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Research Article

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On a generalized theorem of de Bruijn and Erdös in d-dimensional Fuzzy Linear Spaces

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Abstract In this study we follow a new framework for the theory that offers us, other than traditional, a new angle to observe and investigate some relations between finite sets, *F*-lattice *L* and their elements.
The theory is based on the *Fuzzy Linear Spaces* (*FLS*) $S=(N,D)$. In this case, to operate on these spaces the necessary preliminaries, concepts and operations in lattices relative to *FLS* are introduced. Some definitions, such that *k-fuzzy point*, *k-fuzzy line* are given. Then we correspond these definitions to the definitions in usually linear spaces. We investigate some combinatorics properties of *FLS*. In some examples in the case where $|L|=3^{\bullet}$.
We see some differences. In general, taking an ordered lattice $L_n=\{0,a_1,a_2,...,a_n,1\}$ we observe how some combinatorics formulas and properties are changed. In *FLS* the dimension concept is a set. We produce some general formulas by using some trivial examples. Furthermore, we generalize de Bruijn-Erdös Theorem in [2].

Keywords k-fuzzy point; k-fuzzy line; FLS; Generalized de Bruijn-Erdös Theorem

Introduction

k-point, k-line forLinear Spaces, d-dimensional Linear Spaces were studied by some authors like Batten [5] and Barwick [6]. Here, we give a very short proof to well-known the theorem of de Bruijn and Erdös [4,5][↑]. And also, we have been collected all them from the above papers and from [1,2].
In this paper, we extended the Theorem de Bruijn and Erdös. For this we have to give.

Definition 1. Let $S=(N,D)$ be a *FLS* and $X\subset N$.The set

$$\left\{x\in N:\ \forall x_1,...,x_n\in X,\exists d\in D,\ \bigwedge_{\substack{i=1\\ k,m\in\mathbb{N}\text{ ve }2\le k\le m}}^{k} d(x_i)\wedge d(x)\neq\theta\right\}$$

is called *closure* of *X* and denoted by $\langle X\rangle$.

In any $S=(N,D)$ *FLS*, $\langle\varnothing\rangle=\varnothing$, $\langle\{x\}\rangle=\{x\}$ and $\langle S\rangle=S$.

$^{\bullet}|L|$: Number elements of *L*.

↑ De Bruijn and Erdös (1948). Sometimes called the de Bruijn-Erdös and Hanani theorem because of Hanani (1955).

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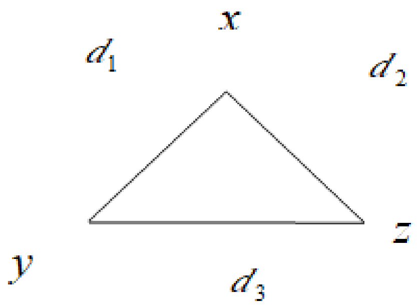
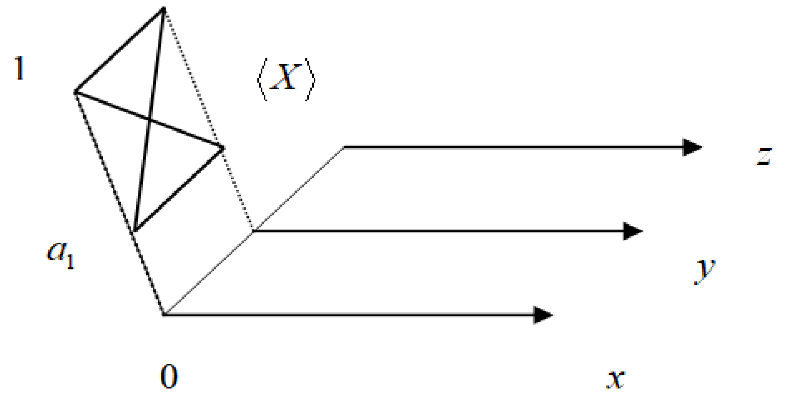
If $\langle X \rangle = B$ then we say that *X* generates *B*.

Example 2. Let $S = (N, D)$ *FLS*, where $N = \{x, y, z\}$ and $D = \{d_1 = \{1,1,0\}, d_2 = \{0,1,1\}, d_3 = \{1,0,1\}\}$.

For $X = \{x, y\}$, $\langle X \rangle = d_1$, which is only one line.



If $L_1 = \{0, a_1, 1\}$, $\langle X \rangle = d_1 \cup \{1, a_1, 0\} \cup \{a_1, 1, 0\} \cup \{a_1, a_1, 0\}$ is not one line.



