



Comprehensive Evaluation of Urban Smart Growth with Application to City Planning

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Abstract Smart growth (SG) emerged to curb continued urban sprawl. First, we construct a SG evaluation metric by developing a comprehensive evaluation system. Second, by utilizing a combined gray relational analysis (GRA) and maximum entropy (ME) model, we establish the weight model and obtain the weights of our proposed SG evaluation metric. We conduct the case study on two cities of Geneva and Wellington. Based on the weights, we calculate the correlation degree between current growth and SG by using the sum of least squares. The results show that the current plan of Geneva is more successful than the one of Wellington according to our metric. Then, we design a growth plan for both cities. We calculate the SG metrics, and find that both metrics of the two cities have improved. Finally, using our metric, we apply fuzzy comprehensive evaluation (FCE) method to rank the individual initiatives within our plan. The ratio of land occupation of both cities is the most potential. The tax rate of Geneva is the least potential, air quality of Wellington as well. Our work contributes to urban planning and may help promote sustainable urban development.

Keywords Smart growth, Comprehensive evaluation, Gray relational analysis, Least squares

1. Introduction

Smart growth (SG) is an urban planning theory that originated in 1990's as a means to curb continued urban sprawl and reduce the loss of farmland surrounding urban centers [1-3]. Smart growth is one of the efforts of controlling the natural resources consumption. Smart City aims to create a high quality of life. In the United States, specific smart-growth practices include federal, state, and local government. Smart Growth America is an organization that applies smart growth to practice, strategically thinking about how to plan and build better cities and communities. Ten principles of smart growth are [2-7]: ① Mix Land Uses; ② Take Advantage of Compact Building Design; ③ Create a Range of Housing Opportunities and Choices; ④ Create Walkable Neighborhoods; ⑤ Foster Distinctive, Attractive Communities with a Strong Sense of Place; ⑥ Preserve Open Space, Farmland, Natural Beauty and Critical Environmental Areas; ⑦ Strengthen and Direct Development Towards Existing Communities; ⑧ Provide a Variety of Transportation Choices; ⑨ Make Development Decisions Predictable, Fair and Cost Effective; ⑩ Encourage Community and Stakeholder Collaboration in Development Decisions. Smart growth focuses on building cities that embrace the E's of sustainability—Economically prosperous, socially Equitable, and Environmentally Sustainable [2]. This task is more important than ever because the world is rapidly urbanizing.

In this paper, we conduct research based on the following assumptions: ① The principle of smart growth does not conflict with the religious in selected cities. ② Based on smart growth planning, cities develop without the impact of the world economic crisis. ③ Smart growth is equally applicable in different continents and cities. ④ The selected city will not experience serious natural disasters in the future. We perform empirical research. Based on ten principles of smart growth and existing SG evaluation systems [5-7], we establish a comprehensive assessment system. Then, we integrate Grey Relation Analysis Model [5,6,8] and Maximum Entropy Model



[7,9] in information theory to seek objective weight of index system. Through searching the relevant data from the Internet and data processing, we calculate the comprehensive assessment value of each candidates. Finally, according to the subjective and objective weights and the relevant statistics, we provide the best SG factor in turn.

2. Comprehensive Evaluation System

The aim of the smart growth is to boost the economic progress, promote social justice and improve environment. Considering long range and sustainable planning goals, we develop a comprehensive evaluation system with three parts--economic benefits, ecological benefits and social benefits. The three parts consist of more detailed indexes[10]--P1:Average Annual Growth Rate of GDP; P2:Tax Rate; P3:Employment Rate; P4:Urban Population Growth Rate; P5:Public Transport Rate; P6:Air Quality; P7:Ratio of Land Occupation; P8:Proportion of Renewable Energy. The system structure is described in Figure 1.

