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Solution of Falkner-Skan Flow and Heat Transfer Over a Wedge by Shooting Method

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Abstract The study of convective heat transfer has generated many interests and become more important recently because of their wide applications in engineering and in several industrial processes. The governing boundary layer equations are transformed into a system of non-dimensional third and second order differential equations. In this work the convective heat transfer equations of the boundary layer with pressure gradient over a wedge are solved simultaneously by a simple and precise iterative formula predicated on Taylor theory utilizing shooting method. This method provides the ability to choose the initial guess function and is utilized to solve the cognate boundary layer quandary. The velocity and temperature profiles for different wedge angle and different Prandtl number are obtained. The results are then compared with published results. The comparison shows an excellent agreement with the results that found in the literature.

Keywords Laminar Convection, Pressure gradient, Heat transfer, Falkner-Skan equation; Nonlinear differential equation, Boundary layer, Shooting Method

1. Introduction

Prelude since its first appearance in the literature in 1908 [1], the Blasius equation describing viscous flow over a flat plate has fascinated physicists, engineers, mathematicians and numerical analysts kindred. This ODE is opulent in physical, mathematical and numerical challenges. Two-dimensional flow over a fine-tuned impenetrable surface engenders a boundary layer as particles move more gradually near the surface than near the free stream. Because of its application to fluid flow, physicists and engineers have a keen interest in solving the Blasius equation and the cognate, but more general, Falkner-Skan (F-S) equation [2].

Since one can elegantly reduce these equations to one-dimensional non-linear ODEs through similarity arguments, mathematicians have found their fulfillment in uncovering the underlying symmetries and proving existence and (non-) uniqueness of its solutions. Thereafter, several authors [3-5] have made significant investigations in generalizing their theoretical study to various situations of practical interest. Such investigations find their applications involving laminar flow heat transfer as in electronic components cooling and plate-type heat exchangers design.

Forced convective in boundary layer include the work of several authors [6-14]. More recently, Mahgoub [15] discussed forced convection heat transfer over a flat plate in a porous medium. Analysis of convective momentum and heat transfer system in boundary layer was done by Escriva and Govannini [16]. The study on heat and mass transfer under various physical situations was carried out by many researchers [17-21]. The study of direct numerical simulation (DNS) of flow over a flat plate was carried out by Wissink and Rodi [22]. Vajravelu et al. [23], investigated the unsteady convective boundary layer flow of a viscous fluid, and this was an extension work of Aydin and Kaya [10].

Unfortunately, a general analytical solution has not been forthcoming; however, for special cases of the F-S equation, several analytical solutions do exist. These prove most beneficial in verifying numerical algorithms.

Numerical analysts, or as they are called by J.P Boyd, "arithmurgists" [24], have had a field-day with these equations. They offer the mystery of nonlinearity - yet the simplicity of unidimensional - and the challenge of solving a boundary value problem through the determinism of an initial value problem. A host of numerical methods has emerged to solve these equations including, but not inhibited to, finite differences and finite elements.

Lin and Lin [25] introduced a homogeneous attribute solution method for the forced convection heat transfer from isothermal or uniform-flux surfaces to fluids of any Prandtl number. The solutions of the resulting kindred attribute Equations are given by the Runge–Kutta scheme. Hsu and Hsiao [26] presented a combination of a series expansion, similarity transformation and finite difference method for the heat transfer problem of a second-grade viscoelastic fluid past a plate fin. Bor-Lih Kuo [27] studied the heat transfer analysis for the Falkner–Skan wedge flow by the differential transformation method; the results were in a good agreement with those provided by other numerical methods.

A list of nearly 150 references (some repeated) for these and other algorithms is found in references 28 and 29.It is also noted that the majority of the numerical techniques are based on Runge-Kutta technique for solving ODE, and therefore have a definite "black box" quality. In spite of the enormous numerical effort however, a truly simple, yet numerically accurate and robust algorithm is still missing. Many, if not all, algorithms to this point seem rather delicate in that their iterative strategies must be carefully tuned to avoid numerical instability. For example, most schemes require the initial guess of the shooting angle to be relatively close to the converged result, which does not make for a robust algorithm. Judging from recent literature, the general lack of numerical agreement to consistent five or more digits is indicative of the need for a reliable algorithm-- the development of which we now address. An approximate solution for second order differential equation based on Taylor expansion is presented in ref. 30. The Solution of the Blasius and Falkner-Skan Boundary Layer Equations based on the technique found in ref. 30 is presented in ref. 31.

