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## Dynamic analysis and optimum route design for snowboarding

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**Abstract** Based on the dynamic analysis, the theoretical and numerical results of snowboarding have been given with the help of theorem kinetic energy. Proper factors are considered in this paper to discuss the influence on snowboarding. It is noted that the mainly effective factors are the initial entry angle and the initial entry velocity. Secondly, it is found that the radius of the U-shaped pool is another effective factor. When the radius becomes larger, the vertical height and the velocity will increase. Another important conclusion in the paper is that the initial entry angle should have a proper range. The snowboarder will crash when this angle is too large, and lose the high speed when it is small. Furthermore, it is noted that the range of the initial angle depends on the radius of the snowboarding pool and the initial entry velocity. Finally, it is suggested the preferred snowboarding route should be approximate "Z" type.

**Keywords** Snowboarding, Dynamic analysis, Optimum route, Velocity

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### Introduction

The sport of snowboarding is becoming popular around the world. The people are enjoyed by the gorgeous actions in snowboarding. Snowboarding pool is shaped like a long half-pipe, made of large amounts of snow shaped into the preferred profile using specially-designed snow groomers. Most people call the pool as U-shaped pool. Players slide side by side in the pool to do a lot of spins and jumps. It is difficult to ski in the U-shaped pool, since the athletes must access high speed to do the spins and jumps. More and more athletes and scientists have given attention to study how to get the maximum twist and vertical height to get the high scores. Williams *et al.* (2007) [1] studied the speeds associated with skiing and snowboarding to reduce the incidence. Subic, Clifton, Kovacs, *et al.* (2010) [2] have presented an original dynamic experimental to evaluate the on-snow performance. Yoneyama, Kitade, Osada *et al.* (2010) [3] have investigated on the ski-snow interaction in a carved turn. Ye and Zhao (2008) [4] have discussed the psychology in snowboarding, by using the methods of investigation and mathematical statistics. Xiao and Gao (2009) [5] have researched of the technical characteristics of halfpipe snowboarding. Shealy, Ettlinger, Johnson *et al.* (2005) [6] measured a numbers of snowboarders at three different ski resorts, and they give the average speed for snowboarders at 38.9 km/h. And plenty of papers have been focused on the injury caused by snowboarding [7-9].

**Table 1:** The maximum vertical height of the skilled snowboarders

Name	Country	Gender	Height (m)
Shaun Whit	America	Male	5.0
Daniel Kas	America	male	4.6
Markku Kosk	Finland	male	4.5
Wang-Cheng Shi	China	male	4.0
Cheng Xu	China	female	3.2



The basic technique of snowboarding is vertical height, which is the maximum vertical distance above the platform of the U-shaped pool. This distance is most important factor to apply to the spins and twists in the air. Tab.1 [5] shows the vertical height of various snowboarders from various countries. It is amazed that the vertical height is about 5 m in height.

The athletes who want to get the high performance in snowboarding will face the problems that, which factor has the most significant effect to the snowboarding, what is the best route of the snowboarding for the large vertical height, how to get the maximum twist in the snowboarding, and which is the preferred course for the snowboarders. Furthermore, to maximize the production of vertical height by a skilled snowboarder, and tailor shape to optimize other possible requirements, such as the maximum twist, it is necessary to determine the main factors, such as the initial entry angle, and the entry speed, or the shape of a snowboard route.

To the authors' knowledge, most papers focus on what the maxim speed is, or how the injuries in snowboarding are. It is then our purpose to discuss the problems we have stated before with the method of dynamic analysis to help the athletes and coaches strive for the peak performance in snowboarding. We have built a dynamic model of snowboarding and derived out the relations of many factors in the sports in Section 2. Then, the numerical results are given to reveal the effects of the factors for snowboarding in Section 3. Finally, the conclusions have been achieved as the summary of our research in Section 4.

