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The Optimization of Production Cost using Linear Programming Solver

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Abstract This study is focused on the optimal production cost of raw materials to its production output. Linear programming solver was applied to solve and to optimize its monthly production output. Based on the result, the monthly optimal production output is 1.2252E-08. The company has to budget at least the optimal result to achieve their monthly cost of production. The result will help the company to eliminate excess waste that incurs in their cost of production.

Keywords Bottle Glass, Optimization, Cost, Production output, Linear Programming, Sodium sulphate, Selenium, Cobalt Oxide, Feldspar, Soda Ash, Limestone and Silica sand

Introduction

The work drives to the optimization of the monthly production output of the case company. Having looked at the case study critically, the researcher begins to observe that there is excess waste of cost of production during the time the product is undergoing its production processes. With this in mind the researcher starts to gather the production process data from the raw material to the finished products over a given period of three years. According to Taha (1992), reveals how linear programming and its application was used to determine the feed mix that will maintain a balanced proportional ratio which includes calcium, protein, and fiber in the right proportion by the Ozark Poultry Farm. Adam et al (1993), shows the application of linear programming model and with its Multi-Band Enterprises. Finding the best mix of citizens band radio (CBs) and portable radio (PRs) to produce is a typical blending and mix determination problem. The application of linear programming to this problem gives a solution values which show that the Multi-Band Enterprises will produce 632.31 of CBs, 126.15 PRs and this will give the maximum profit of $36,661.06. On the resources side, some hours of sub-assembly are unused while the assembly and inspection hours are extensively utilized. Andre et al (2003) applied linear programming to predict nutrients that are potentially low in a child's diet during the complementary feeding period in the sense that the information gathered can be used to direct nutrition intervention programme initiatives because it suggests that foods rich in (foods fortified with) these limiting or problem nutrients should be introduced into local diets. Having gone through several research works, the researcher observes that linear programming solver can help to optimize the best possible cost that can solve the company's monthly production output at any given time of the year.

In solving the case study problem, the objective of the study was developed with the application of linear programming model that will reveal the optimal cost of monthly production of bottle glasses in the case company.

Linear programming (**LP**; also called **linear optimization**) is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. Linear programming is a special case of mathematical programming (mathematical optimization).
More formally, linear programming is a technique for the optimization of a linear objective function, subject to linear equality and linear inequality constraints. Its feasible region is a convex polytope, which is a set defined as the intersection of finitely many half spaces, each of which is defined by a linear inequality. Its objective function is a real-valued affine (linear) function defined on this polyhedron. A linear programming algorithm finds a point in the polyhedron where this function has the smallest (or largest) value if such a point exists.
Linear programs are problems that can be expressed in canonical form as

$$\text{Maximum } \mathbf{c}^T\mathbf{x}$$
$$\text{Subject t } A\mathbf{x} \le \mathbf{b} \text{ and } \mathbf{x} \ge \mathbf{0}$$

where **x** represents the vector of variables (to be determined), **c** and **b** are vectors of (known) coefficients, A is a (known) matrix of coefficients, and $(\cdot)^T$ is the matrix transpose. The expression to be maximized or minimized is called the objective function ($\mathbf{c}^T\mathbf{x}$ in this case). The inequalities $A\mathbf{x} \le \mathbf{b}$ and $\mathbf{x} \ge \mathbf{0}$ are the constraints which specify a convex polytope over which the objective function is to be optimized. In this context, two vectors are comparable when they have the same dimensions. If every entry in the first is less-than or equal-to the corresponding entry in the second then we can say the first vector is less-than or equal-to the second vector.
Linear programming can be applied to various fields of study. It is widely used in business and economics, and is also utilized for some engineering problems. Industries that use linear programming models include transportation, energy, telecommunications, and manufacturing. It has proved useful in modeling diverse types of problems in planning, routing, scheduling, assignment, and design.