Definition 3. Let $S = (N, D)$ be a *FLS*. Then any point $x \in N$ is called *k-fuzzy point* if $\bigwedge_{i=1, d_i \in D}^{k} d_i(x) \neq \theta$.

$\{x\}$ is *0-fuzzy point* for $L = \{0,1\}$. But $\{x\}$ is *1-fuzzy point* for $L_n, n \geq 2$.

Definition 4. Let $S = (N, D)$ be a *FLS*. Then a line $d \in D$ is called *k-fuzzy line* if $\bigwedge_{i=1, x_i \in N}^{k} d(x_i) \neq \theta$.

Lemma 5. Let $S = (N, D)$ be a *FLS* and any line $d \in D$ be a k-fuzzy line. Then the number of k-fuzzy line is $(n+1)^k$.

Proof. There are k points $x_1, \ldots, x_k$ on each k-fuzzy line and $d(x_j) = t$ where $t = a_1, \ldots, a_n, 1$ and $j = 1, \ldots, k$. Then the number of k-fuzzy line is $(n+1)^k$.

Lemma 6. Let $S = (N, D)$ be a *FLS* and any point $x \in N$ be a k-fuzzy point. Then the number of k-fuzzy points is $\prod_{j=1}^{k} (n+1)^{v_j}$ where $v_j = \left| \{ x | d_j(x) \neq \theta, x \in N \} \right|$.

Proof. If there are v_j points on each line d_j from Lemma 5 the number of such line d_j just $(n+1)^{v_j}$, where $j = 1, \ldots, k$. And furthermore since *x* is a k-fuzzy point then the total number of k-fuzzy points is $\prod_{j=1}^{k} (n+1)^{v_j}$

.

Theorem (de Bruijn-Erdös) [5]**.** Let *S* be any finite linear space with $b=|D|>1, |N|=v$. Then

i. $b \geq v$,

ii. If $b=v$, any two lines have point in common. In case (2) either one line has $v-1$ points and all others have two points, or every line has $k+1$ points and every point is on $k+1$ lines, $k \geq 2$.

If any point of *S* has k-fuzzy point then the following proposition will give:

Proposition 7. Let $S=(N,D)$ be a *FLS* such that $|S|=m$, any point $x \in N$ isk-fuzzy point, and $v_j = |\{x | d_j(x) \neq \theta, x \in N\}|$. Then

$$|D| = \frac{\log_{(n+1)} m}{v_j}, n \geq 1.$$

Proof. $|S|=m$ by [2].

$$= \prod_{j=1}^{|D|} (n+1)^{vj}$$

$$= \underbrace{(n+1)^{vj} \ldots .. (n+1)^{vj}}_{|D|\text{-time}}$$

$$= (n+1)^{|D| v_j}$$

$$\log_{(n+1)} m = |D| v_j \log_{(n+1)} (n+1)$$

$$|D| = \frac{\log_{(n+1)} m}{v_j}, n \geq 1.$$

We now extend the Theorem of de Bruijn-Erdös:

Theorem (Hasan KELEŞ)**.** Let $S=(N,D)$ be any finite *FLS* such that with $b=|D|>1, |N|=v \geq 3$. Then $b \geq v$ and any two lines have a point in common. Furthermore, either just one of the lines in *D* is a $(v-1)$-fuzzy line and others are *2*-fuzzy lines, or every line is a $(k+1)$-fuzzy line and every point is $\left[\prod_{j=1}^{k+1} (|L|-1)^{v_j}\right]$-fuzzy point, $k \geq 2$.

Proof. The inequality $b \geq v$ is obvious. The case where $|L|=2$ it is the theorem *de Bruijn Erdös'*. It is clear that *b>v.* The fact that any two lines have a point in common is obtained from the definition of *FLS*. If one of the lines in *D* is $(v-1)$-fuzzy line then $\bigwedge_{i=1}^{v-1} d(x_i) \neq \theta$ and $d(x_v)=\theta$. So $d(x_v) \bigwedge_{i \in \{1,\ldots,v-1\}} d'(x_i) \neq \theta$ for $\forall d' \neq d, d' \in D$ from the definition of *FLS*. Therefore lines d' are *2*-fuzzy lines.

If one of the lines in *D* is not a $(v-1)$-fuzzy line then the other are not *2*-fuzzy lines. Therefore all of them are $(k+1)$-fuzzy lines where $k \geq 2$. So $\bigwedge_{i=1}^{k+1} d(x_i) \neq \theta$. Line *d* has points $(k+1)$. Therefore any point $x_i \in N$ is a $\left[\prod_{j=1}^{k+1} (|L|-1)^{v_j}\right]$-fuzzy point from Lemma 6.

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