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A simple design of a laboratory testing rig for the experimental demonstration and analysis of elastic deflections of beams with different end conditions

Kudzai Wadotiona[1], Takunda Goredema[2], Tawanda Mushiri[3], Stephen Mutuli[4]

[1]Graduate B.Sc. (Honours) Mechanical Engineering Student, Department of Mechanical Engineering, University of Zimbabwe,
[2]Graduate B.Sc. (Honours) Mechanical Engineering Student, Department of Mechanical Engineering, University of Zimbabwe,
[3]Lecturer Department of Mechanical Engineering, University of Zimbabwe,
[4]Visiting Professor on sabbatical leave, Department of Mechanical Engineering, University of Zimbabwe, P.O Box MP167, Mt Pleasant, Harare, Zimbabwe (2014/2015)

Abstract Elastic deflection of beams is one of the major topics in the subject of Solid And Structural Mechanics or Strength of Materials, studied by students pursuing a number of programs in engineering such as Mechanical Engineering, Civil Engineering, Agricultural Engineering, Electrical Engineering, Metallurgical Engineering etc. and the appreciation of the topic by the students is certainly enhanced by doing a number of appropriate and relevant laboratory experiments. A simple design of an affordable beam deflection laboratory testing rig is presented here and an experimental demonstration is carried out on a beam loaded by a combination of loadings comprising point loads, distributed loads and couples under different end conditions, namely, simple end supports; fixed end supports; one end fixed- and one end free (cantilever); and one end fixed- and one end propped, and corresponding deflections are recorded at specific designated points along the length of the beam. A comparison of the experimental results of the deflections and the theoretical results obtained from the elastic beam deflection theory is made.

KeywordsLaboratory, Rig, Experiment, Elastic deflection, Beams

Introduction

A number of new universities offering engineering programs have been established in many developing countries in Africa such as in Kenya, Uganda, Tanzania, Zimbabwe etc., in the last fifteen years and it has been observed that these universities face a big challenge due to lack of appropriate laboratory equipment for the smooth running of the programs. As a result, almost all these new institutions are forced to rely on the older ones for conducting many of the laboratory experiments and workshop practices. Worse still, in a number of cases, even the older institutions have many of the equipment not working due to the constant unavailability of sufficient funds to maintain those that are broken down as well as replace the ones that are obsolete. There is therefore an urgent need for universities in the developing countries to focus on the development of locally designed and fabricated laboratory equipment to help mitigate this problem [1]. The locally designed equipment will certainly be cheaper and easily available and the corresponding maintenance costs will be affordable. In this paper, a simple design of a laboratory testing rig for experimental demonstration and analysis of elastic deflection of beams is presented. This testing rig is able to perform deflection tests on beams with different end supports and carrying a combination of loadings comprising of point loads, distributed loads and couples. It is important to note here that the commonly available testing rig for beam deflection tests (in these mentioned universities) is designed to perform experiments on simply supported beams carrying point loads only.

Theory

The topic of elastic deflections of beams is extensively covered in a number of undergraduate text-books of Solid and Structural Mechanics or Strength of Materials, frequently available in university libraries or bookshops and the used ones have been highlighted under references. The basic differential equation for the deflection of beams is given by:

$$EI\frac{d^2y}{dx^2} = M \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

Where M is the Bending Moment, E the Modulus of Elasticity of the beam, I, the Second Moment of Area of the beam cross-section about its neutral axis and 'y' the deflection of the beam at a point distance 'x' from the left-end of the beam. If the Bending Moment 'M' is expressed as a function of 'x', then successive integrations will first give expressions for $EI\frac{dy}{dx}$ and secondly expressions for EIy, with a constant of integration introduced after each integration. '$\frac{dy}{dx}$' is the expression for the slope of the beam and 'y' is the deflection. The two resulting constants of integration are determined from the boundary conditions and the final expressions for the slope '$\frac{dy}{dx}$'and the deflection 'y' are established as functions of 'x'. Finally expressions for the slope and deflection of the beam at any point (distance 'x') along the length of the beam can be derived [2-11].

A summary of some expressions for the deflections of beams (at specified points) with different loadings and different end supports are hereby given as functions of the loading, the length of the beam, E and I.

Simply supported beams

(a) Simply supported beam of length 'l' carrying a center point load 'P', and the deflection calculated at the center (See Fig 1) [5, 9].

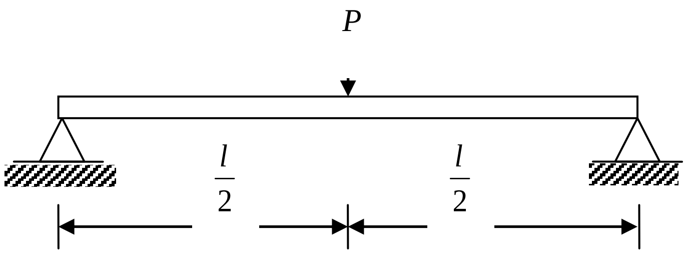
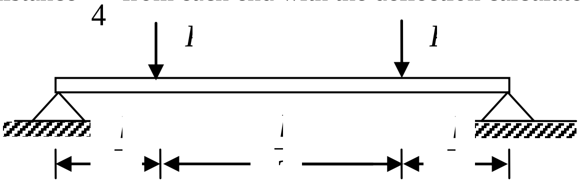


Figure 1

$$y_{x=\frac{l}{2}} = -\frac{Pl^3}{48EI} \text{-----------------------------------} \qquad (2)$$

(b) Simply supported beam of length 'l' supporting two equal point loads, each, of magnitude P and applied at distance $\frac{l}{4}$ from each end with the deflection calculated at the centre. (See Fig 2) [5, 9].



Fiaure 2

$$y_{x=\frac{l}{2}} = -\frac{11Pl}{384E} \qquad \text{-----------------------} \qquad (3)$$

(c) Simply supported beam of length 'l' supporting a point load 'P' and a clockwise couple 'Pe' applied at the center with the deflection calculated at the centre (See Fig 3) [2, 9].

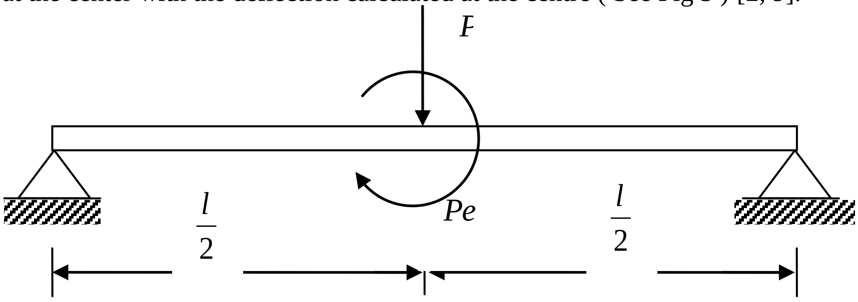


Figure 3

$$y_{x=\frac{l}{2}} = -\frac{Pl^3}{48EI} \quad \text{--} \quad (4)$$

(d) Simply supported beam of length 'l' and carrying a uniformly distributed load of magnitude 'w' N/m over a distance $\frac{l}{4}$ with the deflection calculated at the centre (See Fig 4) [5].

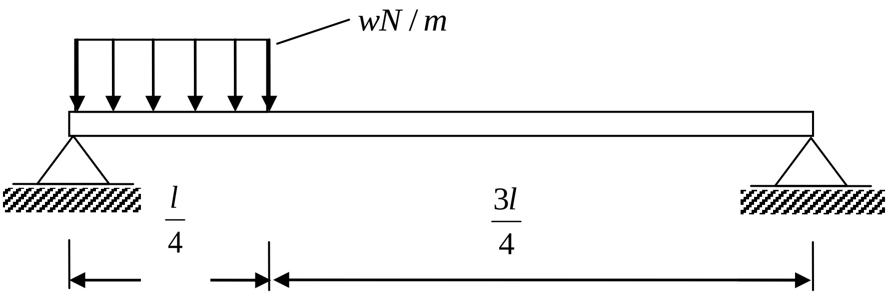


Figure 4

$$y_{x=\frac{l}{2}} = -\frac{23wl^4}{12288EI} \quad \text{------------------------} \quad (5)$$

(c) Simply supported beam of length 'l' and carrying a uniformly distributed load of magnitude 'w' N/m over a distance $\frac{l}{2}$ with the deflection calculated at the centre (see Fig 5)

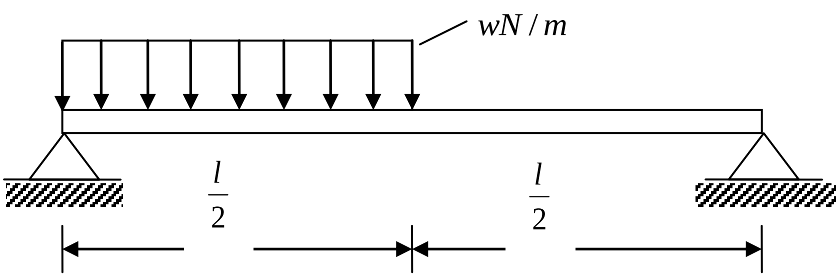


Figure 5

$$y_{x=\frac{l}{2}} = -\frac{5wl^4}{768EI} \quad \text{-----------------------------------} \quad (6)$$

(e) Simply supported beam of length ‘ l ’ carrying a uniformly distributed load of magnitude ‘ w ’ N/m over a distance $\frac{3l}{4}$ with the deflection calculated at the centre (See Fig 6)[5]

