_1}{K_{rw1}} + \frac{\mu_o}{K_{ro1}} X_1 + \frac{\mu_O}{K_{ro1}} (L - X_1) \right] = \frac{\phi_2 \Delta S_{w2} \frac{dx_2}{dt}}{K_2} \left[\frac{\mu_w X_2}{K_{rw2}} + \frac{\mu_o}{K_{ro2}} X_2 + \frac{\mu_O}{K_{ro2}} (L - X_2) \right] \quad (31)$$

$$\phi_1 \Delta S_{w1} \frac{dx_1}{dt} K_2 \left[\frac{\mu_w X_1}{K_{rw1}} + \frac{\mu_o}{K_{ro1}} X_1 + \frac{\mu_O}{K_{ro1}} (L - X_1) \right] = \phi_2 \Delta S_{w2} \frac{dx_2}{dt} K_1 \left[\frac{\mu_w X_2}{K_{rw2}} + \frac{\mu_o}{K_{ro2}} X_2 + \frac{\mu_O}{K_{ro2}} (L - X_2) \right] \quad (32)$$

$$let\ B_1 = \frac{\mu_w}{K_{rw1}} + \frac{\mu_o}{K_{ro1}}, \quad B_2 = \frac{\mu_w}{K_{rw2}} + \frac{\mu_o}{K_{ro2}}, \quad D_1 = \frac{\mu_o}{K_{ro1}}, \quad D_2 = \frac{\mu_o}{K_{ro2}}$$

$$\phi_1 \Delta S_{w1} dx_1 K_2 [B_1 X_1 + D_1 (L - X_1)] = \phi_2 \Delta S_{w2} dx_2 K_1 [B_2 X_2 + D_2 (L - X_2)] \quad (33)$$

Rearranging the above expression

$$K_2 \phi_1 \Delta S_{w1} [B_1 X_1 + D_1 (L - X_1)] dx_1 = K_1 \phi_2 \Delta S_{w2} [B_2 X_2 + D_2 (L - X_2)] dx_2 \quad (34)$$

Integrating

$$K_2 \phi_1 \Delta S_{w1} \int_0^L [B_1 X_1 + D_1 (L - X_1)] dx_1 = K_1 \phi_2 \Delta S_{w2} \int_0^{x_2} [B_2 X_2 + D_2 (L - X_2)] dx_2 \quad (35)$$

$$K_2 \phi_1 \Delta S_{w1} \left[\frac{B_1 X_1^2}{2} + D_1 L X_1 - \frac{D_1 X_1^2}{2} \right]_0^L = K_1 \phi_2 \Delta S_{w2} \left[\frac{B_2 X_2^2}{2} + D_2 L X_2 - \frac{D_2 X_2^2}{2} \right]_0^{x_2} \quad (36)$$

$$K_2 \phi_1 \Delta S_{w1} \left[\frac{B_1 L^2}{2} + D_1 L^2 - \frac{D_1 L^2}{2} \right] = K_1 \phi_2 \Delta S_{w2} \left[\frac{B_2 X_2^2}{2} + D_2 L X_2 - \frac{D_2 X_2^2}{2} \right] \quad (37)$$

$$K_2 \phi_1 \Delta S_{w1} [B_1 L^2 + D_1 L^2 - D_1 L^2] = K_1 \phi_2 \Delta S_{w2} [B_2 X_2^2 + D_2 L X_2 - D_2 X_2^2] \quad (38)$$

$$K_2 \phi_1 \Delta S_{w1} [L^2 (B_1 - D_1) - 2 D_1 L^2] = K_1 \phi_2 \Delta S_{w2} [X_2^2 (B_2 - D_2) + 2 D_2 L X_2] \quad (39)$$

$$\frac{K_2 \phi_1 \Delta S_{w1}}{K_1 \phi_2 \Delta S_{w2}} [L^2 (B_1 - D_1) - 2 D_1 L^2] = [X_2^2 (B_2 - D_2) + 2 D_2 L X_2] \quad (40)$$

$$let\ A = \frac{K_2 \phi_1 \Delta S_{w1}}{K_1 \phi_2 \Delta S_{w2}} \quad (41)$$