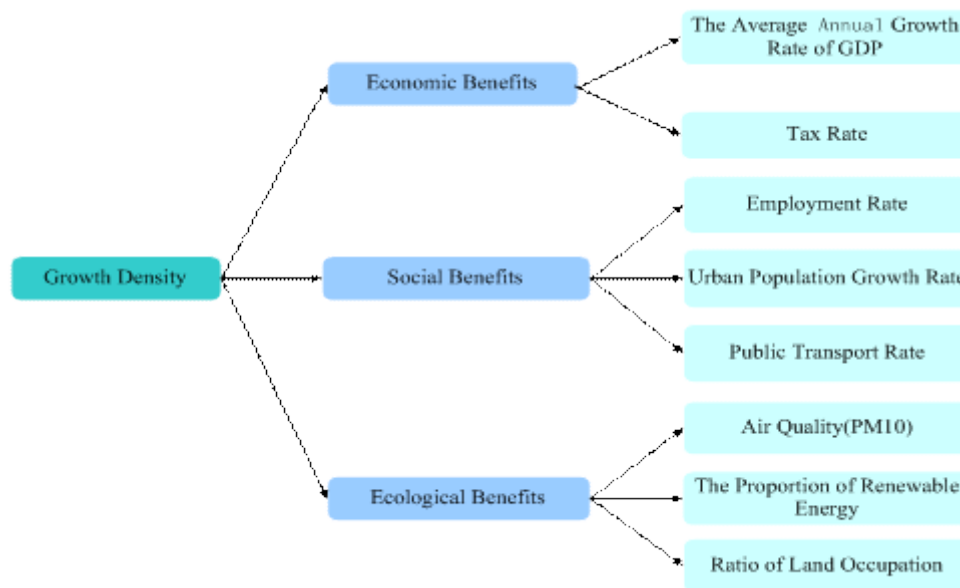


Figure 1: Evaluation System

3. Main Research Methods

We will use the gray relational analysis and maximum entropy model in order to analyze the problem more deeply, dig the data and get the more objective weight value.

3.1. Parameter notification

For readability, we give a description of the mathematical notation as shown in the following table.

3.2. Grey relation analysis based on grey relation depth coefficient

First of all, gray relational analysis is a dynamic analysis process in the process of system development, using gray relational degree as a measure of the degree of correlation between the factors. If the two factors change the trend of consistency, that is, a higher degree of synchronization changes, that is, a higher degree of correlation between the two; on the contrary, the lower.

The maximum entropy model is only known to the knowledge of the model, the unknown things do not make any assumptions, so that the greater the model's entropy, that is, the greater the degree of uncertainty. In information theory, information entropy represents the amount of information, which results in a statistical method that maximizes the use of information. In reality, we always want to retain information and its uncertainty, and in the maximum entropy model, you can infer the possible distribution of information, and can explore the deep information.



Table 1: Parameter Notification

Parameter Definition	Explanation
R	Index matrix
a_{ij}	The j index value of the first I cities in the index matrix
D_j	The value of the j index of urban index vector
r_{ij}	Grey relational coefficient standardization
θ	Grey correlation coefficient
Δ_{\min}	Absolute minimum difference
Δ_{\max}	Absolute maximum difference
Δ_{ij}	The absolute value of jth index difference of ith city
P_j	The weight value of the jth indicator
P_k	The K value of the current growth indicator vector
P'_k	The K - value of the Vector of Smart Growth Index
D	Sum of Least squares

3.3. Metric discriminant matrix

A discriminant matrix of single factors is set up:

$$R = (a_{ij}) = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \quad (1)$$

a_{ij} represents the value of the jth index of the ith city. Because the dimension of the different indicators are different, we need to standardize them.

Then discriminant metric of the normalized single factors obtained:

$$R = (r_{ij}) = \begin{bmatrix} r_{11} & r_{12} & \dots & r_{1n} \\ r_{21} & r_{22} & \dots & r_{2n} \\ \dots & \dots & \dots & \dots \\ r_{m1} & r_{m2} & \dots & r_{mn} \end{bmatrix} \quad (2)$$

Secondly, we calculate the gray relational coefficient (GRC) of this problem. Growth evaluation index vector of the ith city is

$$R_i = \{r_{i1}, r_{i2}, \dots, r_{in}\}, i = 1, 2, \dots, m. \quad (3)$$

The vector of smart growth evaluation metric is

$$R_0 = (r_0(1), r_0(2), \dots, r_0(n)). \quad (4)$$

3.4. Gray relation coefficient

The jth gray correlation coefficient between current growth and smart growth of the ith city is defined as:

$$\theta(R_{ij}, R_0) = \frac{\Delta_{\min} + \rho\Delta_{\max}}{\Delta_{ij} + \rho\Delta_{\max}} \quad (5)$$

where

$$\Delta_{\min} = \min |R_{ij} - R_0|, \quad (6)$$

$$\Delta_i(k) = |R_{ij} - R_0|, \quad (7)$$

$$\Delta_{\max} = \max |R_{ij} - R_0|. \quad (8)$$



The greater the value of ρ is, the less sensitive the gray correlation coefficient is.

The standardized gray correlation coefficient is got

$$\tau_{ij} = \frac{\theta(R_{ij}, R_0)}{\sum_{i=1, j=1}^n \theta(R_{ij}, R_0)}, \quad (9)$$

which is known as the gray correlation coefficient (GRDC).

In order to measure the distribution divergence of the j th index, the following indexes are defined :

$$D(j) = \frac{1}{n} \sum_{j=0}^n \left(\tau_{ij} - \frac{1}{n} \right)^2 \quad (10)$$

According to information theory , the greater the divergence of an index distribution is, the more obvious the personality characteristics of this indicator is.