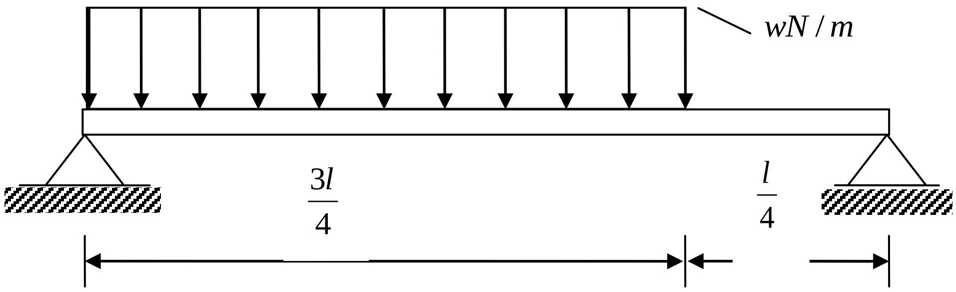


Figure 6

$$y_{x=\frac{l}{2}} = -\frac{137wl^4}{12288EI} \qquad \text{------------------------------} \quad (7)$$

(f) Simply supported beam of length ‘ l ’ carrying a uniformly distributed load of magnitude ‘ w ’ N/m over the length ‘ l ’ with the deflection calculated at the centre (See Fig 7)[5]

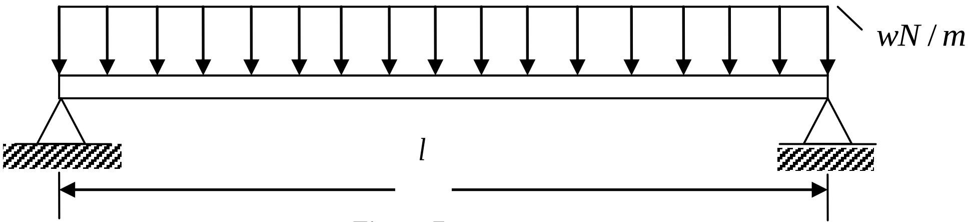


Figure 7

$$y_{x=\frac{l}{2}} = -\frac{5wl^4}{384EI} \qquad \text{----------------------------} \quad (8)$$

(g) Simply supported beam of length ‘ l ’ carrying a uniformly distributed load of magnitude ‘ w ’ N/m over a length $\frac{l}{2}$, together with a point load ‘ P ’ and a couple ‘ Pe ’ applied at a point distance $\frac{l}{4}$ from the right-hand support. The deflection is calculated at the centre of the beam (See Fig 8) [2, 4, 9]

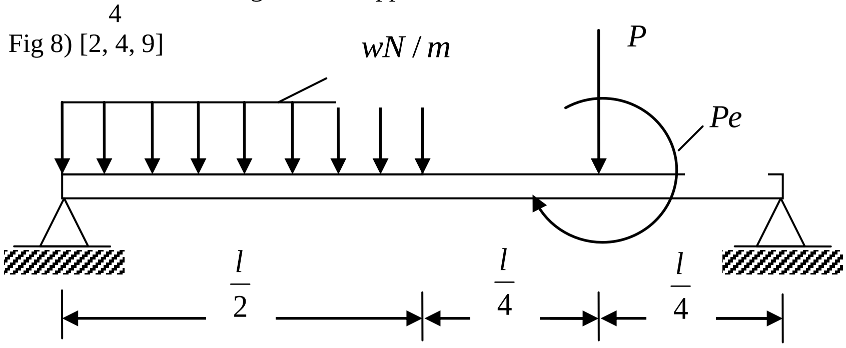


Figure 8

$$y_{x=\frac{l}{2}} = \frac{1}{EI}\left[-\frac{5wl^4}{768} - \frac{11Pl^3}{768} + \frac{3Pel^2}{64}\right] \qquad \text{---------------------} \quad (9)$$

Beams with Fixed Ends

(h) Beam of length ‘l’ with fixed ends carrying a point load ‘ P ’ applied at the centre with the deflection calculated at the centre (See Fig 9) [2-4, 9]

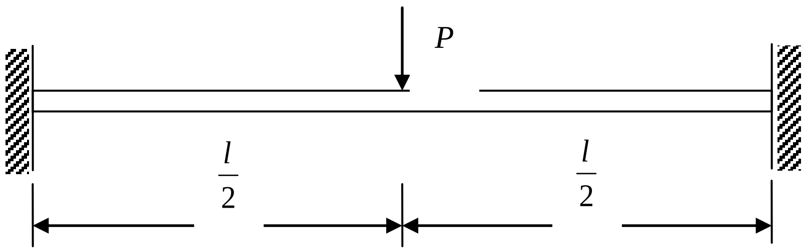


Figure 9

$$y_{x=\frac{l}{2}} = -\frac{Pl^3}{192EI} \quad \text{-----------------------------} \quad (10)$$

Beam of length ‘ l ’ , fixed at the ends and supporting a point load ‘ P ’ and a clockwise couple ‘ Pe ’ applied at the centre with the deflection calculated at the centre (See Fig 10)

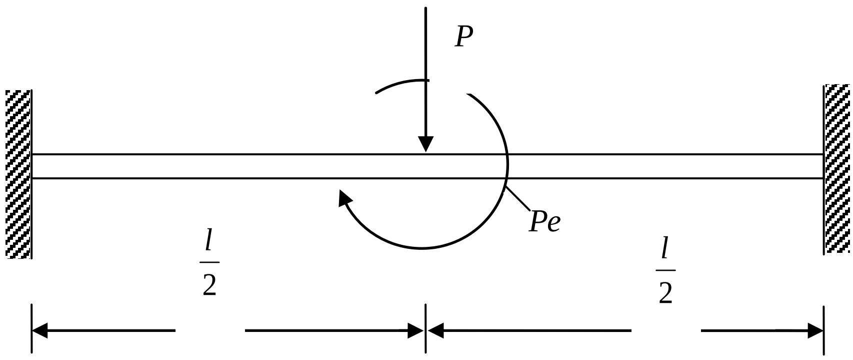
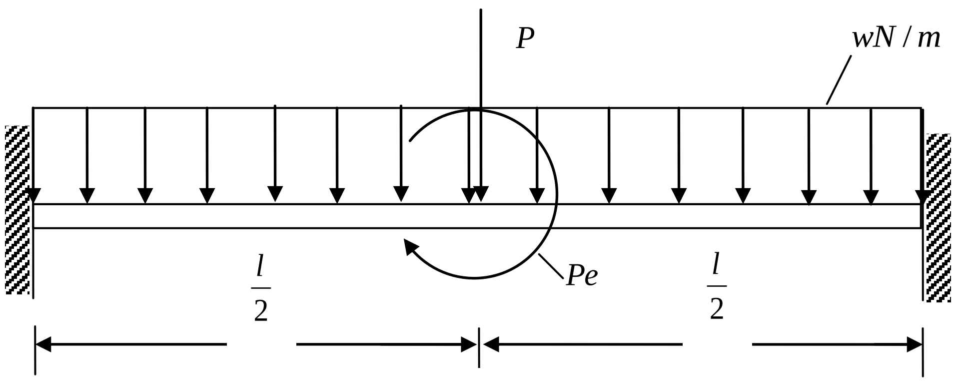


Figure 10

$$y_{x=\frac{l}{2}} = -\frac{Pl^3}{192EI} \quad \text{--------------------------} \quad (11)$$

(i) Beam of length ‘ l ’ fixed at the ends and supporting a uniformly distributed load of magnitude ‘ w ’ N/m over the entire length as well as a point load ‘ P ’ and a clockwise couple ‘ Pe ’ applied at the centre. The deflection is calculated at the centre (See Fig 12) [2, 9].



Fiaure 11

$$y_{x=\frac{l}{2}} = -\frac{wl^4}{384EI} - \frac{Pl^3}{192EI} \quad \text{------------------------} \quad (12)$$

Beams with one end fixed and one end propped

(j) Beam of length 'l' fixed at the left-end, propped at the right end and supporting a load 'P' applied at the centre with the deflection calculated at the centre. (See Fig 12) [2, 9].

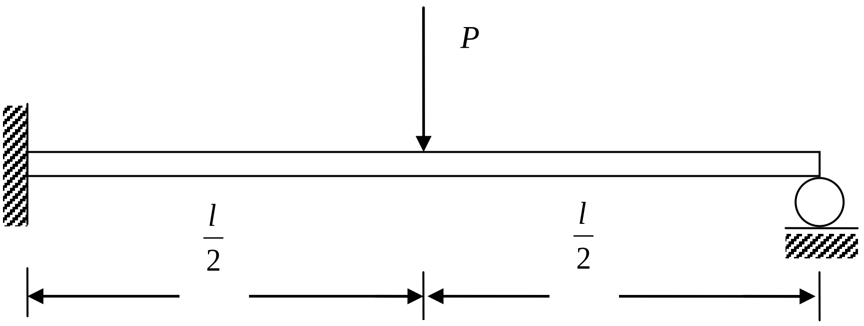


Figure 12

$$y_{x=\frac{l}{2}} = -\frac{7Pl^3}{768EI} \qquad (13)$$

(k) Beam of length 'l' fixed at the left-end, propped at the right end and supporting a point load 'W' acting at a point distance $\frac{l}{4}$ from the left end and a point load 'P' together with a clockwise couple of magnitude 'Pe' applied at the centre. The deflection is calculated at the centre (See Fig 13)[2]

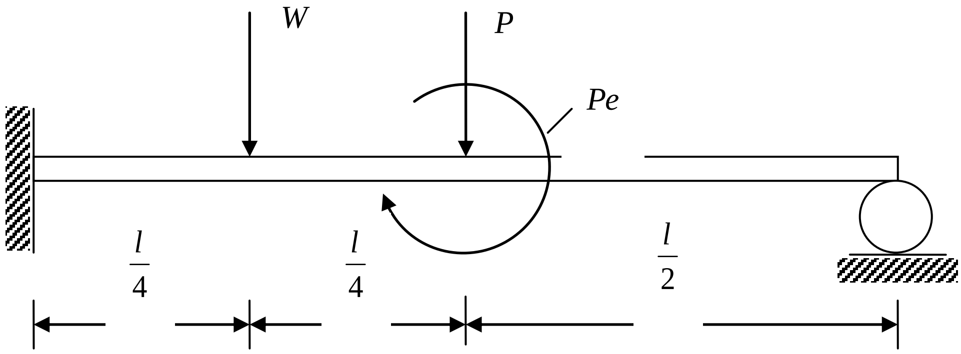


Figure 13

$$y_{x=\frac{l}{2}} = \frac{1}{EI}\left[-\frac{25Wl^3}{6144} - \frac{7Pl^3}{768} - \frac{Pel^2}{128}\right] \qquad (14)$$

(l) Beam of length ‘ l ’ fixed at the left-end, propped at the right –end and supporting a uniformly distributed load of magnitude ‘ w ’ N/m over a distance $\frac{l}{2}$ as well as a point load ‘ P ’ and a clockwise couple of magnitude ‘ Pe ’ applied at a point distance $\frac{l}{4}$ from the right-hand end. The deflection is calculated at the center (See Fig 14) [2, 5].