$$A[L^2 (B_1 - D_1) - 2 D_1 L^2] = [X_2^2 (B_2 - D_2) + 2 D_2 L X_2] \quad (42)$$

$$A[B_1 L^2 - D_1 L^2] = [X_2^2 (B_2 - D_2) + 2 D_2 L X_2] \quad (43)$$

Rearranging equation (3.43)

$$(B_2 - D_2) X_2^2 + 2 D_2 L X_2 - A(B_1 + D_1) L^2 = 0 \quad (44)$$

Divide through by L^2

$$(B_2 - D_2) \left[\frac{X_2^2}{L^2} \right] + 2 D_2 \left[\frac{X_2}{L} \right] - A(B_1 + D_1) = 0 \quad (45)$$

Therefore, the solution of equation (3.45) above can be obtained by using quadratic equation formula:

$$\frac{X_2}{L} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where b = $2D_2$, a = $B_2 - D_2$, c = $-A(B_1 + D_1)$

Therefore the quadratic solution to above equation gives,

$$\frac{x_2}{L} = \frac{D_2 \pm \left(D_2{}^2 + (B_2 - D_2)(B_1 + D_1)A\right)^{1/2}}{(D_2 - B_2)} = X_2 \quad (46)$$

$$A = \frac{K_2\phi_1\Delta S_{w1}}{K_1\phi_2\Delta S_{w2}}$$

Where

$$\Delta S_{w1} = \bar{S}_{w1} - S_{wc1}$$
$$\Delta S_{w2} = \bar{S}_{w2} - S_{wc2}$$
$$\Delta S_{w3} = \bar{S}_{w3} - S_{wc3}$$

Therefore, in general the fractional distance the flood front has moved in layer n at the time there is breakthrough in layer 1 (the most permeable layer) is:

$$\frac{X_n}{L} = \frac{D_n \pm \left(D_n{}^2 + (B_n - D_n)(B_1 + D_1)A\right)^{1/2}}{(D_n - B_n)} \quad (47)$$

Where,

$$B_n = \left[\frac{\mu_w}{K_{rwn}} + \frac{\mu_o}{K_{ron}}\right] @\bar{S}_{wn}$$

$$D_n = \left[\frac{\mu_{on}}{K_{ron}}\right] @Sw_{cn}$$

$$\Delta S_{wn} = \bar{S}_{wn} - S_{wcn}$$

d) Two–Phase Water Injection Rate Modeling

Using figure 1, consider total pressure drop through this layer, ΔP, the fluid injection rate into this layer can be obtained using the Darcy's flow equation

$$\Delta P = \frac{i_w \mu L}{KA} \quad (48)$$

$$i = \frac{KA_1\Delta P}{L} \quad (49)$$

Considering the average mobility in the layer

$$i_{total} = \frac{\left|\frac{\bar{K}}{\mu}\right|_1 A_1\Delta P}{L} \quad (50)$$

$$Since \left|\frac{\bar{K}}{\mu}\right| = K_1 L\left[\frac{\mu_w X_1}{K_{rw1}} + \frac{\mu_o X_1}{K_{ro1}} + \frac{\mu_O}{K_{ro1}}(L - X_1)\right]^{-1} \quad (51)$$

Substitute equation (51) into equation (50)

$$i_{total\ 1} = K_1A_1\Delta P\left[\frac{\mu_w X_1}{K_{rw1}} + \frac{\mu_o X_1}{K_{ro1}} + \frac{\mu_O}{K_{ro1}}(L - X_1)\right]^{-1} \quad (52)$$

$$= \frac{K_1A_1\Delta P}{\left[\frac{\mu_w X_1}{K_{rw1}} + \frac{\mu_o X_1}{K_{ro1}} + \frac{\mu_O}{K_{ro1}}(L - X_1)\right]} \quad (53)$$

$$multiply\ through\ by\ \frac{K_{rw1}}{\mu_w} + \frac{K_{ro1}}{\mu_o} \quad (54)$$

$$i_{total\ 1} = \frac{K_1A_1\Delta P\left[\frac{K_{rw1}}{\mu_w} + \frac{K_{ro1}}{\mu_o}\right]}{\left[\frac{\mu_w X_1}{K_{rw1}} + \frac{\mu_o X_1}{K_{ro1}} + \frac{\mu_O}{K_{ro1}}(L - X_1)\right]} \quad (55)$$