Define p_j as the weight of the j th index. First, p_j must satisfy:

$$p_j = (\min \tau_{ij}, \max \tau_{ij}) \quad k=1,2,\dots,m \quad (11)$$

The variance of the weight should also be limited and related to the dispersion of the index:

$$\frac{1}{n} \sum_{j=0}^n \left(\tau_{ij} - \frac{1}{n} \right)^2 \subset (\min D(j), \max D(j)) \quad (12)$$

3.5. Max entropy model

Based on the above preparations, we can establish the maximum entropy model of the index weight, which is equivalent to the following optimal programming problem, the optimal solution of this planning problem.

Objective function:

$$H = \sum_{j=1}^m p_j \log \frac{1}{p_j} \quad (13)$$

Restrictions:

$$\sum_j^m p_j = 1 \quad (14)$$

$$p_j \subset (0,1) \quad (15)$$

The objective function H is the entropy of the vector of weight distribution. Under the constraint of condition = 1, it can be proved.

4. Data Processing

According to the characteristics of the population, the geography and the economic, we select two mid-sized cities, i.e. Geneva and Wellington, from two different continents, and collect the data for both cities in recent years. The index data is mainly composed of eight parts, such as P1: Average Annual Growth Rate of GDP, P2:Tax Rate, and so on. Our data is collected from different National Statistics Bureaus [10]. The source of data is very reliable and the information of data is adequate, which can support well the credibility of the following models.

5. Model Specification and Calculation

5.1. Result of weight model

In this section, we get the weight vector including eight evaluation indexes. By combining the gray correlation analysis and the maximum entropy model, we get the

$$\{p_j\}_{j=1}^m \quad (16)$$

as the following weight values.



Table 2: Weight Model

Index	Weight
P1:Average Annual Growth Rate of GDP	0.17
P2:Tax Rate	0.11
P3:Employment Rate	0.12
P4:Urban Population Growth Rate	0.08
P5:Public Transport Rate	0.09
P6:Air Quality	0.10
P7:Ratio of Land Occupation	0.20
P8:Proportion of Renewable Energy	0.13

5.2. Comparative analysis between the two cities

We get the growth rate of Geneva and Wellington's current growth indicators through data processing. We use the weight model of smart growth to calculate the growth rate of each metric of smart growth. Finally, we calculate the matching degree of the both cities by using the least squares method.

We obtain data of Geneva and Wellington in recent years, and calculate the growth rates of the metric of current growth of the two cities, see Table 3.

Table 3: The Current Growth Rate of Geneva and Wellington

City	P1	P2	P3	P4	P5	P6	P7	P8
Geneva	2.42	0.06	-0.92	1.22	2.72	1.59	1.02	1.23
Wellington	2.09	0.03	-0.80	1.26	1.14	1.63	0.79	4.33

Based on the demographic characteristics, economic growth and urban geography of Geneva and Wellington, we utilize the weighting model of smart growth design the smart growth plan of them. The growth rates of the two cities in the smart growth plan are calculated, see Table 4.

Table 4: The Smart Growth Rate of Geneva and Wellington

City	P1	P2	P3	P4	P5	P6	P7	P8
Geneva	2.51	0.02	0.31	0.52	3.02	1.78	0.70	1.39
Wellington	2.31	0.01	0.10	0.76	2.57	1.83	0.58	4.12

We calculate the sum of squares of growth rates by utilizing the *least squares* method:

$$D = \frac{1}{n} \sum_k^n (P_k - P'_k)^2 \tag{17}$$

From the Tables 3 and 4, we find that Geneva meet basically the smart growth, however, the employment rate of Geneva current growth is lower than the growth rate of smart growth. Wellington's current growth does not meet smart growth, its employment rate is relatively is lower, and its proportion of public transport relatively high.