Our motivation in the present study is to obtain the solution of the convective heat transfer equations of boundary layer with pressure gradient over a wedge. The system of equations is solved simultaneously by a simple and accurate iterative formula based on Taylor theory using shooting method. This method provides the ability to choose the initial guess function and is used to solve the related boundary layer problem. The velocity and temperature profiles for different wedge angle and different Prandtl number are obtained. The results are then compared with published results. Comparison shows an excellent agreement with the results that found in the literature. Results are obtained using Matlab software and compared with that published in the literature. Comparison shows an excellent agreement between the proposed technique and the published one.

2. Mathematical Formulation

A steady laminar boundary-layer problem is studied. Let the free stream velocity U_∞ which is not constant and depends on the pressure gradient along the flat plate. Assume the free stream temperature T_∞ be constant and let all fluid properties be constant. The velocity distribution is unchanged by the temporal changes in temperature. The boundary layer flow over a flat plate is governed by the equations: continuity and Navier-Stokes equations. The associated partial differential equations are:

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0 \tag{1}$$

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}=-\frac{dp^*}{dx^*}+\frac{\partial^2 u^*}{\partial y^{*2}} \tag{2}$$

$$u^*\frac{\partial T}{\partial x^*}+v^*\frac{\partial T}{\partial y^*}=k\frac{\partial^2 T}{\partial y^{*2}} \tag{3}$$

subject to the boundary conditions;

$$u^*(0,y^*)=1,\quad u^*(x^*,0)=0,\quad u^*(x^*,\infty)=1,\ v^*(x^*,0)=0$$
$$T(0,y^*)=1,\ T(x^*,0)=1,\quad T(x^*,\infty)=1 \tag{4}$$

Where x^*andy^* are the Cartesian coordinates measured along the surface of the flat plate starting from the leading edge of the flat plate. The coordinate y^*measured normal to the flat plate. u^* and $v^*$are the velocity components along $x^*$and y^* directions, whiles T is the fluid temperature.

The following dimensionless variables are introduced as follows:

$$u = \frac{u^*}{U_\infty},\ v = \frac{v^*}{U_\infty},\ p = \frac{p^*\rho}{U_\infty^2},\quad \theta = \frac{T - T_\infty}{T_w - T_\infty},\quad x = \frac{x^*}{L},\quad y = \frac{y^*}{L} \tag{5}$$

The dimensionless equations are obtained as follows;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{6}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \frac{\partial^2 y}{\partial y^2} \tag{7}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = k\frac{\partial^2 \theta}{\partial y^2} \tag{8}$$

The stream function $\psi(x, y)$ is introduced and defined as;

$$u = \frac{\partial \psi}{\partial x},\qquad v = -\frac{\partial \psi}{\partial y} \tag{9}$$

Converting the set of partial differential equations (6)–(9) into ordinary differential equations by using the similarity transformation

$$\psi = \sqrt{\nu U_\infty x}\, f(\eta) \qquad \eta = \sqrt{\frac{U_\infty}{\nu x}} \tag{10}$$

Where η is the similarity variable, f is the similarity function and ψ is the stream function, and simply by replacing u and v components of velocity by a single function. Let define the free stream velocity as $U_\infty = Cx^m$ where m is the Falkner-Skan power-law parameter. The quantity β in (11) is related to the wedge angle, where the wedge angle is given by βπ/2 as shown in Figure 1. The case m = 0 is for a flat plate, and m = 1 is for the wedge half-angle 90°, which is two-dimensional stagnation flow known as Hiemenz flow;

