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A Box-Jenkins Method Based Subset Simulating Model for Daily Ugx-Ngn Exchange Rates

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Abstract A 149-point realization of daily exchange rates of the Uganda shilling (UGX) – Nigerian naira (NGN) from 4th October, 2014 to 1st March, 2015, is analyzed by Box-Jenkins methods. By a new fitting algorithm, it is concluded that the time series follows the subset SARIMA $(1,1,0)x(1,1,0)_7$ model. Daily exchange rates between the two currencies may be simulated or forecasted by the model.

Keywords Ugandan shilling, Nigerian naira, foreign exchange rates, SARIMA models

Introduction

Foreign exchange is a major issue in the discussion of world economy. Any trade relationship between the country Uganda and the country Nigeria is based on the relative value of the Uganda Shilling (UGX) and the Nigerian Naira (NGN). In this write-up the daily exchange rates shall be modelled by Box-Jenkins methods. The particular approach shall be the seasonal autoregressive integrated moving average (SARIMA) approach proposed by Box *et al.* [1].

In recent times, many authors have adopted the SARIMA modelling approach to model real life data. Jianfeng (2013) noticed that SARIMA modelling results in closer forecasts to the real data than dynamic linear modelling in forecasting monthly cases of mumps in Hong Kong. He fitted a SARIMA$(2,1,1)x(1,1,1)_{12}$ model to the time series [2] . Li et al. (2013) modelled monthly outpatient numbers in China by a SARIMA$(0,1,1)x(0,1,1)_{12}$ [3]. Kibunja et al. (2014) forecasted monthly precipitation in Mount Kenya region using a SARIMA$(1,0,1)x(1,0,0)_{12}$ model [4]. Valipour (2015) observed that SARIMA modelling outdid its non-linear counterpart ARIMA in long-term runoff forecasting [5]. Hassan and Mohamed (2015) found that a SARIMA$(0,0,5)x(1,0,1)_{12}$ was the most adequate in the simulation of monthly rainfall drought in the Gadaref region of Sudan [6]. Gikungu et al. (2015) fitted a SARIMA$(0,1,0)x(0,0,1)_4$ to quarterly Kenyan inflation rates [7].

The purpose of this write-up is to fit a model to the daily exchange rates of Ugandan shilling (UGX) and Nigerian Naira (NGN). Because of the observed seasonal nature of the series, a SARIMA approach is adopted.

Material and Methods

Data

The data for this work are 149 values of daily UGX / NGN exchange rates of October 4, 2014 through March 1, 2015. They were obtained from the website www.exchangerates.org/UGX-NGN-exchange-rate-history.html accessed on March 2, 2015. These numbers are interpreted as the quantities of NGN per UGX.

Seasonal Autoregressive Integrated Moving Average (SARIMA) Models

The definition of a SARIMA model as proposed by Box *et al.* [1] is as follows:

A stationary time series $\{X_t\}$ is said to follow a multiplicative seasonal autoregressive integrated moving average model of order p, d, q, P, D, Q, s designated SARIMA$(p,d,q)x(P,D,Q)_s$ if

$$A(L)\Phi(L^s)\,\nabla^d\nabla_s^D X_t = B(L)\Theta(L^s)\varepsilon_t \quad (1)$$

where A(L) is a p-order polynomial in L and is called the autoregressive (AR) operator; B(L) is a q-order polynomial in L and is called the moving average (MA) operator; Φ(L) is a P-order polynomial in L called the seasonal AR operator; Θ(L) is a Q-order polynomial in L called the seasonal MA operator. The numbers d and D are the non-seasonal and the seasonal differencing orders respectively. L is the backward shift operator defined by $L^kX_t = X_{t-k}$. the number s is the period of the seasonality of the time series. ∇ and ∇_s are the non-seasonal and the seasonal differencing operators respectively. $\{\varepsilon_t\}$ is a white noise process.