#### The method of dynamic analysis and the basic equations

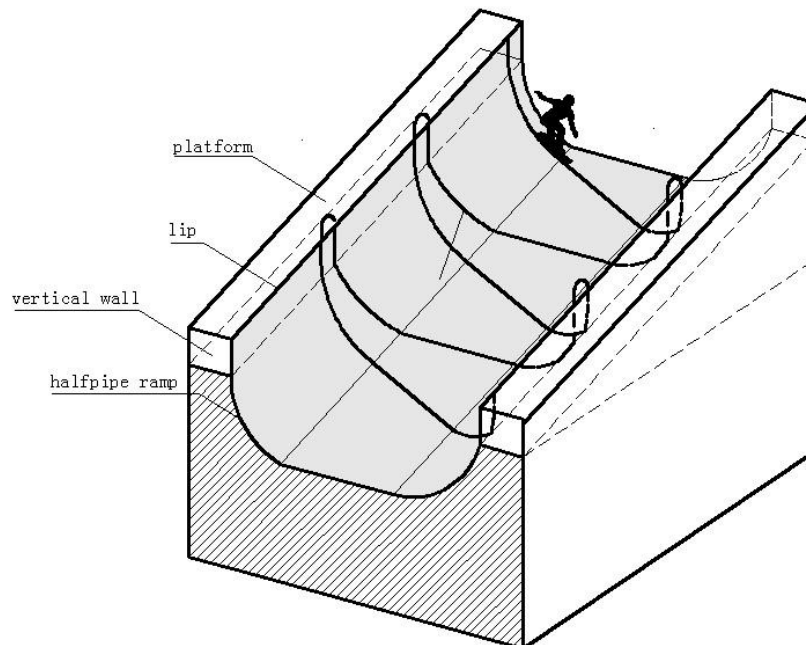


Figure 1: The possible snowboard route in U-shaped pool

In the snowboarding, the snowboarders slide side by side in the pool to do several technical actions, after entering the U-shaped pool from platform with a certain initial velocity (shown in Fig.2). For the high qualities of the performance, they should slide to get the high speed which they can control. Furthermore, the snowboarders will try their best to do the maximum twists and large vertical height, and avoid falling down in the procedure.

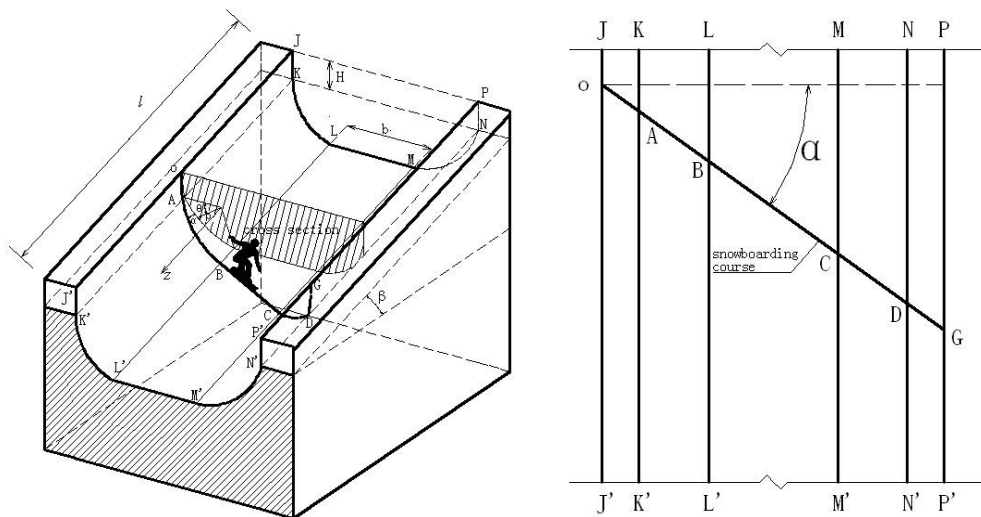
Table 2: The nomenclature

$m$	the mass of athlete and snowboard
$g$	the gravity acceleration
$\mu$	the coefficient of friction