$$m = \frac{\beta}{2-\beta}\ and\quad \beta = \frac{2m}{m+1} \tag{11}$$

If $m > 0, then\ \frac{dP}{x} < 0$ ⇒ (favourable) forward pressure gradient

If $m < 0, then\ \frac{dP}{x} > 0$ ⇒adverse pressure gradient

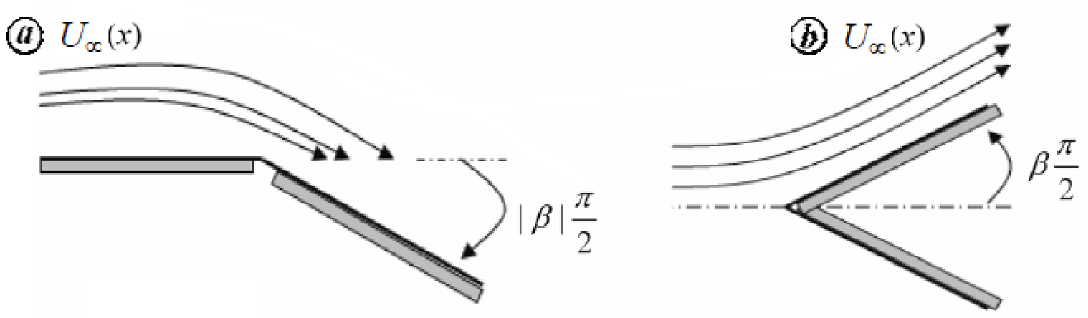

Figure 1: Different potential flows over a wedge. (a), Flow around a corner (diffusion). (b), Wedge flow

By substitution, using equations (10) & (11) in equation (6)-(9), and thus these become;

$$f'''(\eta) + \frac{m+1}{2}f(\eta)f''(\eta) + m(1 - f'(\eta)^2] = 0\ \ ; \eta \in [0, \infty] \tag{12}$$

$$\theta''(\eta) + \frac{Pr\ (m+1)}{2}\ f(\eta)\theta'(\eta)] = 0\ \ ; \eta \in [0, \infty] \tag{13}$$

With the boundary conditions

$$f(0) = 0 \qquad f'(0) = 0 \qquad lim_{\eta\to\infty} f'(\eta) = 1 \tag{14}$$

$$\theta(0) = 1 \qquad lim_{\eta\to\infty}\theta(\eta) = 0 \tag{15}$$

Where Pr is the Prandtl number, which is equal to the ratio of the momentum diffusivity of the fluid to its thermal diffusivity. Equations (12) and (13) along with the boundary conditions (14) and (15) present a system of ordinary differential equations for the Falkner-Skan boundary layer problem. Simultaneous solution of these two equations yield the velocity and temperature profiles for the flow of a viscous fluid passing over a wedge.

3. Method of Solution

Equations (12) and (13) can be solved as initial value problem using the shooting method. Through a specification of an additional initial condition to replace the condition at infinity, the boundary value problem transforms into an equivalent iterative initial value problem. In this case equations (13) and (14) are subjected to the following initial conditions:

$$f(0) = 0 \qquad f'(0) = 0 \qquad f''(0) = \alpha \tag{16}$$
$$\theta(0) = 1 \qquad \theta'(0) = \gamma \tag{17}$$

3.1. Shooting Method

One method for solving boundary-value problems; is the shooting method which is based on converting the boundary-value problem into an equivalent initial-value problem. Generally, the equivalent system will not have sufficient initial conditions and so a guess is made for any undefined values. These guesses are changed until the final solution satisfies all the boundary conditions.

The shooting method is the preferred way to treat the F-S boundary value problem. The boundary condition $\lim_{\eta\to\infty} f'(\eta) = 1$is replaced by the initial condition $f''(0) = \alpha$, where $\alpha$is the skin friction coefficient. To be equivalent, the shooting angle α must be determined such that$\lim_{\eta\to\infty} f'(\eta) = 1$. Except where noted for a particular range of β, the solution is assumed unique [7]. In this method it is most important to choose the appropriate finite values of $\eta \to \infty$. The solution process is repeated with another large value of $x \to \infty$ until two successive values of $f''(0)$ differ only after a desired digit signifying the limit of the boundary alongη. The last value of $\eta \to \infty$ is chosen as appropriate value of the limit $\eta \to \infty$for that particular set of parameters. Then the value of $f''(0) = \alpha$ is refined until the exact value of $\alpha$is determined.

Similarly for the variable θ,The boundary condition $\lim_{\eta\to\infty} \theta(\eta) = 0$ is replaced by the initial condition $\theta'(0) = \gamma$. As done with the Falkner-Skan equation, after choosing the appropriate finite values of $\eta \to \infty$. The solution process is repeated until a refined value of γ is determined as the exact one.