SARIMA Modelling

Generally the model (1) is estimated beginning with the determination of the orders: p, d, q, P, D, Q and s. The AR orders p and P are estimated by the non-seasonal and seasonal cut-off lags of the partial autocorrelation function, respectively. Similarly the MA orders q and Q are estimated by the non-seasonal and the seasonal cut-off lags of the autocorrelation function respectively. The seasonal period often suggests itself by the known nature of the series. Otherwise it may be suggestive by the correlogram or an analytical inspection of the series. The differencing orders d and D are such that they sum up to 2 at most.

In this work the subset SARIMA modelling algorithm proposed by Etuk and Ojekudo (2015) shall be used [8]. It is the autoregressive-moving-average-duality-based version of the algorithm of Suhartono [9].

Suhartono's algorithm is as follows:

Fit to $\{X_t\}$ the following SARIMA$(0,0,1)x(0,0,1)_s$ model

$$X_t = \varepsilon_t + \beta_1\varepsilon_{t-1} + \ldots + \beta_s\varepsilon_{t-s} + \beta_{s+1}\varepsilon_{t-s-1} \quad (2)$$

If $\beta_{s+1} = 0$ then the model is said to be additive. Otherwise if $\beta_{s+1} = \beta_1\beta_s$, then the model is said to be multiplicative. Otherwise it is said to be subset.

Etuk and Ojekudo's algorithm which is the dual version of (2) is as follows:

Fit to $\{X_t\}$ the following SARIMA$(1,0,0)x(1,0,0)_s$ model

$$X_t + \alpha_1X_{t-1} + \cdots + \alpha_sX_{t-s} + \alpha_{s+1}X_{t-s-1} \quad (3)$$

If $\alpha_{s+1} = 0$ the model is said to be additive. If not, if $\alpha_{s+1} = \alpha_1\alpha_s$, the model is said to be multiplicative. Otherwise it is said to be subset. Additivity is ascertained if

$\widehat{\alpha_{s+1}} < 2SE(\widehat{\alpha_{s+1}})$

where SE(.) is the standard error of and ^ denotes the estimate of Multiplicativity is ascertained if

$T = (\widehat{\alpha_{s+1}} - \widehat{\alpha_s}\,\widehat{\alpha_1})/SE(\widehat{\alpha_{s+1}})$

is not statistically significant where T is t-distributed.

Estimation of the model parameters is done via a non-linear optimization process for the mixed ARMA process. The Eviews software which uses the least squares technique is to be used for this work.

Results and Discussion

The time-plot of Figure 1 shows a generally positive trend depicting relative depreciation of the Naira within the time period of interest. A seven-day differencing yields a series with the time-plot of Figure 2 which depicts a generally horizontal trend and a correlogram of figure 3 showing a seasonal nature of period 7 days. A further non-seasonal differencing yields a series with the plot of Figure 4 and the correlogram of Figure 5. Evident is a stationary nature and a correlation structure suggestive of a SARIMA$(1,1,0)x(1,1,0)_7$. Applying the algorithm of Etuk and Ojekudo (3) becomes naturally suggestive [8].