Substitute equation (20) into equation (55),

$$i_{total\ 1} = \frac{K_1A_1\Delta P\left[\frac{K_{rw1}}{\mu_w} + \frac{K_{ro1}}{\mu_o}\right]}{X_1 + (L - X_1)m_{tp}} \quad (56)$$

Divide through by $\frac{m_{tp}}{m_{tp}}$

$$i_{total\ 1} = \frac{K_1A_1\Delta P\frac{K_{ro1}}{\mu_o}}{\frac{X_1}{m_{tp}} + (L - X_1)} \quad (57)$$

$$= \frac{K_1 A_1 \Delta P \lambda_o}{\frac{X_1 + m_{tp} L - m_{tp} X_1}{m_{tp}}} \quad (58)$$

$$i_{total\ 1} = \frac{K_1 A_1 \Delta P \lambda_o}{\frac{m_{tp} L - X_1(m_{tp} - 1)}{m_{tp}}} \quad (59)$$

$$i_{total\ 1} = \frac{K_1 A_1 \Delta P \lambda_o}{L\left(1 - \left(\frac{m_{tp} - 1}{m_{tp}}\right) X_1\right)} \quad (60)$$

Therefore, for a stratified reservoir with N-number of layers, water injection rate into layer i can be given as:

$$i_{wi} = \frac{k_i A_i \lambda_0^0 \Delta P_t}{L(1 - (\frac{m_{tp} - 1}{m_{tp}}) X_i} \quad (61)$$

Where i represents layer under consideration (i= 1,2,3,4 ……. N).

The differences of this model than that proposed by Deppe [3] and Muskat [4] are in the assumption of saturation gradient behind the flood front, two phase mobility ratio instead of unit mobility ratio and the inclusion of position of displacement flood front which represent the real field physical condition. This model can be generalized to predict injectivity in any heterogeneous or stratified reservoir with any number of layers.

3. The Case Study and Application of the New Model

The developed models are applied to predict the waterflood injection rate into a ten layers reservoir from Niger-Delta marginal oil Field, Nigeria as case study. The result of the model was compared with the 17 years waterflood production and injection history of the field in other to validate the new model. To compare our new model with other analytical methods, calculations and results from the works of Muskat [4] and Deppe [3] was presented and compared with our new model and field data.

The data showing the layer's characteristic for the case studied reservoir is given in Table 1. The results obtained are as shown in figs. 3 through 14.

Table 1: Characteristics of 10 Layers Stratified Reservoir

Characteristics/Layer	1	2	3	4	5	6	7	8	9	10
Permeability, k, [md]	1000	795.0	500	432.0	348.5	280.5	230.0	188.0	149.0	110.0
Oil End point relative permeability, kroe	0.85	0.9	0.75	0.8	0.75	0.75	0.8	0.68	0.80	0.85
Water End point relative permeability, krwe	0.35	0.5	0.7	0.4	0.35	0.37	0.23	0.2	0.28	0.3
Initial oil saturation, Soi	80	70	70	75	80	75	70	85	60	80
Connate water saturation, Swc	20	30	30	25	20	25	30	15	40	20

4. Results and Discussion

In most forecasting situation, accuracy is treated as the overriding criterion for selecting a model. In many instance the word "accuracy" refers to "goodness of fit," which in turn refers to how well the forecasting model is able to reproduce the data from the actual field.

The results of the application of our new model to forecast waterflood injectivity performance of Niger- Delta reservoir and the new model performance comparison with 2 existing available models and actual reservoir field injection rate are as shown in fig. 3 and fig. 4 respectively.

Figure 3 presents the comparison of the Total water injection rate based on the new model and actual Field performance. It can be noted that, the Total injection rate of the new model is in very good agreement and closely fit with the field performance curve as shown in figure 4. From this figure, it is depicted that the new analytical method is highly accurate and agrees very well with the field data.