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- $\rho$  the radius of the halfpipe
  - $\theta$  the angle of local cylinder coordinate
  - $z$  the axis of local cylinder coordinate
  - $h$  the height of the vertical wall of U-shaped pool
  - $l$  the length of U- shaped pool
  - $b$  the width of the flat
  - $\beta$  the slope angle of U-shaped pool
  - $\alpha$  the initial entry angle
  - $s$  the length of the snowboard course
  - $H$  the vertical air
  - $v$  the velocity of athlete and snowboard
  - $v_o$  the initial velocity of entering the U-shaped pool
  - $v_B$  the velocity at point B
  - $v_C$  the velocity at point C
  - $v_D$  the velocity at point D
  - $v_G$  the velocity at point G
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To discuss the problem, we have given the nomenclature used for the dynamic analysis in Tab.2 firstly.



(a) Stereographic projection (b) Unfold surface of the U-shaped pool

Figure 2: The model of snowboarding for the dynamic analysis

Based on the analysis of the procedure of the snowboarding, we have made several simplifications and assumptions that, the body sizes and the body motions of the athletes are not taken into consideration, since we only study the external factors for the snowboarding. In addition, we also assume that the route  $s$  in the unfold view (see Fig.2b) is a straight line with the initial entry angle  $\alpha$ . we have the relationship  $z = \rho\theta \tan \alpha$  of the coordinate  $\rho, \theta, z$  (shown in Fig.2a).

The most powerful tool to solve the problem is the theorem of kinetic energy [10]. When snowboard slide from the platform with the initial speed  $v_o$  and the entry angle  $\alpha$  is known, the speed  $v$  of the snowboarder at any point on the route  $s$  can be determined without any difficulties.

On the route OA, the gravity gives the work of  $W_G$  as

$$W_G = mgs \cos \alpha \quad (1)$$

We emphasize that there is no friction work, for pressure between the snowboard and the vertical wall is zero. Therefore, we have

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_o^2 + mgs \cos \alpha \quad (2)$$

On the route of AB, the work of gravity reads

$$W_G = mg(\rho \sin \theta + z \sin \beta) + mgh \quad (3)$$

For the work  $W_f$  of friction on AB,

$$W_f = -\int_A^B \mu F_N ds = -\int_A^B \mu mg \sin \theta \cos \beta ds \quad (4)$$

where  $F_N$  is the normal force of the halfpipe wall. It is noted that, we neglect the inertia force for the normal force  $F_N$ , since that the friction force is not the main factor which will affect the maximum velocity of the snowboard. It is obviously that

$$ds = \sqrt{d_z^2 + (\rho d\theta)^2} = \sqrt{(\rho \tan \alpha d\theta)^2 + (\rho d\theta)^2} = \sqrt{\rho^2 (1 + \tan^2 \alpha)} d\theta \quad (5)$$

Therefore, the  $W_f$  on the course AB is

$$W_f = -\int_0^\theta mg \sin \theta \cos \beta \mu \frac{\rho}{\cos \alpha} d\theta = \frac{\rho mg \mu \cos \beta}{\cos \alpha} \cos \theta \Big|_0^\theta \quad (6)$$

With the help of Eqs.(4), (5) and (6), we have

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_o^2 + mg(\rho \sin \theta + \rho \theta \tan \alpha \sin \theta + h) + \frac{\rho mg \mu \cos \theta}{\cos \alpha} \cos \theta \Big|_0^\theta \quad (7)$$

Therefore, the velocity  $v$  can be given as

$$v = \left[ v_o^2 + 2g(\rho \sin \theta + \rho \theta \tan \alpha \sin \theta + h) + \frac{2\rho g \mu \cos \beta}{\cos \alpha} (\cos \theta - 1) \right]^{\frac{1}{2}} \quad (8)$$