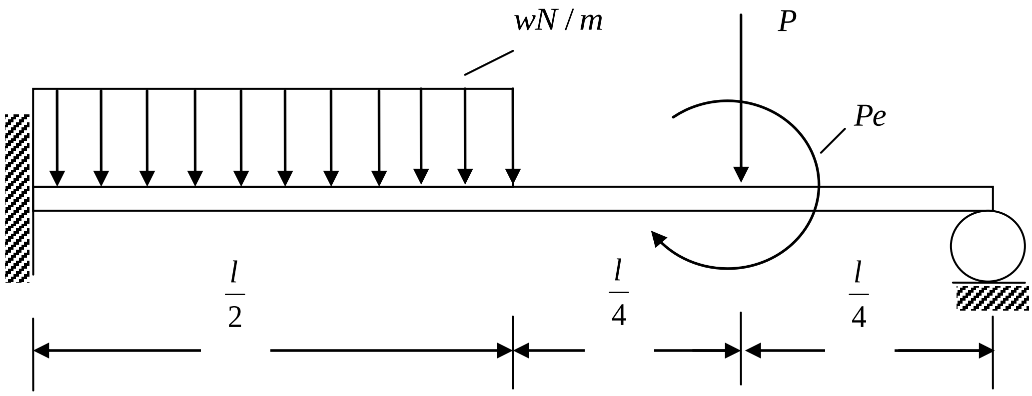


Figure 14

$$y_{x=\frac{l}{2}} = \frac{1}{EI}\left[-\frac{43Pl^3}{6144}+\frac{11Pel^2}{512}-\frac{13wl^4}{6144}\right] \qquad (15)$$

Beams one end fixed and the other end free
(Cantilever beams)

(m) Cantilever beam of length ‘ l ’ fixed at the left end and supporting a point load ‘ P ’ and a clockwise couple ‘ Pe ’ applied at the centre with the deflection calculated at the centre (See Fig 15) [2, 5].

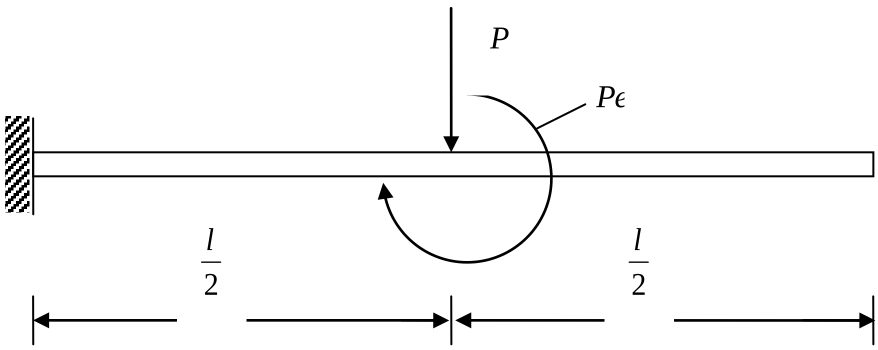


Figure 15

$$y_{x=\frac{l}{2}} = -\frac{Pl^3}{24EI}-\frac{Pel^2}{8EI} \qquad (16)$$

(n) Cantilever beam of length ‘ l ’ fixed at the left end and supporting a uniformly distributed load of magnitude ‘ w ’ N/m over its entire length as well as a point load ‘ P ’ together with a clockwise couple of magnitude ' Pe ' applied at the right end. The deflection is measured at the centre (See Fig 16) [2, 5].

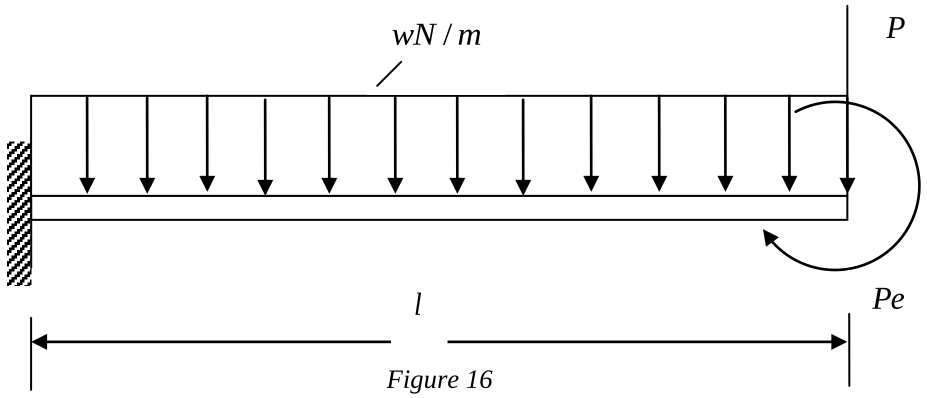


Figure 16

$$y_{x=\frac{l}{2}} = \frac{1}{EI}\left[-\frac{17wl^4}{384} - \frac{5Pl^3}{48} - \frac{Pel^2}{8}\right] \quad \text{---------------------} \quad (17)$$

Design, Materials And Methods

Design of rig framework

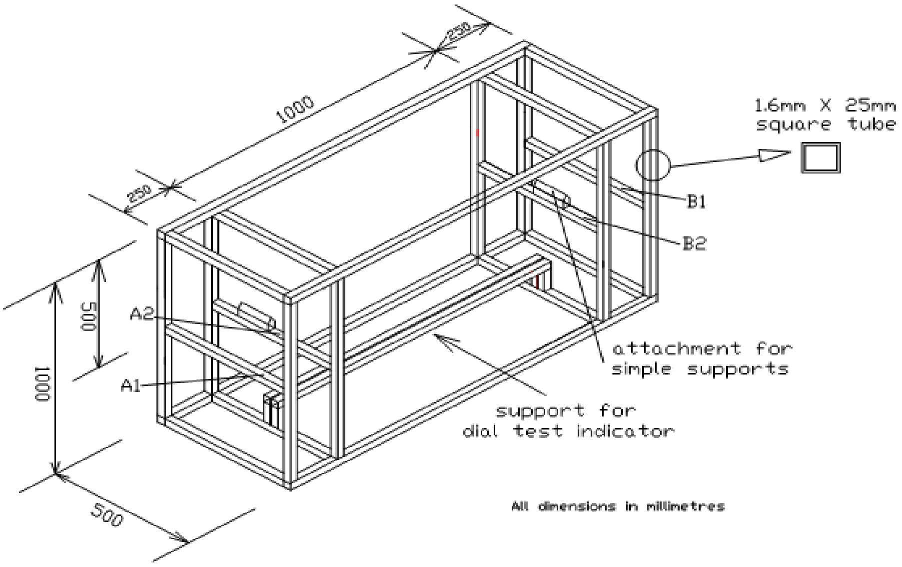


Figure 17

Fig 17 shows an isometric view of the designed rig frame. The frame has been assembled by welding together pieces of 25mm × 25mm × 1.6mm steel square tubes. The attachment for simple supports are indicated on the diagram and the beam simply rests on the indicated knife edges. On the right-hand side of the simple supports points A_1 and A_2 are indicated on one side of the frame while points B_1 and B_2 are on the other side. The points indicated are positions where accessories to facilitate fixed end conditions; one end fixed and one end propped; and finally one end fixed and one end free (cantilever) are placed as indicated below.

(a) Fixed – End position

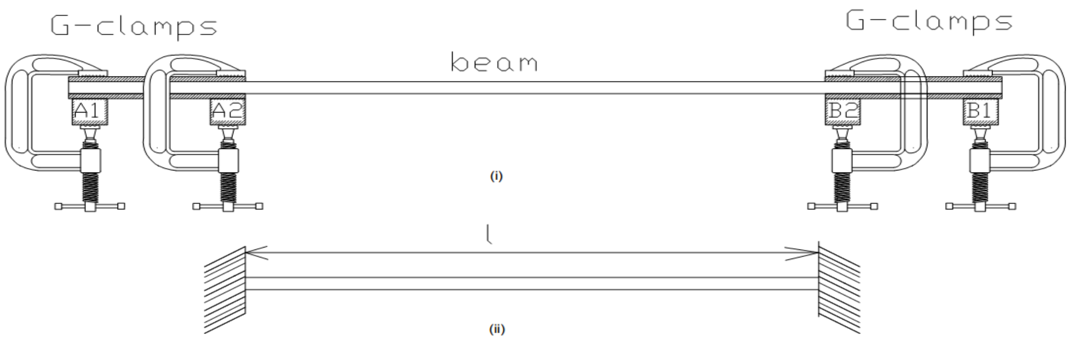


Figure: 18

Fig 18(i) shows a diagram indicating fixed end condition. Four G-clamps are used to firmly grip the beam at points A_1 , A_2 , B_1 and B_2 finally resulting in a beam of length 'l' with both ends fixed (see Fig 18(ii)).