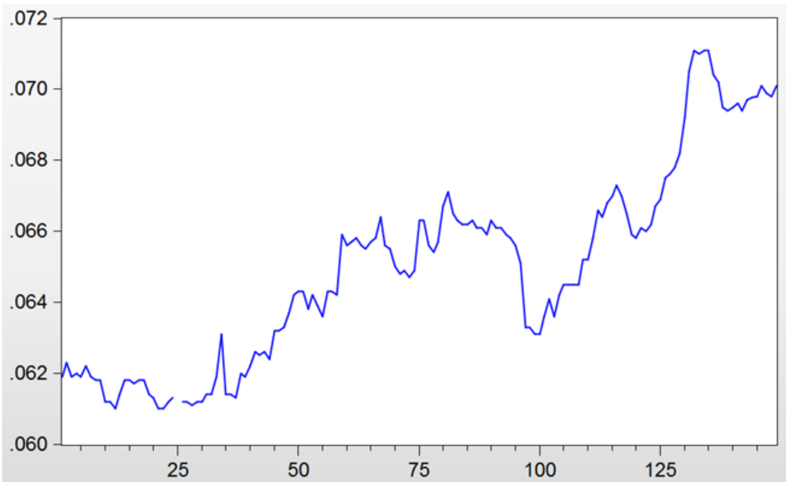


Figure 1: Time Plot of Exchange Rates

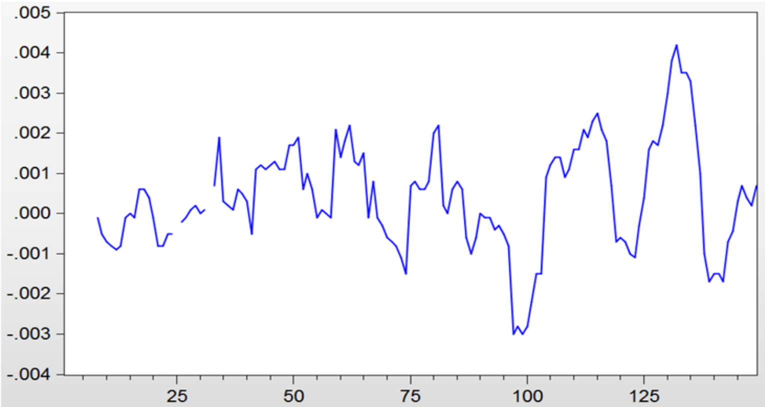
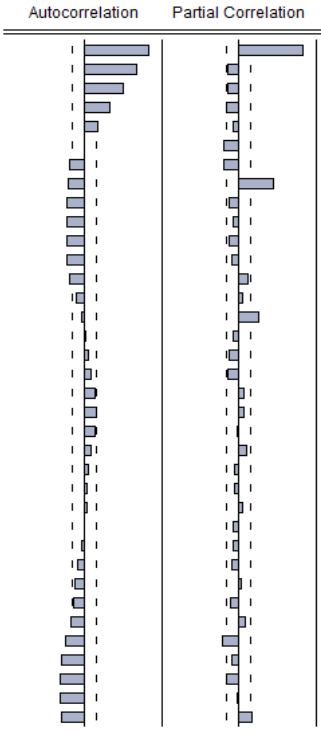


Figure 2: Time Plot of the seasonal differences



	AC	PAC	Q-Stat	Prob
1	0.860	0.860	105.76	0.000
2	0.703	-0.142	176.86	0.000
3	0.530	-0.147	217.66	0.000
4	0.345	-0.159	235.09	0.000
5	0.174	-0.076	239.55	0.000
6	-0.008	-0.193	239.56	0.000
7	-0.191	-0.189	245.04	0.000
8	-0.218	0.466	252.18	0.000
9	-0.237	-0.127	260.72	0.000
10	-0.236	-0.080	269.24	0.000
11	-0.233	-0.130	277.58	0.000
12	-0.241	-0.094	286.60	0.000
13	-0.191	0.131	292.32	0.000
14	-0.108	0.041	294.18	0.000
15	-0.045	0.276	294.50	0.000
16	0.023	-0.081	294.58	0.000
17	0.064	-0.120	295.24	0.000
18	0.096	-0.145	296.75	0.000
19	0.154	0.071	300.64	0.000
20	0.163	0.074	305.03	0.000
21	0.138	-0.022	308.20	0.000
22	0.097	0.101	309.80	0.000
23	0.055	-0.058	310.31	0.000
24	0.035	-0.053	310.52	0.000
25	0.038	0.053	310.77	0.000
26	-0.003	-0.072	310.77	0.000
27	-0.049	-0.077	311.19	0.000
28	-0.091	-0.093	312.67	0.000
29	-0.121	0.033	315.28	0.000
30	-0.154	-0.107	319.58	0.000
31	-0.186	0.087	325.86	0.000
32	-0.260	-0.206	338.34	0.000
33	-0.300	-0.089	355.02	0.000
34	-0.325	-0.167	374.85	0.000
35	-0.327	-0.021	395.12	0.000
36	-0.312	0.185	413.69	0.000