For the snowboarder on the route of BC (flat), the work of gravity  $W_G$  is

$$W_G = mgs \sin \beta \sin \alpha \quad (9)$$

The work of friction  $W_f$  is

$$W_f = -\mu mgs \cos \beta \quad (10)$$

Consequently, with the help of the theorem of the energy, we have



$$\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 + mg(\rho \sin \theta + \rho \theta \tan \alpha \sin \theta + h) \left| \begin{matrix} \theta = \frac{\pi}{2} + \frac{\rho \mu mg \cos \beta}{\cos \alpha} \cos \theta \\ \theta = 0 \end{matrix} \right|_{\theta = \frac{\pi}{2}}^{\theta = 0} + mgs \sin \alpha \sin \beta - \mu mgs \cos \beta \quad (11)$$

and

$$v = \left[ v_0^2 + 2g\left[\left(\rho + \frac{\pi}{2} \rho \tan \alpha + h\right) + s(\sin \alpha \sin \beta - \mu \cos \beta)\right] - \frac{\rho \mu g \cos \beta}{\cos \alpha} \right]^{\frac{1}{2}} \quad (12)$$

Similarly, for the velocity at the point on the course of CD, we have

$$v = \left[ v_0^2 + 2g\left[\rho \tan \alpha \left(\frac{\pi}{2} - 1 - \theta \cos \theta + \sin \theta\right) + b \tan \alpha \sin \beta + \rho \sin \theta + h - \mu \frac{\cos \beta}{\cos \alpha} (\rho + b - \rho \cos \theta)\right] \right]^{\frac{1}{2}} \quad (13)$$

Furthermore, we have give the speed at the point G as

$$v_G = \left[ v_0^2 + 2g\left[\left(\frac{3\pi}{2} - 1\right)\rho \tan \alpha - \frac{\mu \cos \beta}{\cos \alpha} (2\rho + b) + b \tan \alpha \tan \beta\right] \right]^{\frac{1}{2}} \quad (14)$$

With the help of energy consideration, the vertical height H can be easily calculated out

$$H = \frac{1}{2g} v_0^2 + \left[\left(\frac{3\pi}{2} - 1\right)\rho \tan \alpha - \frac{\mu \cos \beta}{\cos \alpha} (2\rho + b) + b \tan \alpha \tan \beta\right] \quad (15)$$

**The numerical experiments**

Only theoretical analytical solution is not enough to show the whole aspect of snowboarding. The numerical calculations are given in this section.

For the standard U-shaped pool [11], we have the length  $100m \leq l \leq 140m$ , the radius of halfpipe of flat  $2.7m \leq b \leq 4.2m$ , the width of flat  $5.5m \leq b \leq 12.5m$  and the average slope angle  $14^\circ \leq \beta \leq 18^\circ$ .

The main constant of the dimension of the U-shaped pool we use are  $l = 120$ ,  $\rho = 3.2$ ,  $b = 8.5$ ,  $h = 0.3$  and  $\beta = 17^\circ$ , which is just the size of the one used in Olympic Winter Games 2010.

There are many factors that influence snowboarding, such as initial entry angle  $\alpha$ , coefficient of friction  $\mu$  and slope angle  $\beta$ . Firstly, we have given some numerical calculating to show which factors have the more significant effect to the snowboarding.

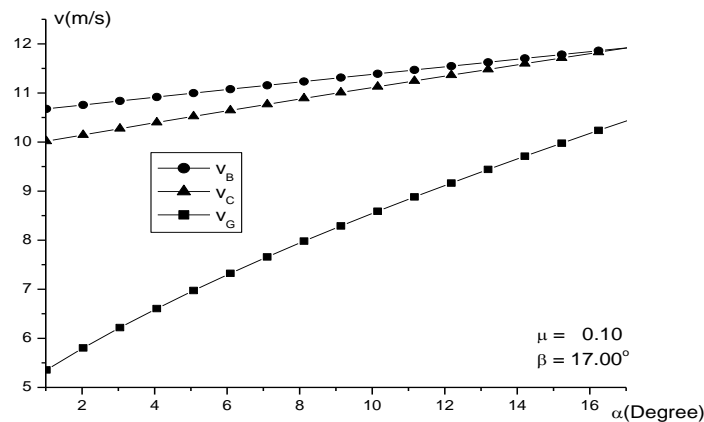


Figure 3: The velocities on the special point B, C and G, according to the angle  $\alpha$