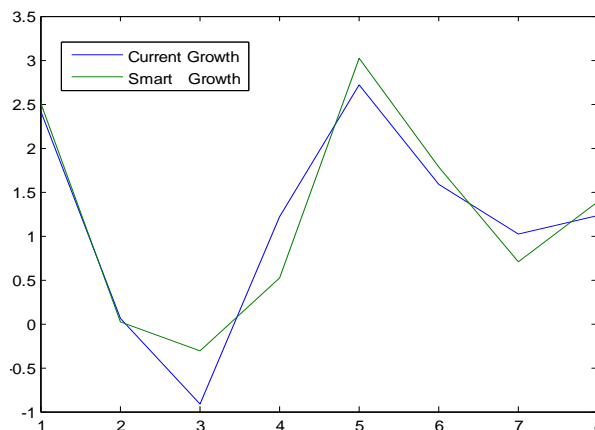


Figure 2: The Fit between Geneva's Current Growth Rates and Smart Growth Rates

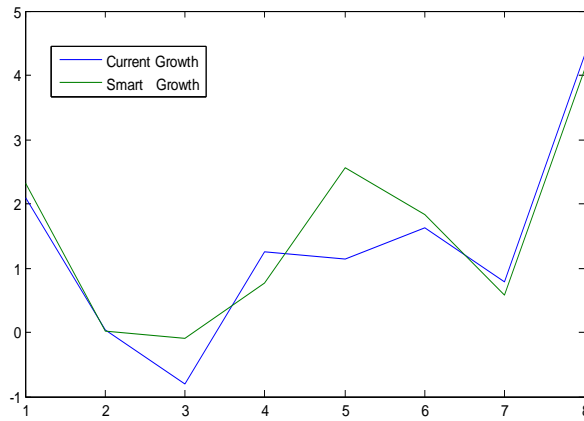


Figure 3: The Fit between Wellington's Current Growth Rates and Smart Growth Rates

The sum of squares of growth rates of Geneva: D1= 1.1259; The sum of squares of growth rates of Wellington: D2=2.9619. Because D1 <D2, the smart growth plan of Geneva is more successful than one of Wellington, see Figures 2 and 3.

5.3. Evaluating success of smart growth plans

Choosing the components and initiatives of smart growth plan based on the geography, expected growth rates, and economic opportunities of selected cities, and based on smart growth principles, we design smart growth plan to develop both cities over the next few decades. Then we evaluate the success of smart growth plans by improving the weight model of smart growth.

Based on the principle of smart growth to develop Geneva and Wellington, the two cities of the smart growth rate of the indicators, see Table 5.

Table 5: Using Smart Growth Principles to Develop both Cities in Next Decade

City	P1	P2	P3	P4	P5	P6	P7	P8
Geneva	2.49	0.04	0.65	1.05	2.91	1.40	1.00	1.43
Wellington	2.40	0.21	0.82	1.06	2.14	1.67	0.49	4.33

We also use equation (17) to computer. The sum of squares of growth rates of Geneva : D3=0.6454; The sum of squares of growth rates of Geneva : D4=1.0592. Because D3 <D1, The smart growth plan of Geneva is successful; Because D4 <D2, The smart growth plan of Wellington is successful.

6. Fuzzy Comprehensive Evaluation

Comprehensive evaluation model is to consider the impact of all factors and get the correct evaluation results. According to the weight value of smart growth metric, A is defined as the list of the metrics:

$$A = \{p_j\} = \{a_1, a_2, \dots, a_n\} \tag{18}$$

Combining the discriminant matrix with the weight value, the comprehensive judgment metric is obtained:

$$B = A * R = (a_1, a_2, \dots, a_n) * \begin{bmatrix} r_{11} & r_{12} & \dots & r_{1n} \\ r_{21} & r_{22} & \dots & r_{2n} \\ \dots & \dots & \dots & \dots \\ r_{m1} & r_{m2} & \dots & r_{mn} \end{bmatrix} = (b_1, b_2, \dots, b_n) \tag{19}$$

Geneva and Wellington's smart growth plan indicator construction discriminant matrix, smart growth weight value construction weight list:

$$A=\{0.17,0.11,0.12,0.08,0.09,0.1,0.2,0.13\};$$

The weighting set is substituted into the formula and the following result is got.

Ranking in Geneva : P7>P6>P4>P5>P8>P1>P3>P2

Ranking in Wellington : P7>P5>P8>P4>P1>P3>P2>P6



Table 6: Fuzzy Evaluation of Geneva and Wellington

City	P1	P2	P3	P4	P5	P6	P7	P8
Geneva	0.2343	0.1113	0.1283	0.5432	0.5428	0.6532	0.8473	0.3448
Wellington	0.5267	0.1523	0.1934	0.5322	0.6392	0.1031	0.7043	0.5389

From Table 6, results show that the most potential metric of the two cities is the proportion of land occupation, the lowest potential metric of Geneva is tax rate, and the least potential metric of Wellington is air quality.

7. Conclusion

We have constructed a SG evaluation metric by developing a comprehensive evaluation system. We conducted the case study on two cities of Geneva and Wellington. By utilizing a combined GRA and ME model, we obtained the weights of our proposed SG evaluation metric. Based on the weights, we calculated the correlation degree between current growth and SG by using the sum of least squares. We also designed a growth plan for both cities, and found that both metrics of the two cities have improved. Finally, we applied FCE method to rank the individual initiatives within our plan. The ratio of land occupation of both cities is the most potential. The tax rate of Geneva is the least potential, air quality of Wellington as well. Our work contributes to urban planning and may help promote sustainable urban development.

In this paper, the two cities are medium-sized cities, not strong representative. The comprehensive evaluation system does not consider the condition that the development of science and technology has great effect on economic. We leave these for further work.

Acknowledgments

This work was supported by University Students' Innovative Training Program of SUES under Grant No. cx1721004.

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