Figure 3: Correlogram of the seasonal differences

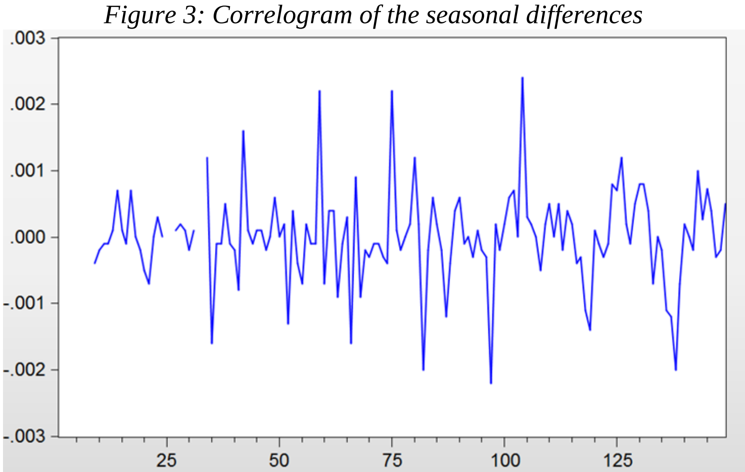
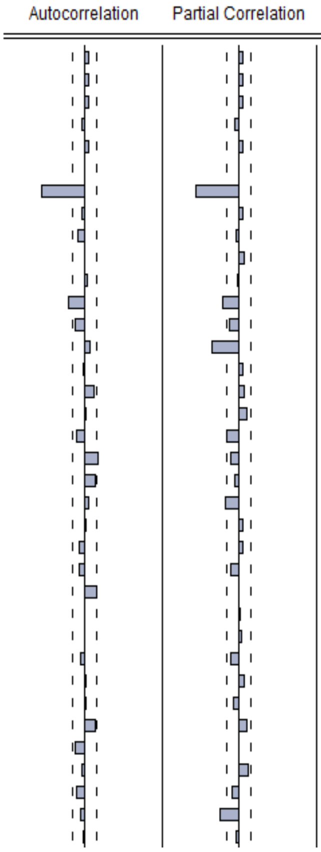


Figure 4: Time Plot of Difference of the seasonal Differences

Applying algorithm (3) yields as estimated in Table 1 the SARIMA(1,1,0)x(1,1,0)$_7$ model

$$X_t = 0.1616X_{t-1} - 0.5983X_{t-7} + 0.1802X_{t-8} + \varepsilon_t \quad (4)$$
$$(\pm 0.0932) \quad (\pm 0.0740) \quad (\pm 0.0920)$$

Clearly the model is neither additive nor multiplicative. It is subset. From Figure 6, by the Jarque-Bera test, the residuals may be said to follow a normal distribution at 1% level of significance. This implies that the model may be considered as adequate.



	AC	PAC	Q-Stat	Prob
1	0.064	0.064	0.5813	0.446
2	0.051	0.047	0.9430	0.624
3	0.053	0.048	1.3498	0.717
4	-0.044	-0.053	1.6266	0.804
5	0.045	0.047	1.9243	0.860
6	0.007	0.004	1.9321	0.926
7	-0.570	-0.578	49.483	0.000
8	-0.026	0.061	49.586	0.000
9	-0.097	-0.042	50.986	0.000
10	0.002	0.076	50.987	0.000
11	0.032	-0.017	51.145	0.000
12	-0.208	-0.217	57.731	0.000
13	-0.135	-0.133	60.542	0.000
14	0.066	-0.353	61.225	0.000
15	-0.017	0.055	61.268	0.000
16	0.118	0.071	63.458	0.000
17	0.010	0.114	63.473	0.000
18	-0.101	-0.163	65.118	0.000
19	0.186	-0.106	70.690	0.000
20	0.133	-0.056	73.558	0.000
21	0.057	-0.180	74.089	0.000
22	0.016	0.048	74.132	0.000
23	-0.074	0.059	75.036	0.000
24	-0.082	-0.105	76.157	0.000
25	0.172	-0.002	81.170	0.000
26	0.002	0.020	81.171	0.000
27	-0.006	0.039	81.178	0.000
28	-0.060	-0.101	81.803	0.000
29	0.010	0.073	81.822	0.000
30	0.010	-0.066	81.839	0.000
31	0.152	0.113	86.013	0.000
32	-0.137	-0.003	89.412	0.000
33	-0.031	0.115	89.588	0.000
34	-0.108	-0.083	91.738	0.000
35	-0.050	-0.262	92.213	0.000
36	-0.011	-0.035	92.235	0.000