(b) One end fixed, one end propped

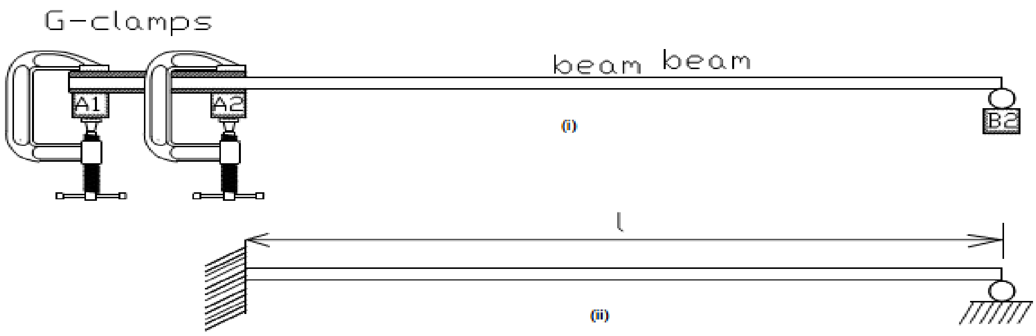


Figure: 19

Fig 19(i) shows a diagram indicating a propped cantilever beam. Two G-clamps are used to firmly grip the beam at points A_1 and A_2. At point B_2 an attachment for propping the beam is added as shown in the figure. Fig 19(ii) is a schematic representation of Fig 19(i).

(c) One end fixed, one end free

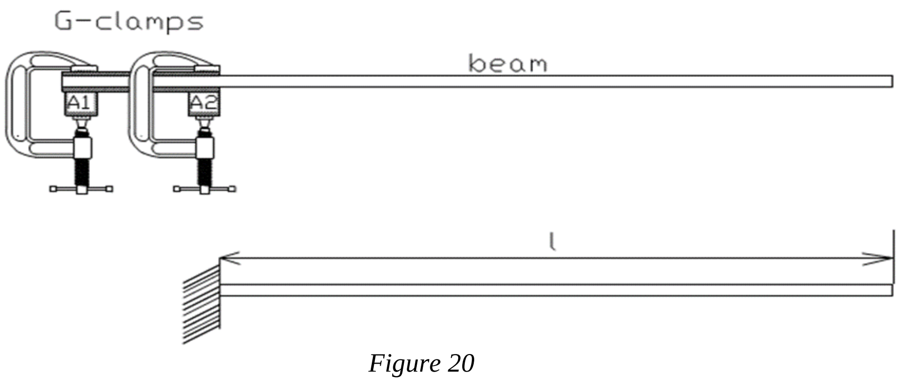


Figure 20

Fig 20(i) shows a diagram indicating a beam fixed at one end and free at the other end (cantilever). Two G-clamps are used to firmly grip the beam at points A_1 and A_2 the other end being left free. Fig 20(ii) is a schematic representation of Fig 20(i).

Simply supported

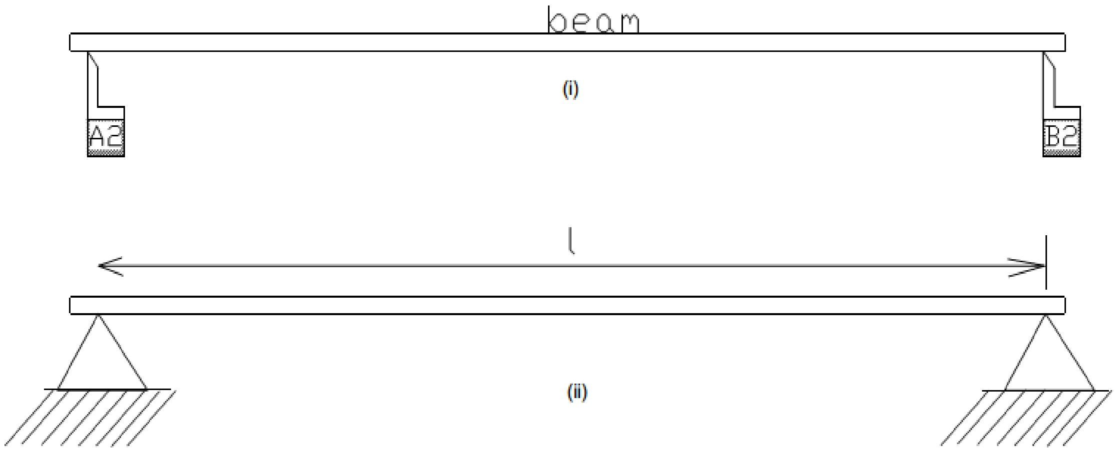


Figure 21

The simple supports are indicated on the frame shown in (Fig 17). A beam that is simply supported on the two knife edges is shown above in Fig 21(i) and a schematic representation is shown in Fig 21 (ii).

Design of the accessory for application of point load and couple

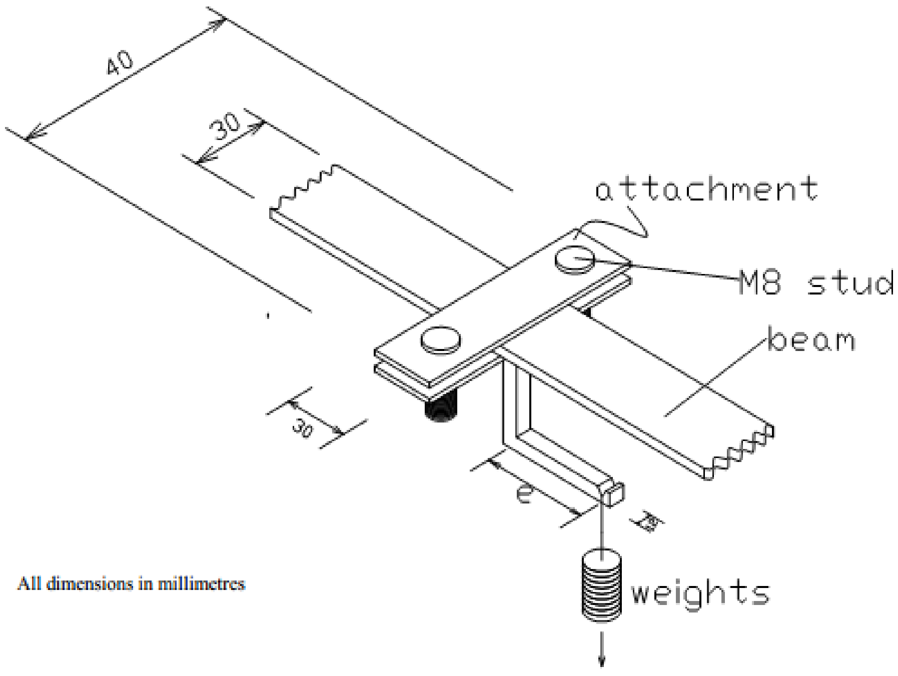
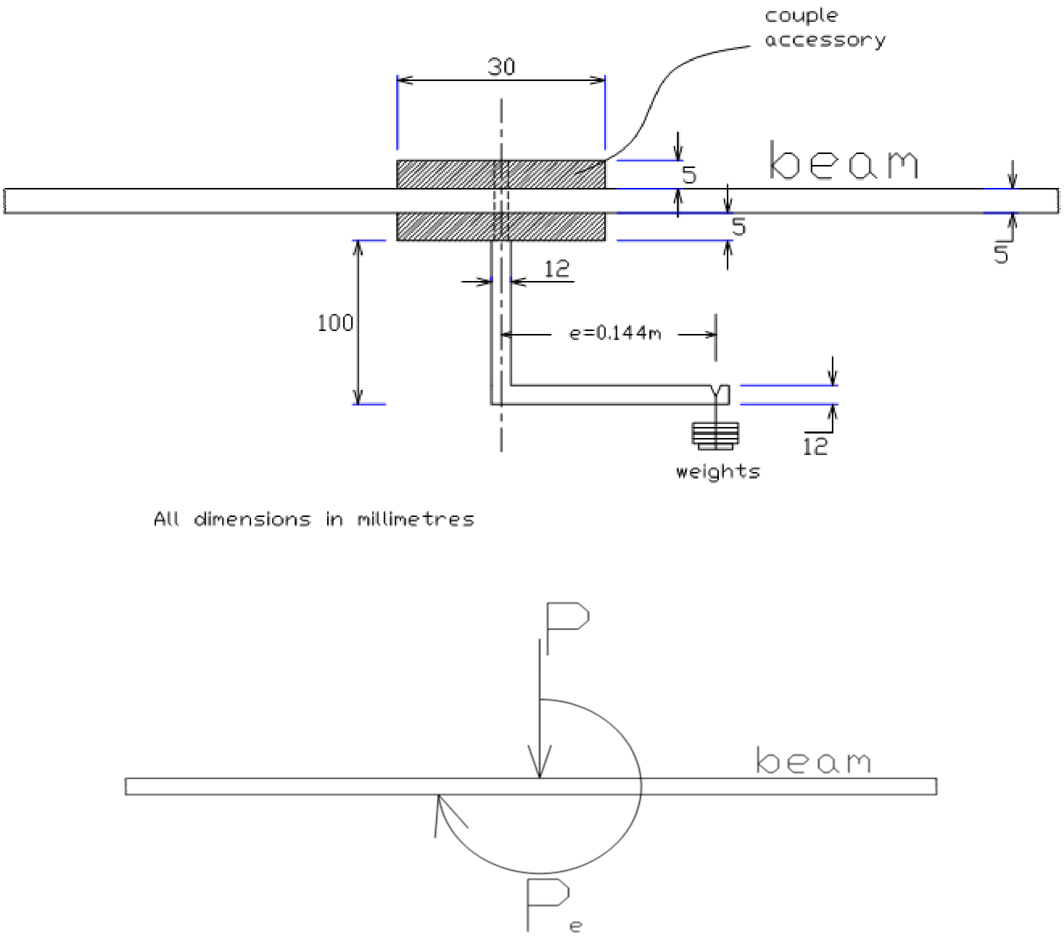


Figure 22

Fig.22 above and Fig 23(i) below show diagrams of an attachment that is fixed at any preferred point on the beam. A point load P applied with an eccentricity 'e' finally produces a point load 'P' together with a resulting couple 'Pe' (see Fig 23(ii)).