3.2. Methodology of the Proposed Technique

In this technique, the differential equation (12) is rearranged as follows:

$$f'''(\eta) = -\frac{m+1}{2} f(\eta) f''(\eta) - m(1 - (f'(\eta))^2) \tag{18}$$

By direct substitution of the initial conditions given in (11), (12) and (13) the third derivative $f'''(0)$atstarting point can be written as:

$$f'''(0) = -\frac{m+1}{2} f(0) f''(0) - m(1 - f'(0)^2) \tag{19}$$

The approximate function at$\eta + \Delta\eta$ is obtained using Taylor expansion (1) up to the fourth term:

$$f(\eta + \Delta\eta) = f(\eta) + f'(\eta)\Delta x + \frac{1}{2} f''(\eta)\Delta\eta^2 + \frac{1}{6} f'''(\eta)\Delta\eta^3 \tag{20}$$

The first and second derivatives of the function$f(\eta)$ at $\eta + \Delta\eta$are obtained using the central difference approximation of Taylor expansion as:

$$f'(\eta + \Delta\eta) = \frac{f(\eta+2\Delta\eta) - f(\eta)}{2\Delta\eta} - \frac{f'''(\eta)}{6}\Delta\eta^2 \tag{21}$$

$$f''(\eta + \Delta\eta) = \frac{f(\eta+2\Delta\eta) - 2f(\eta+\Delta\eta) + f(\eta)}{\Delta\eta^2} \tag{22}$$

Then, the approximate third derivative at time $\eta + \Delta\eta$ is obtained using equation (18) as:

$$f'''(\eta + \Delta\eta) = -\frac{m+1}{2} f(\eta + \Delta\eta) f''(\eta + \Delta\eta) - m(1 - f'(\eta + \Delta\eta)^2) \tag{23}$$

So, the first iteration is obtained from equations (20) through (23) as:

$$f(\Delta\eta) = f(0) + f'(0)\Delta\eta + \frac{1}{2} f''(0)\Delta\eta^2 + \frac{1}{6} f'''(0)\Delta\eta^3 \tag{24}$$

$$f(2\Delta\eta) = f(0) + 2f'(0)\Delta\eta + 2f''(0)\Delta\eta^2 + \frac{4}{3} f'''(0)\Delta\eta^3 \tag{25}$$

$$f'(\Delta\eta) = \frac{f(2\Delta\eta) - f(0)}{2\Delta\eta} - \frac{f'''(0)}{6}\Delta\eta^2 \tag{26}$$

$$f''(\Delta\eta) = \frac{f(2\Delta\eta) - 2f(\Delta\eta) + f(0)}{\Delta\eta^2} \tag{27}$$

$$f'''(\Delta\eta) = -\frac{m+1}{2} f(\Delta\eta) f''(\Delta\eta) - m(1 - f'(\Delta\eta)^2) \tag{28}$$

The recurrence formula of this technique to solve the Falkner-Skan equation can be written as:

$$f_n = f_{n-1} + f'_{n-1}\Delta\eta + \frac{1}{2}f''_{n-1}\Delta\eta^2 + \frac{1}{6}f'''_{n-1}\Delta\eta^3 \quad (29)$$

$$f_{n+1} = f_{n-1} + f'_{n-1}\Delta\eta + 2f''_{n-1}\Delta\eta^2 + \frac{4}{3}f'''_{n-1}\Delta\eta^3 \quad (30)$$

$$f'_n = \frac{f_{n+1}-f_{n-1}}{2\Delta\eta} + \frac{f'''_{n-1}}{6}\Delta\eta^2 \quad (31)$$

$$f''_n = \frac{f_{n+1}-2f_n+f_{n-1}}{\Delta\eta^2} \quad (32)$$

$$f'''_n = -\frac{m+1}{2}f_n f''_n - m\left(1 - f'^2_n\right) \quad (33)$$

Similarly, the recurrence formula to solve the convection problem through boundary layer can be written as:

$$\theta_n = \theta_{n-1} + \theta'_{n-1}\Delta t + \frac{1}{2}\theta''_{n-1}\Delta t^2 \quad (34)$$

$$\theta_{n+1} = \theta_{n-1} + 2\dot{\theta}'_{n-1}\Delta t + 2\theta''_{n-1}\Delta t^2 \quad (35)$$

$$\theta'_n = (\theta_{n+1} - \theta_{n-1})/(2\Delta t) \quad (36)$$

$$\theta''_n = -\frac{\Pr(m+1)}{2}f_n\ \theta'_n \quad (37)$$

4. Results and Discussion

In the absence of an analytical solution of a problem, a numerical solution is indeed an obvious and a natural choice. Thus, the nonlinear third-order Falkner–Skan equations (12,13) with boundary conditions (14, 15), are solved using the present technique with shooting algorithm. The present technique was derived based on Taylor Expansion. To assess the validity and accuracy of the present method, comparison with previously reported data available in the literature has been made via Table 1 and Table 2.