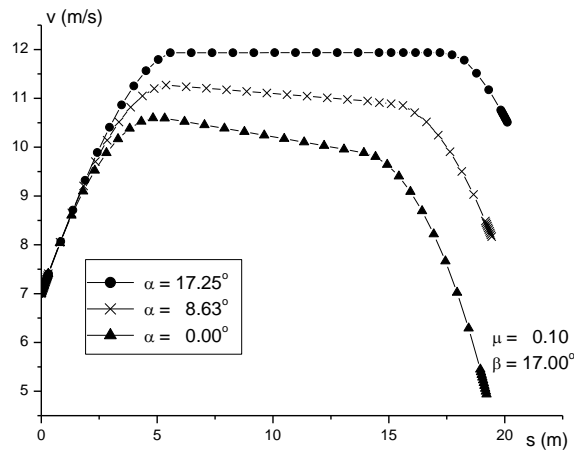


Figure 4: The velocity along the course  $S$  from  $O$  to  $G$  with different initial angle  $\alpha$

From Figures 3 and 4, the velocity raise quickly when the angle  $\alpha$  increases. We can see that the initial entry angle  $\alpha$  has the significant effect on the speed of snowboarding. Another conclusion here is that maximum speed of the snowboard is at the point B for the fixed angle  $\alpha$ .

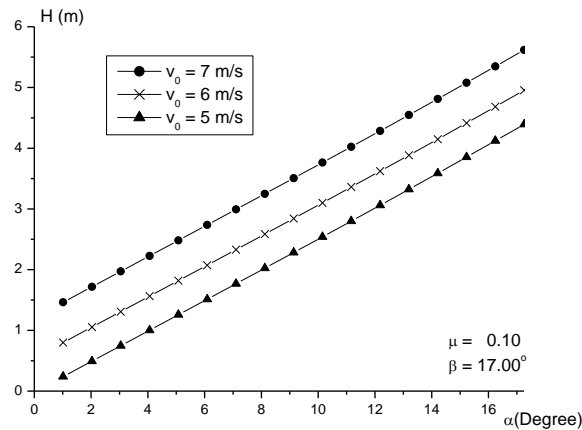


Figure 5: The vertical height  $H$  according to the angle  $\alpha$  with different initial speed  $v_0$

In Fig.5, we have given the relationship between the vertical height  $H$  and the main factor  $\alpha$ . One can easily concluded that the vertical air  $H$  increases rapidly when the initial entry angle  $\alpha$ . Furthermore, it is also shown that higher initial velocity  $v_0$  has the larger effects on the height and the velocity in snowboarding.

In order to maximize the vertical air  $H$ , the snowboarder should enlarge the initial entry angle  $\alpha$  and increase the initial speed  $v_0$ .

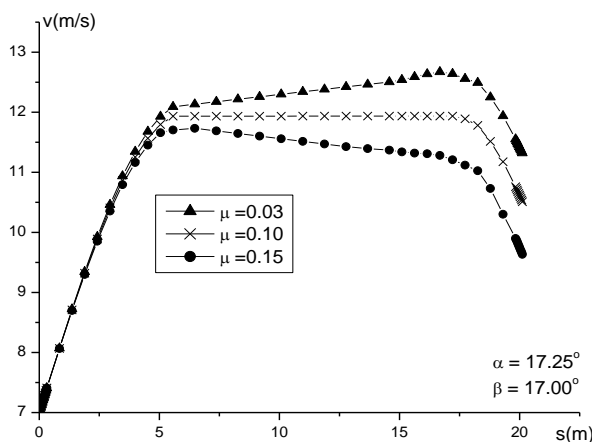


Figure 6: The velocities along the course  $S$  from  $O$  to  $G$  with different  $\mu$

It is emphasized that the initial entry angle must be not larger than a certain degree, since the maximum speed  $v$  is about 12 m/s [6]. If the maximum speed is too high, the snowboarders could not control, and would fall down.