(ii)

Figure 23

Design of uniformly distributed loads

The concept of representing a uniformly distributed load can easily be understood by considering the sequence of diagrams indicated in Fig 24. Fig 24(i) shows a block of weight W and length l to be placed on top of a beam to produce a distributed load of magnitude $\omega\left(=\frac{W}{l}\right)N/m$

This load W is applied on the beam as shown in Fig 24(ii) and as the beam deflects, the load touches the beam at points X and Y. The load is unable to embrace the profile of the deflecting beam. The resulting loading is simply two point loads each of magnitude $W/_2$ acting at points X and Y instead of the expected uniformly distributed load$\omega\left(=\frac{W}{l}\right)N/m.$

On the other hand, if this load is imagined split into smaller segments as show in Fig 24(iv), the small segments of this load end up embracing the profile of the beam thereby producing the expected uniformly distributed load.

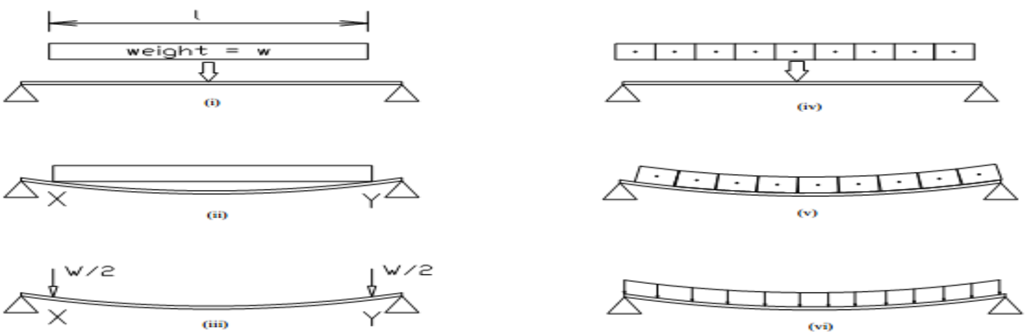


Figure 24

Fabrication of uniformly distributed load

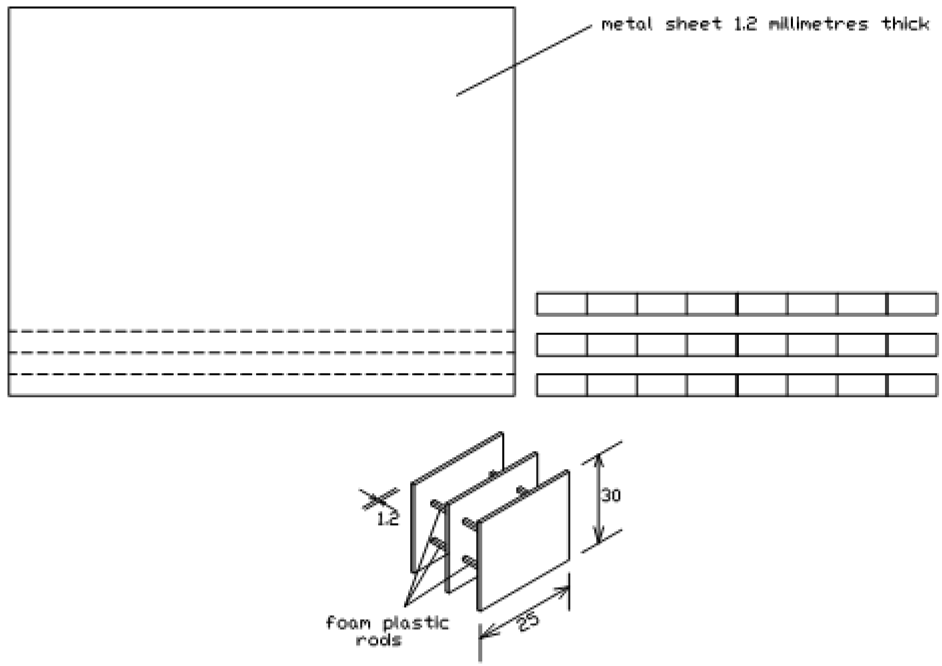


Figure 25

Fig 25 shows the steps taken in the fabrication of a uniformly distributed load. A metal sheet of thickness 1.2mm is marked as shown in Fig 25(i) and cut using guillotine to produce three identical strips. The three strips are marked and split into smaller identical pieces as shown in Fig 25(ii).

Three identical pieces are joined together to form a small block using foam plastic and adhesive (See Fig 25 (iii)) and a number of identical blocks are produced in this way. The blocks are weighed and placed on the beam one next to the each other and the final representation of a uniformly distributed load is obtained by dividing the total weight by the length covered.

Properties of Beam Specimen

The beam specimen had a length of 1.5m with a rectangular cross section of 30mm by 5mm.The value of the Modulus of Elasticity of the beam was $201GN/m^2$ and this value was also confirmed by tests conducted at Standards Association of Zimbabwe (SAZ).

Experimental procedure

A series of experiments on beam deflection was conducted as described below.

a) **Simply supported beam carrying a centre point load.**
The cross-section of the beam was measured using Vernier calipers and the beam was placed on two knife edges 1000 millimetres apart for simple supports. A dial gauge on magnetic stand was positioned to measure the deflection at the centre.
A series of increasing loads, P was applied through an attachment at the centre (see Fig.1) and corresponding deflections were recorded. A comparison of the experimental results was made with the theoretical results predicted using Equation 2.

b) **Simply supported beam of length 'l' carrying two equal loads each of magnitude P and applied at a distance $^{l}/_{4}$ from each end.**
The beam was placed on two knife edges 1000 millimetres apart for simple supports and attachments for application of two equal point loads were placed at points 250 millimetres from each end. A dial gauge on magnetic stand was positioned to measure the deflection at the centre of the beam. A series of increasing equal loads, P was applied at two points and the corresponding deflections were recorded at the centre of the beam. A comparison of the experimental results was made with the theoretical results predicted using Equation 3.

c) **Simply supported beam of length 'l' supporting a point load P and a clockwise couple 'Pe' at the centre.**
The beam was placed on two knife edges 1000 millimetres apart for simple supports and the attachment for the application of point load and couple (see Fig 22) was attached on the beam at the centre. A dial gauge on magnetic stand was positioned to measure the deflection at the centre of the beam. Increasing values of point loads 'P' were applied and the corresponding deflection at the centre of the beam recorded. A comparison of the experimental results was made with the theoretical results predicted using Equation 4.

d) **Simply supported beam of length 'l' carrying a uniformly distributed load of magnitude ω N/m and deflection measured at the centre.**
The beam was placed on two knife edges 1000 millimetres apart for simple supports and a dial gauge on magnetic stand was positioned to measure the deflection at the centre of the beam. Identical blocks were weighed and placed on the beam progressively to cover quarter length, half length, three-quarters length and full length. (See figures Fig 4, Fig 5, Fig 6, and Fig 7) For each of these loadings, the corresponding deflections were recorded at the centre. A comparison of the experimental results was made with the theoretical results from Equation 5 for quarter-length coverage, Equation 6 for half-length coverage, Equation 7 three-quarters length coverage and Equation 8 for full length coverage.

e) **Simply supported beam of length 'l' carrying a uniformly distributed load of magnitude ω N/m over a length of $l/_2$ together with a point load P and a couple 'Pe' applied at a distance $l/_4$ from the right hand support.**
The beam was placed on knife edges 1000 millimetres apart for simple supports and the fixture for point load and couple attached at a point 250 millimetres from the right hand end. A dial gauge on magnetic stand was positioned to measure the deflection at the centre of the beam. Identical blocks to produce a uniformly distributed load were placed to cover half the length of the beam. A series of increasing point loads P was then applied on the fixture producing a point load P and couple 'Pe' (See Fig 26), and the corresponding deflections were recorded at the centre of the beam. A comparison of the recorded experimental results and the theoretical results from Equation 9 was made

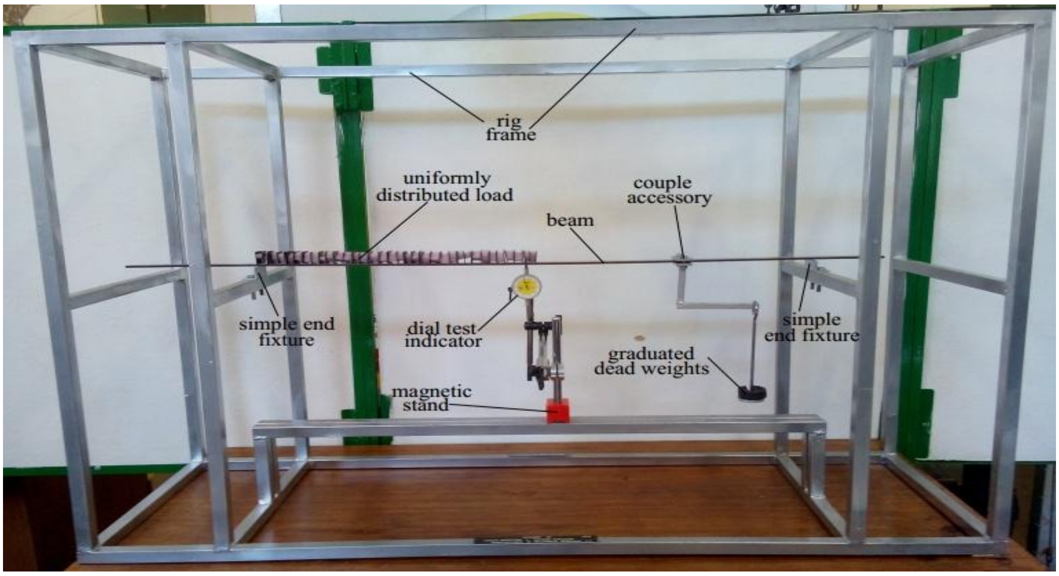


Figure 26: A simply supported beam supporting a uniformly distributed load and a point load and a couple

f) **A beam of length l with fixed ends carrying a point load P applied at the centre**
The beam of length 1000 millimetres is held fixed at both ends using four G-clamps (See Fig 18(i)) and a fixture for application of point loads is attached at the centre of the beam. A dial gauge on magnetic stand is positioned to measure the deflection at the centre. A series of increasing point loads P is applied and the corresponding deflections are recorded. A comparison of experimental results is made with the theoretical results from Equation 10.

g) **A beam of length 'l' fixed at both ends and supporting a point load P and a couple 'Pe' at the centre.**

The beam of length 1000 millimetres is held fixed at both ends by G-Clamps and a fixture for the application of point load and couple is fixed at the centre of the beam. A dial gauge on magnetic stand is positioned to measure the deflection at the centre of the beam. A series of increasing loads P is applied on the fixture and the corresponding deflections are recorded. A comparison of the results is made with the theoretical results from Equation 11.

h) A beam of length 'l' fixed at both ends and supporting a uniformly distributed load of magnitude ω N/m over the entire length as well as a point load P and a clockwise couple 'Pe' at the centre.