Table 1: Comparison of Values of $f''(0) = \alpha$ for different values of m

		values of $f''(0) = \alpha$	
m	$\beta = \frac{2m}{m+1}$	**Cebeci (1988) [32]**	**Present**
-0.0904	-0.19877	0.00000	0.000000
-0.0654	-0.13995	N/A	0.163800
-0.0500	-0.10526	0.21351	0.213300
0.0000	0.00000	0.33206	0.331980
0.333333	0.50000	0.75745	0.757595
1.0	1.0	1.23259	1.233142

Table 2: Comparison of Values of $\theta'(0) = \gamma$ for different values of m and different values of Prandtl Number

m	$\beta = \frac{2m}{m+1}$	**f''(0)**	**θ'(0)**					
			Pr=0.7		**Pr=1**		**Pr =10**	
			Louis[33]	**Present**	**Louis**	**Present[33]**	**Louis**	**Present**
-0.0753	-0.16286	0.1243	-0.242	-0.24212	-0.272	-0.27119	-0.570	-0.5571
0	0	0.33198	-0.292	-0.29253	-0.332	-0.33183	-0.730	-0.72704
0.111	0.19982	0.5117	-0.331	-0.33100	-0.378	-0.37748	-0.851	-0.8488
0.333333	0.5	0.757595	-0.384	-0.38391	-0.44	-0.43975	-1.013	-1.0096
1.0	1	1.233142	-0.496	-0.49549	-0.57	-0.56995	-1.344	-1.33555
4.0	1.6	2.408136	-0.813	-0.81260	-0.938	-0.93765	-2.236	-2.2311

The present Technique is used to solve the Falkner-skan third order equation for different number of pressure gradient; -0.0904, -0.0654, -0.05, 0.0, 1/3, 1. The f''(0) and θ'(0) are obtained using shooting technique and presented in Table 3.

Table 3: Values of $\theta'(0) = \gamma$ for different values of m and different values of Prandtl Nmber

m	$\beta = \frac{2m}{m+1}$	f''(0) Present	θ'(0)		
			Pr=0.6	Pr=1	Pr=15
-0.0904	-0.19877	0	-0.19205	-0.21973	-0.44061
-0.0654	-0.13995	0.1638	-0.24032	-0.28422	-0.67672
-0.05	-0.10526	0.2133	-0.25175	-0.2992	-0.7269
0.0	0	0.33198	-0.27688	-0.33183	-0.83265
0.333333	0.5	0.757595	-0.36174	-0.43975	-1.16255
1.0	1.0	1.233142	-0.466	-0.56995	-1.5413

The velocity profile of the wedge flow for different pressure gradient is shown in Figure 2. Large pressure gradient hinders boundary layer growth and resulted in a reduced boundary layer thickness. In figure 2, when the pressure gradient parameter is m=-0.0904, the separation point is reached and the fluid will not be in contact with the surface anymore. It is also observed that the velocity increases with the increase in the pressure gradient parameter m. The pressure gradient parameter suppresses the boundary layer growth, and when the pressure gradient parameter is m=1, this is on a two-dimensional stagnation flow.