Figure 4 presents comparison of the performance of newly developed model with two most commonly used existing available models: Muskat and Deppe and actual field performance. It can be seen that Muskat method under-predicted the field waterflood injection rate while Deppe method over predict the injection rate performance of the reservoir for the 17 years water injection period. This shows that results of these two methods were optimistic, whereas there is a goodness of fit between new model result and actual field performance. This further affirms the accuracy of the new analytical model.

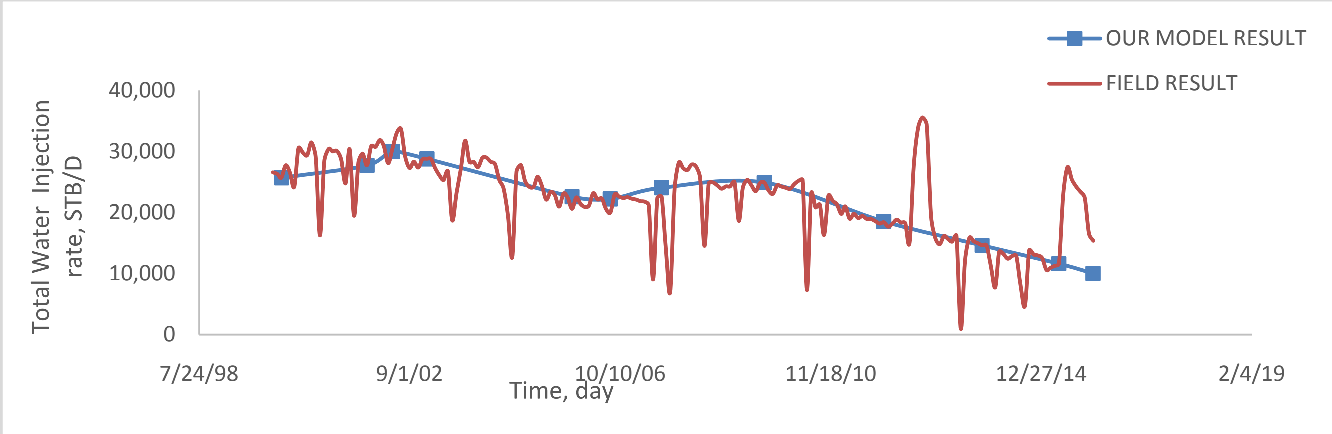

Figure 3: Comparison of New Model Total Injection Rate with Actual Niger- Delta Reservoir Field Performance