In order to reveal the effect of the friction coefficient  $\mu$ , the numerical example is given in Fig.6. It is shown that velocity at any point on the snowboard course with different the coefficient  $\mu$ . It is obvious that the friction has smaller effect on the speed than that of initial angle  $\alpha$ .

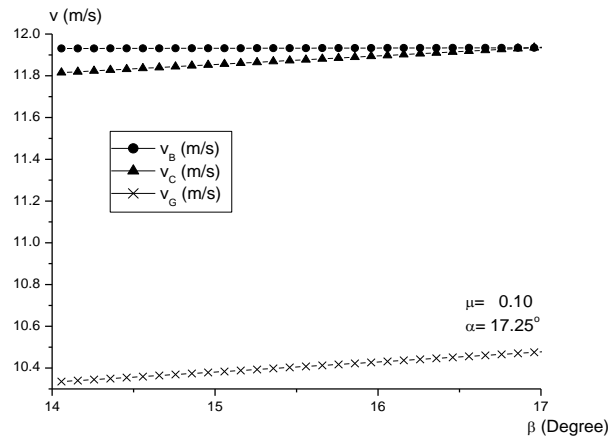


Figure 7: The velocities on the special point B, C and G, according to the angle  $\beta$

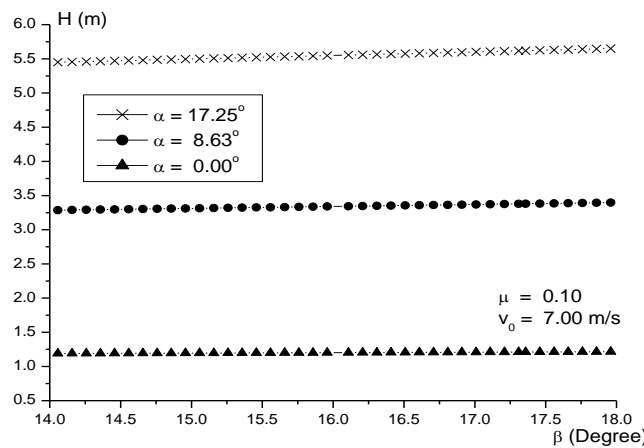


Figure 8: The vertical air  $H$  according to  $\beta$  with different angle  $\alpha$

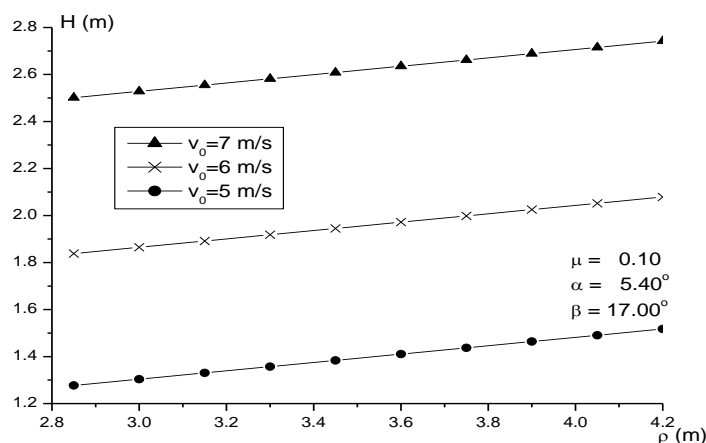


Figure 9: The vertical air  $H$  according to the halfpipe radius  $\rho$  with the different speed  $v_0$

In Figures 7 and 8, we can see that the slope angle  $\beta$  has little effect to the velocity  $v$  and the vertical air  $H$ .