The beam of length 1000 millimetres is held fixed at both ends using four G-Clamps and the fixture for application of point load and couple attached at the centre. A dial gauge on magnetic stand is positioned to measure deflection at the centre.

Identical blocks to produce a uniformly distributed load are first placed along the length of the beam and the corresponding deflection was recorded at the centre of the beam. A series of increasing loads P is then applied on the fixture and the corresponding deflections are recorded at the centre of the beam (See Fig.27). A comparison of the experimental results is made with the theoretical results obtained from equation 12.

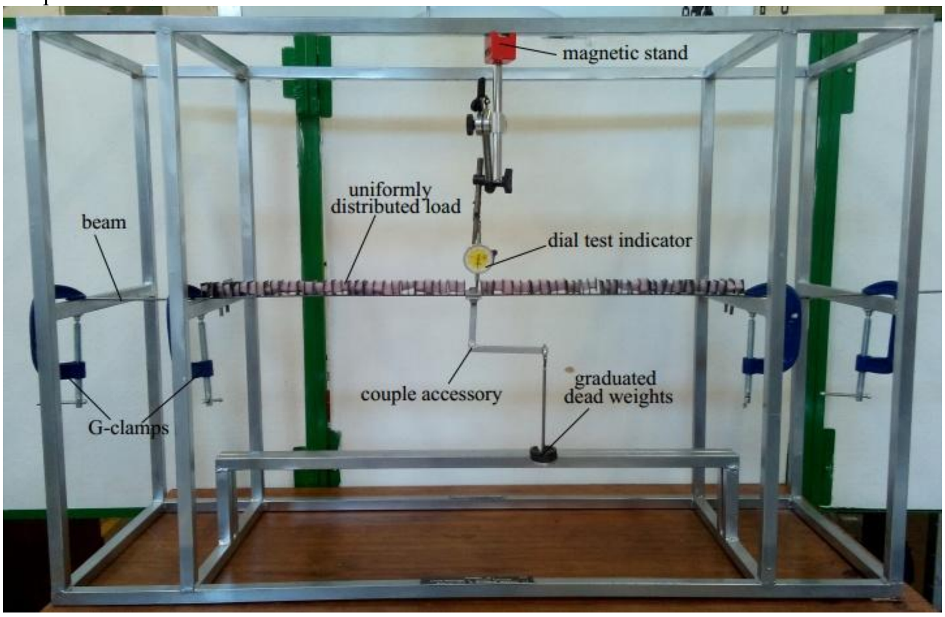


Figure 27: A beam fixed on both ends supporting a uniformly distributed load and a point load and a couple

i) A beam of length 'l' fixed at the left end and propped at the right end, supporting a centre point load P

The beam of length 1000 millimetres is held fixed on one end using two G-Clamps and propped at the right hand end (see Fig.19). A dial gauge on magnetic stand is positioned to measure deflections at the centre of the beam. A series of increasing point loads P is applied at the centre and the corresponding deflections at the centre of the beam are recorded. A comparison of the experimental results is made with the theoretical results obtained from Equation 13.

j) A beam of length 'l' fixed at the left end and propped at the right end, supporting a point load W acting at distance $^{l}/_{4}$ from the end together with a point load P and a couple 'Pe' applied at the centre.

The beam of length 1000 millimetres is held fixed on one end using two G-Clamps and propped on the other end. A fixture for application of point load is placed at a point 250 millimetres from the left end, and the fixture for the application of point load and couple is attached at the centre of the beam. A dial gauge on magnetic stand is positioned to measure deflections at the centre of the beam. A load W (= 5.886 N) is first applied and the deflection at the centre is recorded, then a series of increasing loads P are applied and the corresponding deflections at the centre are recorded. A comparison of the experimental results is made with the theoretical results obtained from Equation 14.

k) **A beam of length 'l' fixed at the left end and propped at the right hand end, supporting a uniformly distributed load of magnitude ω N/m over a distance $^{l}/_{2}$ as well as a point load P and a clockwise couple 'Pe' applied at a distance $^{l}/_{4}$ from the right hand end.**

The beam of length 1000 millimetres is held fixed on one end using two G-Clamps and propped on the right hand end. A fixture for application of point load and couple is attached at a point 250 millimetres from the propped end and a dial gauge on magnetic stand is positioned to measure deflections at the centre of the beam. Identical blocks to produce uniformly distributed load are first placed along the length of the beam together. A series of increasing point loads P are applied on the fixture and the corresponding deflections at the centre of the beam are recorded (See Fig.28). A comparison of the experimental results is made with the theoretical results obtained from Equation 15.

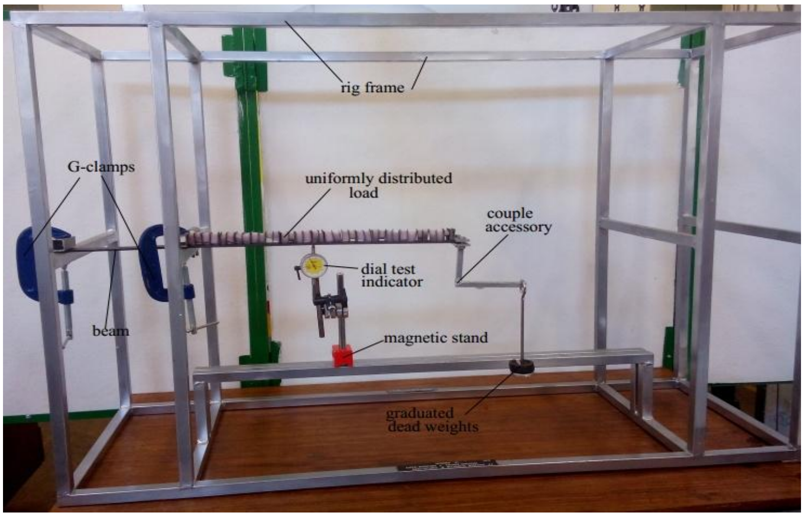


Figure 28: A beam fixed on one end and propped at the other end supporting a uniformly distributed load and a point load and a couple.

l) A cantilever beam of length 'l' supporting a point load P and a clockwise couple 'Pe' applied at the centre of the beam.

The beam of length 550 millimetres is held fixed on one end using two G-Clamps and a fixture for application of point load and couple fixed at the centre. A series of increasing point loads P are applied on the fixture and the corresponding deflections at the centre of the beam are recorded. A comparison of the experimental results is made with the theoretical results obtained from Equation 16.

m) A cantilever beam of length 'l' supporting a uniformly distributed load of magnitude ω N/m over its entire length as well as appoint load P and a clockwise couple 'Pe' applied at the free end.

The beam of length 550 millimetres is held fixed on one end using two G-Clamps and the fixture for application of point load and couple is attached at the free end. Identical blocks to produce a uniformly distributed load are first placed along the entire length of the beam. A dial gauge on magnetic stand is positioned to measure deflections at the centre of the beam. A series of increasing point loads P are applied on the fixture and the corresponding deflections at the centre of the beam are recorded. A comparison of the experimental results is made with the theoretical results obtained from Equation 17.

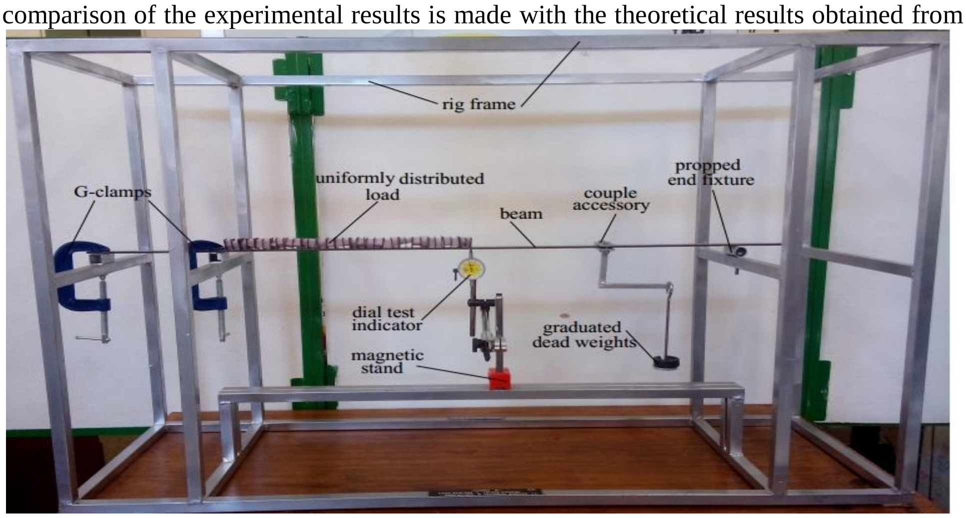


Figure 29: A cantilever beam supporting a uniformly distributed load and a couple and point load

Experimental Results

Table 1: Simply supported beam of length 'l' carrying a center point load 'P', and the deflection measured at the center (See Fig 1).