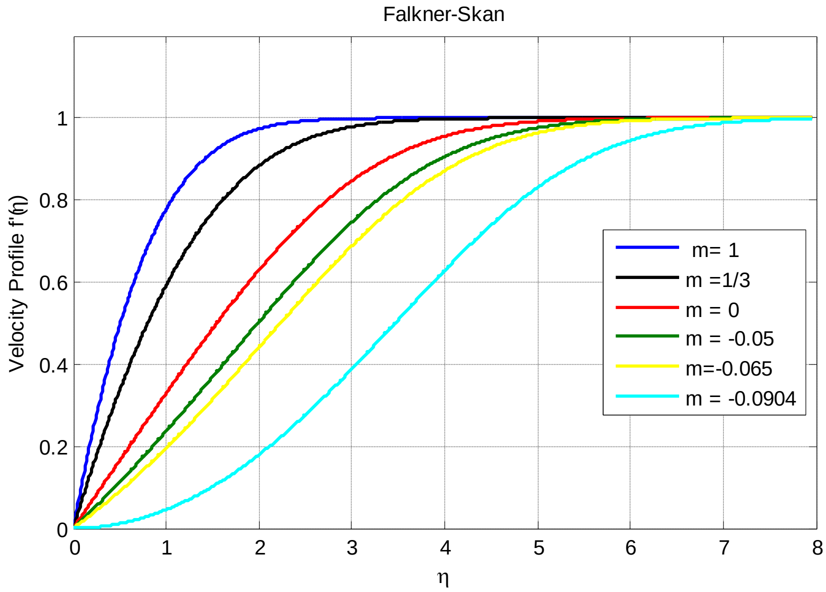

Figure 2: Velocity Profile $f'(\eta)$ for various values of m

The influence of Prandtl number Pr at fixed pressure gradient m on temperature profile θ(η) for forced convection flow is shown in figures 3 to 8. The Prandtle number values Pr = 15, 1, and0.6. The Prandtl number with value 15 represents a large Prandtl number which means that heat wave penetration is less in fluid for example in oil. The Prandtl number with value 0.6 for example liquid metals (Mercury). The pressure gradient values m = -0.094, -0.065, -0.05, 0, 1/3 and 1.In convective heat transfer for different pressure gradient, the Prandtl number controls the relative thickness of the momentum and the thermal boundary layer. For a small Prandtl number, the heat is transferred slower, which makes the temperature drops slowly. For large Prandtl number, the temperature shows a sharp fall. The thermal boundary layer thickness decreases sharply with significant increases in Prandtl number. The reason is that, the value of Prandtl number is larger, the thermal diffusivity decreases. It will result in decrease of energy transfer ability and causing the thermal boundary layer to decrease.

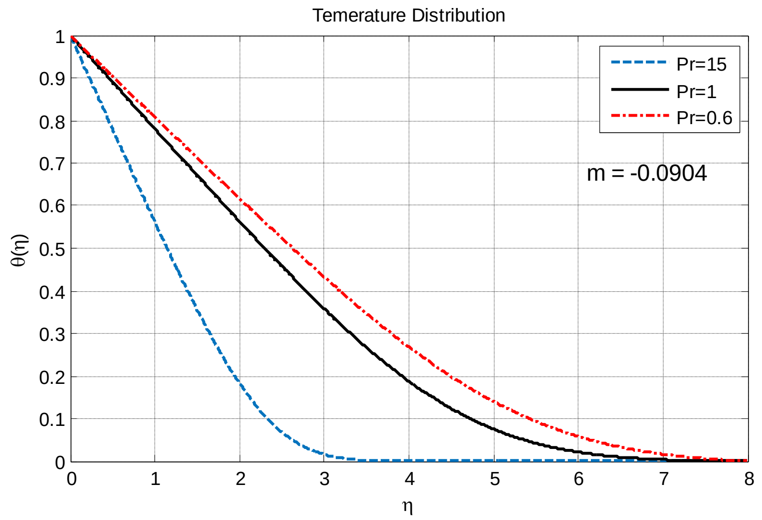

Figure 3: Temperature Profile θ for various values of Prandtl Number at m=-0.0904