Therefore, the coefficient of friction  $\mu$  (0.03~0.1)[5] and the slope angle  $\beta$  ( $14^\circ \sim 18^\circ$ ) [6] has little effect to the snowboarding. Consequently, we can get a clear conclusion that the main factors of snowboarding are the initial entry angle  $\alpha$  and the initial velocity  $v_o$ .

Another factor induced the vertical height  $H$  is the size of the U-shaped pool. A numerical example is shown in Fig.9. It is obvious that, the U-shaped pool of maximum radius  $\rho$  has little effect on the vertical height  $H$ . But, we emphasis that the initial entry angle  $\alpha$  is related to this radius, which will be discussed in the next.

One will doubt that whether there is a maximum angle  $\alpha_{max}$  and minimum angle  $\alpha_{min}$  for the initial angle  $\alpha$ . The answer of this question is sure. Generally speaking, the large entry angle  $\alpha$  causes the high velocity  $v$  and vertical air  $H$ . But the snowboard can only control at a limited velocity  $v_{max}$ . If the velocity is larger than  $v_{max}$ , the snowboarder will fall down. On the contrary, if the entry angle is smaller, the snowboarder will lose the speed when he or she slides in the U-shaped pool.

The factors which influence the range from  $\alpha_{max}$  to  $\alpha_{min}$  are mainly the initial entry speed  $v_o$  and the halfpipe radius  $\rho$ . For simplicity, we only give the range of the angle  $\alpha_{max}$  and  $\alpha_{min}$  according to the radius  $\rho$ .

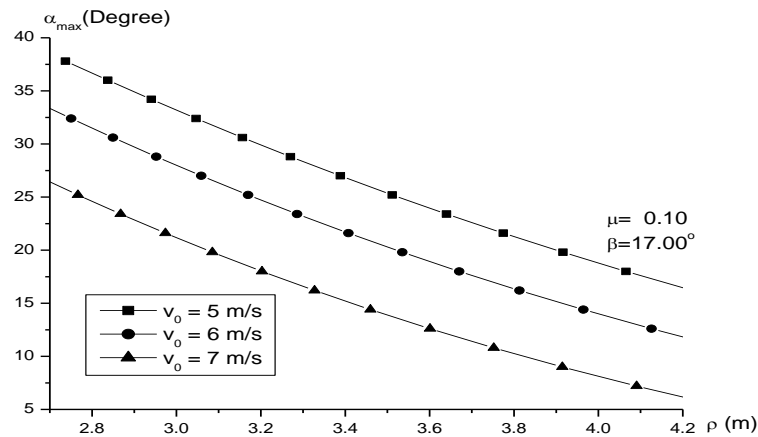


Figure 10: The maximum initial angle  $\alpha_{max}$  according to the halfpipe radius  $\rho$  with the different initial speed  $v_o$

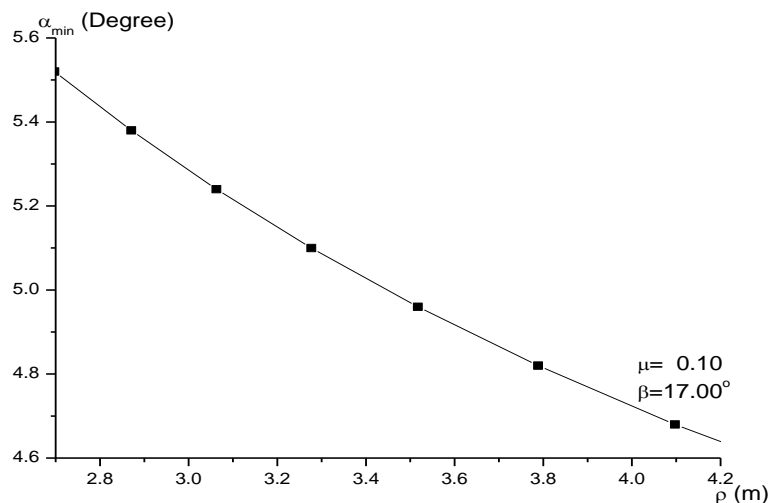


Figure 11: The minimum initial angle  $\alpha_{min}$  according to the halfpipe radius  $\rho$

From Figures 10 and 11, we can see that the angle range  $\alpha_{max}$  or  $\alpha_{min}$  decrease when  $\rho$  increase. And the maximum angle  $\alpha_{max}$  should be reduced when the initial velocity increase.