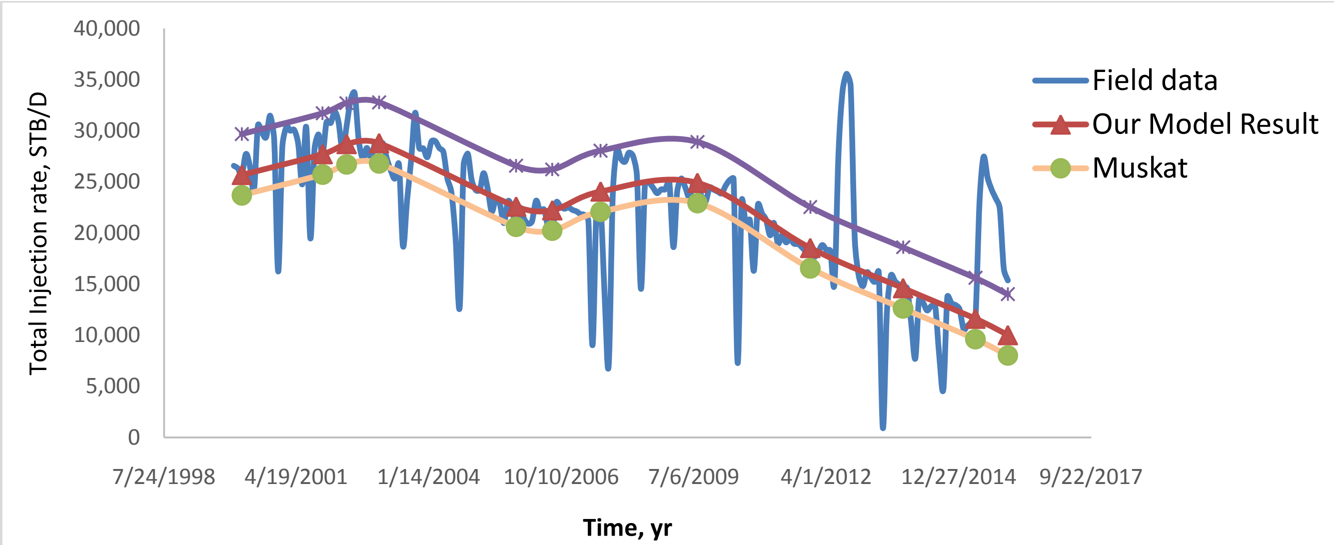

Figure 4: Comparison of Niger- Delta Field Reservoir Injection Rate Performance with Our New Correlation and Two Different Analytical Methods: Muskat and Deppe

5. Conclusion

From the analysis done in this research, the following conclusions could be deduced:

A mathematical model is developed for predicting reservoir layer injection rate for a stratified reservoir taking into account saturation gradient behind the flood front of each layer, physical properties of each layer of the stratified system, introducing the new concept of two-phase fluid mobilities ahead and behind the flood front for each layer, Bottom-hole injection pressure, producing well pressure, and average reservoir pressure at the start of injection and flood front advancement in successive layers of the reservoir at a real point in time. This model gives accurate result that with goodness of fit with the field data.

An improved and effective two-phase mobility ratio model that is based on the average total fluid mobility in the invaded zone has been developed. This model also accounts for variable saturation behind the displacement front. This effective two-phase mobility ratio model gives an accurate injectivity prediction result as compared to other available existing methods that used conventional single phase mobility ratio models and assume piston displacement mechanism.

Determination of the relative location of displacement front in successive layers of stratified reservoir at any point in time and its incorporation into injectivity model gives an accurate estimation of injection rate into each of the layers of the stratified reservoir at that particular reservoir pressure at that real time.

It was also observed from the analysis of this research work that, the injectivity ratio increases as the reservoir heterogeneity and viscosity ratio increase.

Nomenclature

B_{oi} = formation volume factor for oil layer i, RB/STB
B_{wi} =for formation volume factor for water layer i, RB/STB
$I_{wi} = I_{total\ i}$ =The injection rate into layer i at breakthrough, STB/D
$K_i = A$bsolute permeability for layer i, md
K_{roi} = relative permeability of layer i to water
K_{roi} = relative permeability of layer i to oil
L =Distance between injector and producer, ft
M_{tp} = Two − Phase Mobility Ratio at breakthrough, fraction
ΔP_t = Difference between injection pressure and producing well bottom hole pressure, psi
P_{wf} =Bottom hole pressure of the producer well, Psi
P_{inj} =Injection pressure at the injector well, Psi
S_{wci} =Connate or initial water saturation for layer i, fraction,
ΔS_{wi} =Change in water saturation across the displacement front for layer i
S_{ori} =Residual oil saturation in layer i, fraction
S_{wci} =Initial or connate water saturation in layer i, fraction
S_{oi} =Initial oil saturation in layer i, fraction
S_{wi} =Water saturation in layer i at a particular time
$\bar{s}$ = Average saturation in the swept area, fraction
x_i =Distance travel by displacement front in layer i, ft
$X_i = \left(\frac{x_1}{L}\right)$ =, fractional distance travel by displacement front in layer i, fraction
λ_0^0 =, mobility of oil at end point water saturation (Swc), cp^{-1}
$\mu_w = viscosity\ of\ water$, cp
$\mu_o = viscosity\ of\ oil$, cp
ϕ =Porosity, fraction
$\bar{\lambda}_t$ =Is the average total two phase mobility behind the front.

Subscript

i =Layer under consideration
$n = i + 1$
2 = as for layer 2
bt = at breakthrough
D = dimensionless
o = oil
r = relative
t = total
w = water

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