Load (N)	Experimental deflection $\times 10^{-3}m$	Theoretical deflection $\times 10^{-3}m$
0.981	0.31	0.33
1.962	0.66	0.65
2.943	0.98	0.98
3.924	1.30	1.30
4.905	1.62	1.63
5.886	1.97	1.95

Table 2: Simply supported beam of length 'l' supporting two equal point loads, each, of magnitude P and applied at distance $\frac{l}{4}$ from each end with the deflection measured at the centre. (See Fig 2)

Load (N)	Experimental deflection $\times 10^{-3}m$	Theoretical deflection $\times 10^{-3}m$
0.981	0.45	0.45
1.962	0.93	0.90
2.943	1.39	1.34
3.924	1.85	1.79
4.905	2.27	2.24
5.886	2.76	2.68

Table 3: Simply supported beam of length 'l ' supporting a point load 'P 'and a clockwise couple 'Pe ' applied at the center with the deflection measured at the centre (See Fig 3)

$Load, P$(N)	$Couple, P_e$ $/Nm$	Experimental deflection $\times 10^{-3}$m	Theoretical deflection $\times 10^{-3}$m
0.981	0.142	0.30	0.33
1.962	0.284	0.66	0.65
2.943	0.427	0.98	0.98
3.943	0.572	1.29	1.31
4.905	0.711	1.63	1.63
5.886	0.853	1.95	1.95

Table 4: Simply supported beam of length 'l' and carrying a uniformly distributed load of magnitude 'ω' Nm over varying distance l' metres with the deflection measured at the centre (See Figs 4, 5, 6 and 7)

Uniformly distributed load *N/m*	Length covered by uniformly distributed load=l' (m)	Experimental deflection $\times 10^{-3}$m	Theoretical deflection $\times 10^{-3}$m
9.81	0.25 (Fig.4)	0.23	0.29
9.81	0.50 (Fig.5)	0.93	1.01
9.81	0.75 (Fig.6)	1.72	1.74
9.81	1.00 (Fig.7)	2.04	2.03

Table 5 Simply supported beam of length 'l' carrying a uniformly distributed load of magnitude 'ω' N/m over a length$\frac{l}{2}$, together with a point load 'P' and a couple 'Pe' applied at a point distance $\frac{l}{4}$ from the right-hand support. The deflection is measured at the centre of the beam (See Fig 8)

$Load, P$ (N)	Couple, Pe (Nm)	Experimental deflection $\times 10^{-3}$m	Theoretical deflection $\times 10^{-3}$m
0.981	0.142	1.10	1.13
1.962	0.284	1.27	1.25
2.943	0.427	1.36	1.37
3.943	0.572	1.53	1.49
4.905	0.711	1.64	1.61
5.886	0.853	1.76	1.72

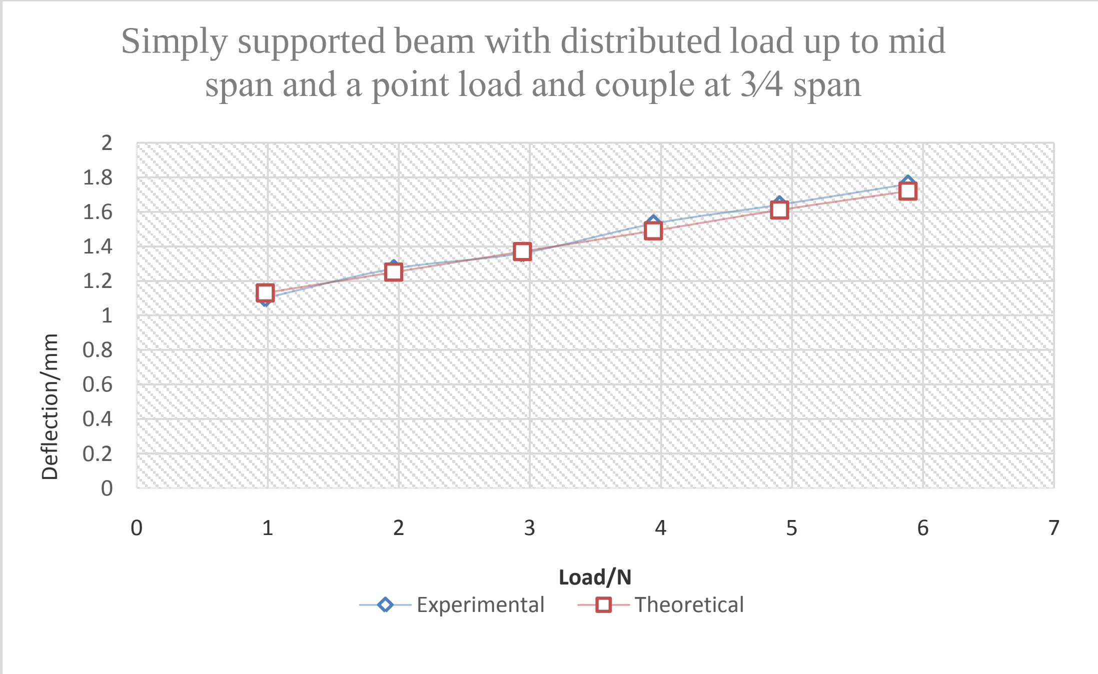


Figure 30

Table 6: Beam with fixed ends carrying a point load 'P' applied at the centre with the deflection measured at the centre (See Fig 9).

Load (N)	Experimental deflection $\times 10^{-3}$m	Theoretical deflection $\times 10^{-3}$m
0.981	0.07	0.09
1.962	0.15	0.18
2.943	0.26	0.27
3.924	0.35	0.37
4.905	0.44	0.47
5.886	0.53	0.55

Table 7: Beam of length 'l', fixed at the ends and supporting a point load 'P' and a clockwise couple 'Pe' applied at the centre with the deflection measured at the centre (See Fig 10)

Load (N)	Couple, Pe (Nm)	Experimental deflection $\times 10^{-3}$m	Theoretical deflection $\times 10^{-3}$m
0.981	0.142	0.10	0.08
1.962	0.285	0.19	0.16
2.943	0.427	0.23	0.24
3.942	0.572	0.36	0.33
4.905	0.711	0.44	0.41
5.886	0.853	0.53	0.49

Table 8 Beam of length 'l' fixed at the ends and supporting a uniformly distributed load of magnitude 'w' N/m over the entire length as well as a point load 'P' and a clockwise couple 'Pe' applied at the centre. The deflection is measured at the centre (See Fig 11)

Load (N)	Couple, Pe (Nm)	Experimental Deflection $\times 10^{-3}$m	Theoretical Deflection $\times 10^{-3}$m
0.981	0.142	0.09	0.12
1.962	0.285	0.17	0.20
2.943	0.427	0.26	0.28
3.924	0.572	0.35	0.37
4.905	0.711	0.45	0.45
5.886	0.853	0.54	0.53

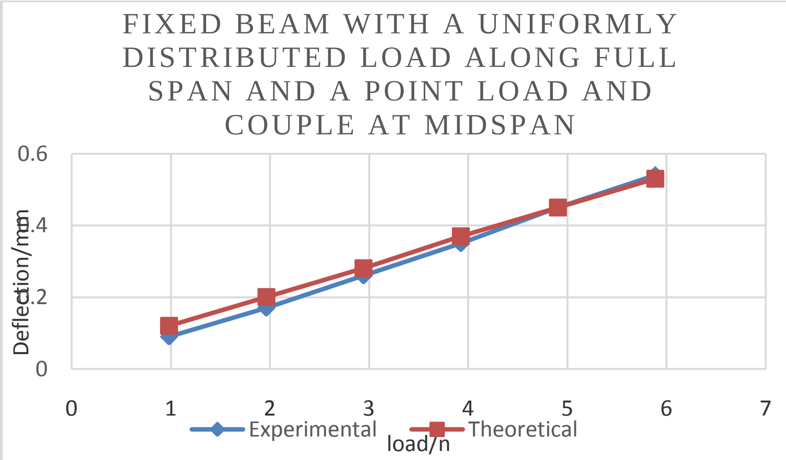


Figure 31

Table 9: Beam of length 'l' fixed at the left-end, propped at the right end and supporting a load 'P' applied at the centre with the deflection measured at the centre. (See Fig 12)

Load (N)	Experimental Deflection/ $\times 10^{-3}$m	Theoretical Deflection/$\times 10^{-3}$mm
0.981	0.13	0.14
1.962	0.32	0.28
2.943	0.48	0.42
3.924	0.65	0.56
4.905	0.81	0.71
5.886	0.98	0.85

Table 10: Beam of length 'l' fixed at the left-end, propped at the right end and supporting a point load W'' acting at a point distance $\frac{1}{4}$ from the left end and a point load 'P' together with a clockwise couple of magnitude '$\frac{l}{2}$' applied at the centre. The deflection is measured at the centre (See Fig 13)

Load (N)	Experimental Deflection $\times 10^{-3}$m	Theoretical Deflection $\times 10^{-3}$m
0.981	0.13	0.14
1.962	0.32	0.28
2.943	0.48	0.42
3.924	0.65	0.56
4.905	0.81	0.71
5.886	0.98	0.85

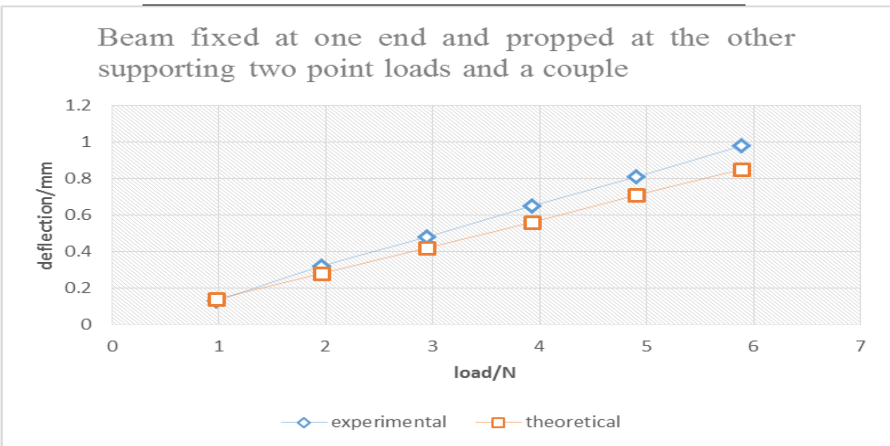


Figure 32

Table 11: Beam of length ‘ ’ fixed at the left-end, propped at the right –end and supporting a uniformly distributed load of magnitude ‘ω’ N/m over a distance $\frac{l}{2}$ as well as a point load ‘ P ’ and a clockwise couple of magnitude ‘ Pe ’ applied at a point distance $\frac{1}{4}$ from the right-hand end. The deflection is measured at the center (See Fig 14)