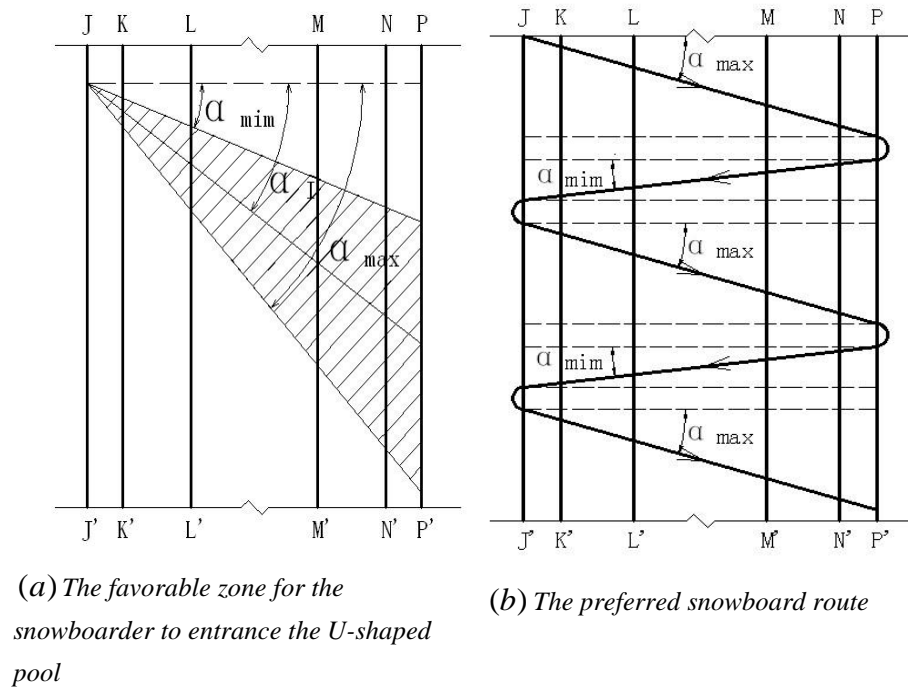


Figure 12: The optimum route for snowboarding

Therefore, the suggestions for the best quality performance of snowboarding are that,

- The authors should enter the U-shaped pool in the favorable zone with the initial angle about 3~4 Degrees (shown in Fig.12a);
- The preferred snowboard course is approximate to “Z” shape (shown in Fig.12b).

### Conclusion Remarks

Based on the theorem of kinetic energy, we have given the theoretical and numerical analysis of snowboarding. Some clear conclusions are achieved correctly. Many factors considered in our paper will influence snowboarding. We find that the mainly effective factors are the initial entry angle  $\alpha$  and the initial entry speed  $v_o$ . When  $\alpha$  and  $v_o$  increase, the possible requirements, such as vertical air  $H$  and the maximum twist will get higher quality performance.

Another important result of ours is that the angle  $\alpha$  should be in a proper range of  $[\alpha_{\min}, \alpha_{\max}]$ . In addition, it noted that the range of  $\alpha$  depends on the initial entry angle  $\alpha$  and the radius  $\rho$  of the half pipe. The snowboarder will crash when  $\alpha$  is too large, or lose the high speed, when  $\alpha$  is too small. All of our results of the problem are proper correct, and high coincident with the actual conditions. The preferred snowboard route is approximate “Z” type.

The conclusions achieved in this paper are helpful to the athletes and coaches of snowboarding.

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