Figure 5: Correlogram of Difference of the Seasonal Differences

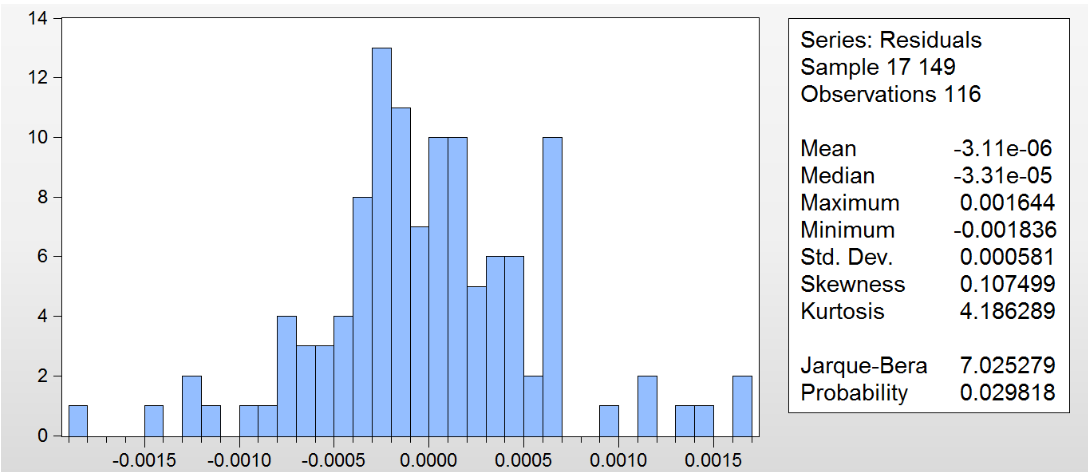


Figure 6: Histogram of The Sarima(1,1,0)X(1,1,0)$_{12}$ Residuals

Table 1: Estimation of the Sarima(1,1,0)X(1,1,0)$_7$ Model

Method: Least Squares
Date: 10/03/15 Time: 16:09
Sample (adjusted): 17 149
Included observations: 116 after adjustments
Convergence achieved after 3 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.161555	0.093165	1.734076	0.0856
AR(7)	-0.598328	0.074033	-8.081868	0.0000
AR(8)	0.108211	0.092019	1.175960	0.2421

R-squared	0.373022	Mean dependent var	6.90E-06
Adjusted R-squared	0.361926	S.D. dependent var	0.000733
S.E. of regression	0.000586	Akaike info criterion	-12.02211
Sum squared resid	3.88E-05	Schwarz criterion	-11.95090
Log likelihood	700.2825	Hannan-Quinn criter.	-11.99320
Durbin-Watson stat	1.984381		

Inverted AR Roots	.83-.40i	.83+.40i	.20+.91i	.20-.91i
	.18	-.58+.73i	-.58-.73i	-.93

Conclusion

It may be concluded that daily UGX-NGN exchange rates follow a subset SARIMA(1,1,0)x(1,1,0)$_7$ model. Forecasting and simulation of the series may therefore be based on the proposed model (4).

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