Load (N)	Experimental Deflection $\times 10^{-3}$m	Theoretical Deflection $\times 10^{-3}$m
0.981	0.068	0.07
1.962	0.11	0.15
2.943	0.15	0.21
3.924	0.20	0.29
4.905	0.24	0.36
5.886	0.28	0.43

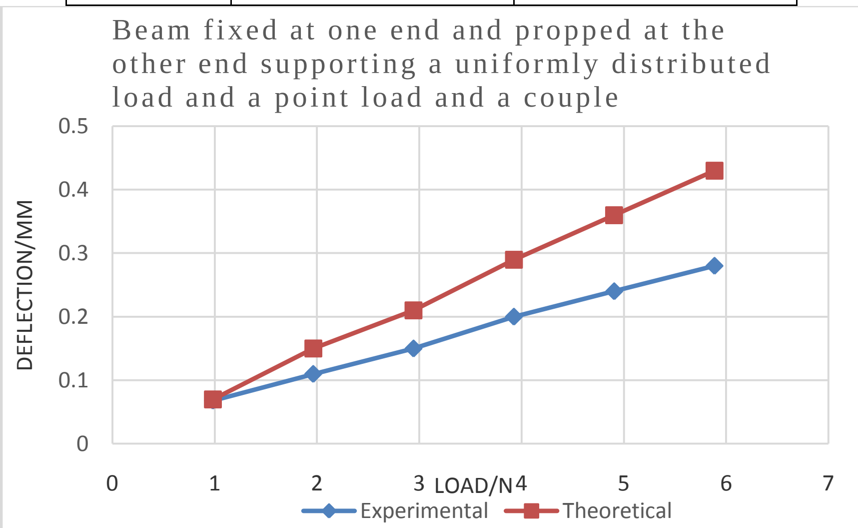


Figure 33

Table 12: Cantilever beam of length ‘ l ’ fixed at the left end and supporting a point load ‘ P ’ and a clockwise couple ‘ Pe ’ applied at the centre with the deflection measured at the centre (See Fig 15)

Load (N)	Experimental Deflection $\times 10^{-3}$m	Theoretical Deflection $\times 10^{-3}$m
0.981	0.52	0.63
1.962	1.10	1.32
2.943	1.83	2.03
3.924	2.50	2.71
4.905	3.26	3.48
5.886	4.09	4.45

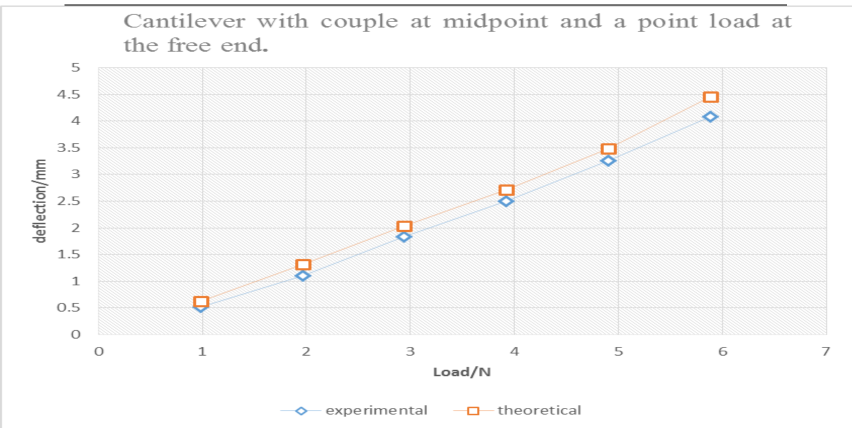


Figure 34

(Uniformly distributed load = 9.81 N/m)

Table 13: Cantilever beam of length 'l' fixed at the left end and supporting a uniformly distributed load of magnitude 'w' N/m over its entire length as well as a point load 'P' together with a clockwise couple of magnitude 'Pe' applied at the right end. The deflection is measured at the centre (See Fig 16)

Load (N)	$Couple, Pe$ (Nm)	Experimental Deflection $\times 10^{-3}$m	Theoretical Deflection $\times 10^{-3}$m
0.981	0.142	0.48	0.99
1.962	0.284	0.94	1.34
2.943	0.427	1.40	1.70
3.943	0.572	1.86	2.06
4.905	0.711	2.34	2.41
5.886	0.853	2.83	2.77

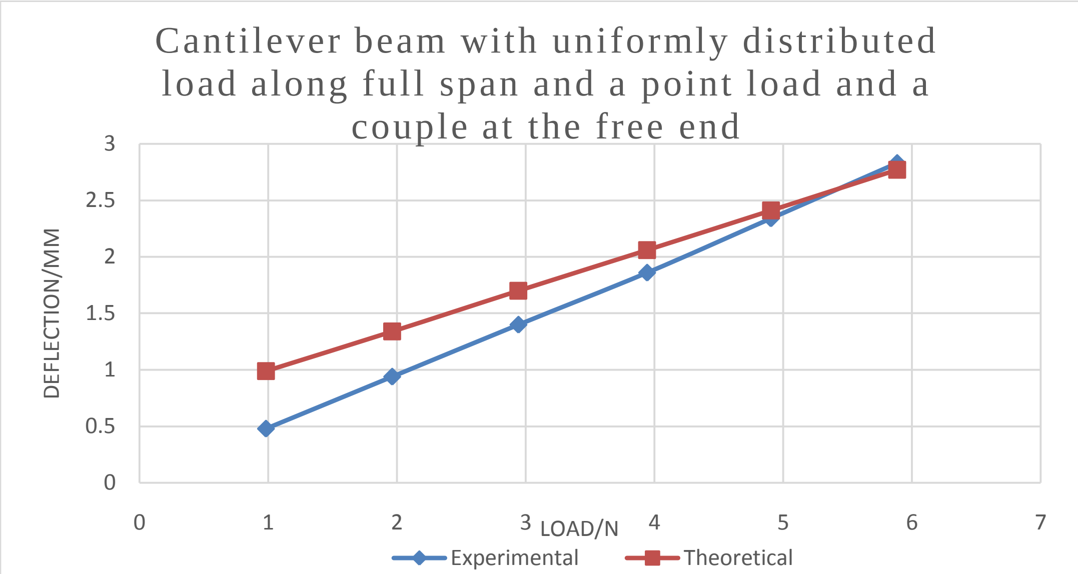


Figure 35

The experimental results are indicated in Tables 1 to 13 and the corresponding results from theoretical calculations are also indicated. In general it can be said that that there was a good correlation between the experimental and the theoretical results. Table 4 shows results for a simply supported beam carrying a uniformly distributed load over quarter length, half length, three-quarter length and full length. The results for the deflection at the centre correlate well between the experimental and theoretical results.

Table 5 shows results for a simply supported beam carrying a uniformly distributed load over half the length, a point load and a couple applied at quarter length from the right hand end support. The deflection was measured at the centre and again a good correlation of experimental and theoretical results was observed. Fig. 30 shows a graph plotted for the results.

Table 8 shows results for a beam fixed at both ends and supporting a uniformly distributed load of magnitude w, over a length entire length of the beam together with a point load and couple applied at the centre with the deflection measured at the centre as well. Fig 31 is a graph showing these results and again a good correlation is observed.

Table 11 shows results for a beam fixed at one end and propped at the other end carrying uniformly distributed load of magnitude w N/m over half the length of the beam as well as a point load and couple a quarter length from the propped end. Fig 33 shows a graph indicating the experimental results and theoretical results with deflection measured at the centre. In this case there was a small variation between the experimental and the theoretical results.

Table 13 shows results for a cantilever beam fixed at the left end and supporting a uniformly distributed load of magnitude w over the entire length of the beam together with a point load and couple applied at the free end and deflection measured at the centre. Again here a small discrepancy is shown between the experimental and the theoretical results for smaller loads. The results tend to become closer as the load increases. Fig. 35 is a graph showing these results.

Discussion

In general there was good correlation between the experimental and the theoretical results for most of the experiments carried out. In a few cases mentioned, a variation of results was noticed. There are a number of sources of error that would have contributed to the variations observed which include the inhomogeneity of the beam material and the beam not being perfectly straight initially. Efforts were made to achieve the expected uniformly distributed load by splitting the load into smaller sections, this could have also contributed to some of the errors noticed. The theoretical equations assumes that the point load and a couple are acting at a point. However in these experiments, the attachment used covered a small contact area and this could have caused some errors. The dial gauges used had not been recently serviced and calibrated. The human limitations in reading accurately could have contributed to the errors as well.

Conclusion

The laboratory testing rig designed has been able to demonstrate the elastic deflection of beams with different end conditions and a combination of different loadings. The design of the uniformly distributed load presented here has been shown to work and the concept of the design of the attachment of a point load and a couple has worked very well. In general, the design of the testing rig has been made simple and easy to use. Simple concepts have been used for the application of a couple as well as the uniformly distributed load. The testing rig and its accessories were easy to manufacture and the overall cost was significantly low in comparison to the importation of similar equipment. The success of this project should inspire the design and manufacture of other laboratory equipment locally.

Acknowledgements

We would like to thank Professor Mutuli for allowing us to do this research at University of Zimbabwe. It was a success and the rig will be used in our laboratory as an asset.

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