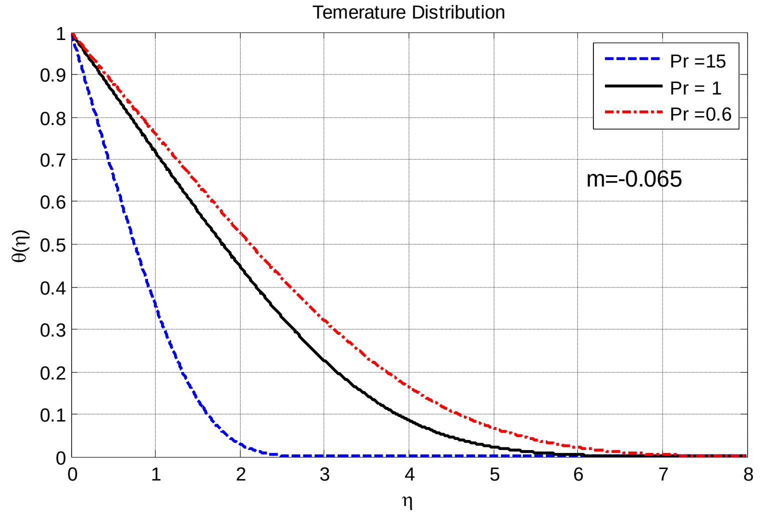

Figure 4: Temperature Profile θ for various values of Prandtl Number at m=-0.065

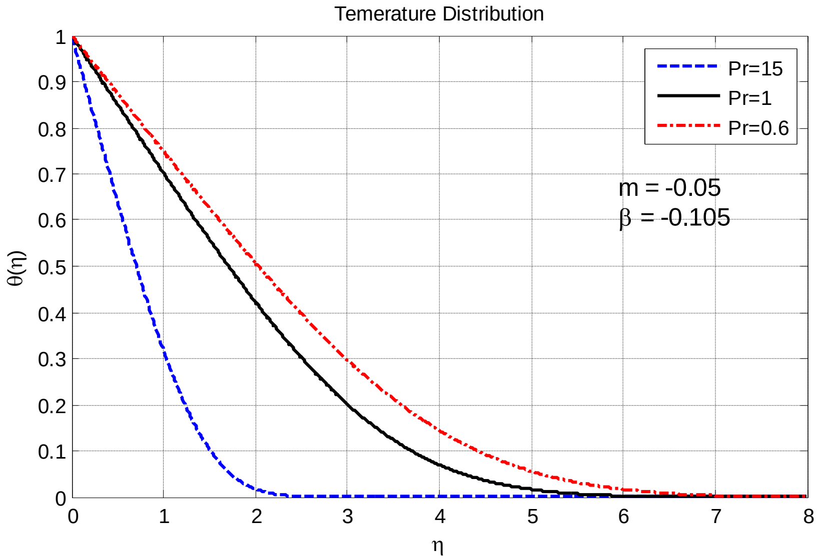

Figure 5: Temperature Profile θ for various values of Prandtl Number at m= -0.05

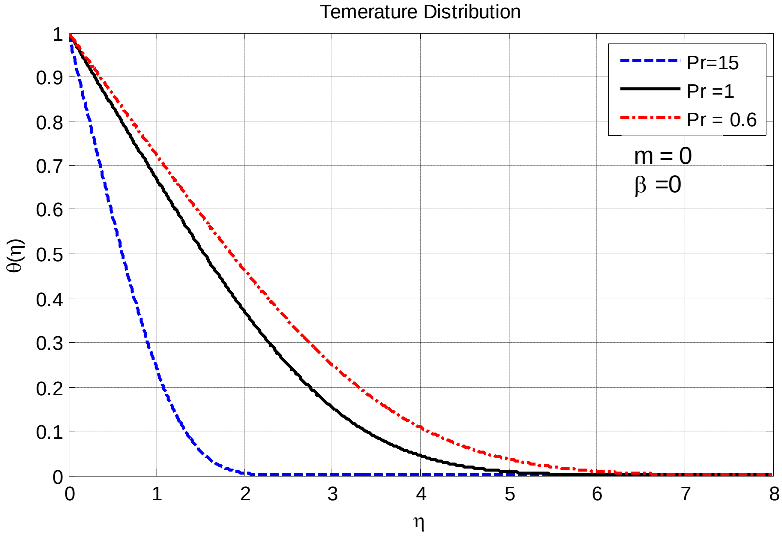

Figure 6: Temperature Profile θ for various values of Prandtl Number at m=0

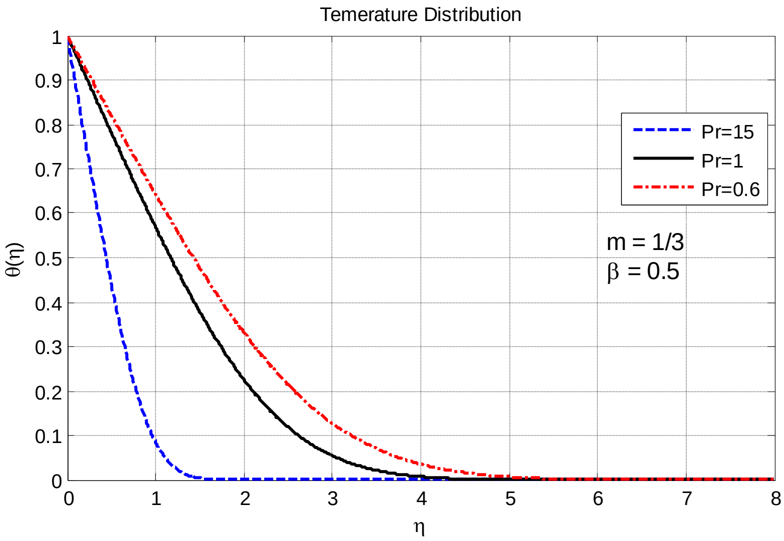

Figure 7: Temperature Profile θ for various values of Prandtl Number at m=1/3

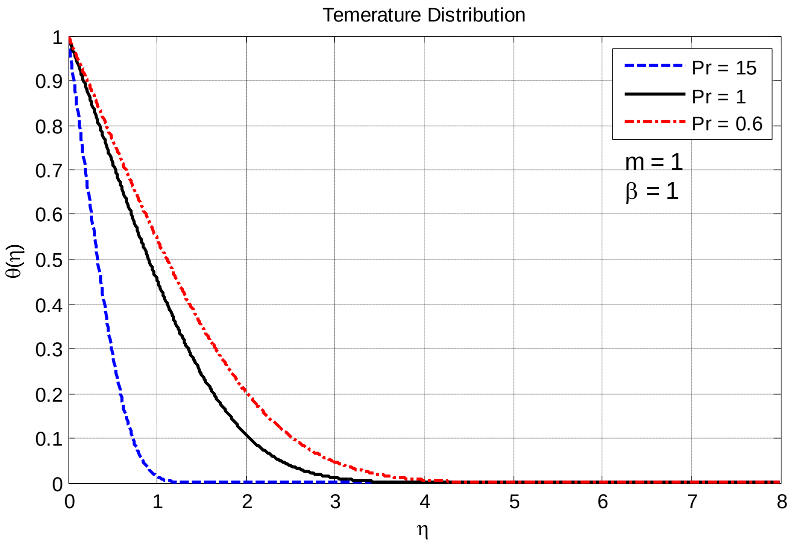

Figure 8: Temperature Profile θ for various values of Prandtl Number at m= 1

The influence of the different pressure gradients m with various values of Prandtl parameter Pr, on temperature profile is shown in Figure9. It is observed that the rate temperature variation increases with the increase in the values of pressure gradient parameter m and Prandtle number Pr.

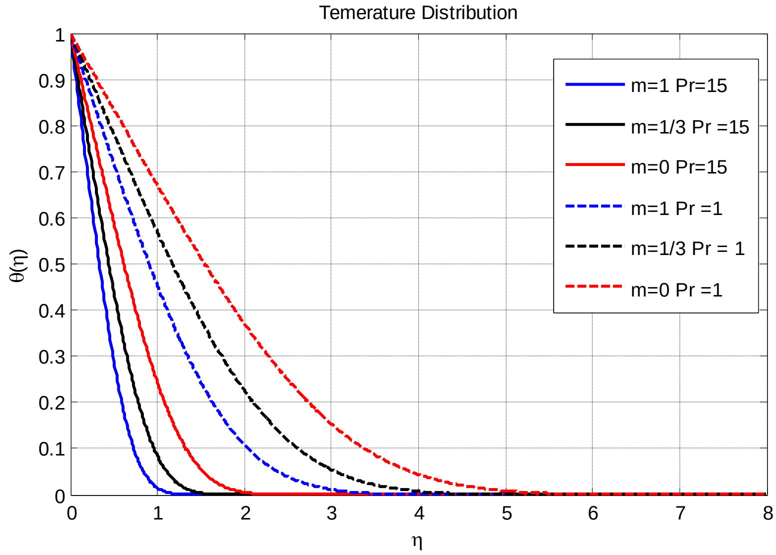

Figure 9: Temperature Profile θ for various values of Prandtl Number and m

5. Conclusion

In this work the Convective heat transfer equations of boundary layer with pressure gradient over a wedge are solved simultaneously by a simple and accurate iterative formula based on Taylor theory using shooting method. This method provides the ability to choose the initial guess function and is utilized to solve the related boundary layer problem. The velocity and temperature profiles for different wedge angle and different Prandtl number are obtained. The results are then compared with published results. Comparison shows an excellent agreement with the results that found in the literature. Using the suggested technique, there is no need to transform the higher order differential equations to state space as in Runge-kutta technique.

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