History of Linear programming

The problem of solving a system of linear inequalities dates back at least as far as Fourier, who in 1827 published a method for solving them (Kantorovich, 1940), and after whom the method of Fourier–Motzkin elimination is named.
The first linear programming formulation of a problem that is equivalent to the general linear programming problem was given by Leonid Kantorovich in 1939, who also proposed a method for solving it (Hitchcock, 1941). He developed it during World War II as a way to plan expenditures and returns so as to reduce costs to the army and increase losses incurred by the enemy. About the same time as Kantorovich, the Dutch-American economist T. C. Koopmans formulated classical economic problems as linear programs. Kantorovich and Koopmans later shared the 1975 Nobel prize in economics (Kantorovich, 1940). In 1941, Frank Lauren Hitchcock also formulated transportation problems as linear programs and gave a solution very similar to the later Simplex method (Hitchcock, 1941).
During 1946-1947, George B. Dantzig independently developed general linear programming formulation to use for planning problems in US Air Force. In 1947, Dantzig also invented the simplex method that for the first time efficiently tackled the linear programming problem in most cases. When Dantzig arranged meeting with John von Neumann to discuss his Simplex method, Neumann immediately conjectured the theory of duality by realizing that the problem he had been working in game theory was equivalent. Dantzig provided formal proof in an unpublished report "A Theorem on Linear Inequalities" on January 5, 1948. Postwar, many industries found its use in their daily planning. The theory behind linear programming drastically reduces the number of possible solutions that must be checked.
The linear programming problem was first shown to be solvable in polynomial time by Leonid Khachiyan in 1979, but a larger theoretical and practical breakthrough in the field came in 1984 when Narendra Karmarkar introduced a new interior-point method for solving linear-programming problems.

Uses of Linear programming

Linear programming is a widely used field of optimization for several reasons. Many practical problems in operations research can be expressed as linear programming problems. Certain special cases of linear programming, such as network flow problems and multi commodity flow problems are considered important enough to have generated much research on specialized algorithms for their solution. A number of algorithms for other types of optimization problems work by solving LP problems as sub-problems. Historically, ideas from linear programming have inspired many of the central concepts of optimization theory, such as duality, decomposition, and the importance of convexity and its generalizations. Likewise, linear programming is heavily used in microeconomics and company management, such as planning, production, transportation, technology and other issues. Although the modern management issues are ever-changing, most companies would like to maximize profits or minimize costs with limited resources. Therefore, many issues can be characterized as linear programming problems.

The research method is the use of linear programming solver as a tool in Goal programming software was applied to observe the optimal cost necessary to achieve monthly glass production output.

Table 1: Production Data for Bottle Glass Monthly Production Record

Soda Ash	Sand	Feldspar	Limestone	Salt cake	Selenium	Cobalt Oxide	Total Amount of Monthly production
1.09E+08	3102406	1550775	7455698	2575356	1381794	44628.1	125285217
1.01E+08	2860189	1429700	6873603	2374288	1273912	41143.8	115503716
1.08E+08	3057443	1528300	7347645	2538032	1361768	43981.3	123469489
1.03E+08	2915304	1457250	7006056	2420040	1298460	41936.6	117729447
1.06E+08	3012481	1505825	7239591	2500708	1341742	43334.5	121653762
1.06E+08	3024084	1511625	7267476	2510340	1346910	43501.4	122122336
1.11E+08	3147368	1573250	7563752	2612680	1401820	45274.9	127100945
1.1E+08	3124887	1562013	7509725	2594018	1391807	44951.5	126193082
1.06E+08	3024084	1511625	7267476	2510340	1346910	43501.4	122122336
1.08E+08	3079924	1539538	7401672	2556694	1371781	44304.7	124377354
1.06E+08	3002328	1500750	7215192	2492280	1337220	43188.5	121243759
1.09E+08	3102406	1550775	7455698	2575356	1381794	44628.1	125285217
1.09E+08	3102406	1550775	7455698	2575356	1381794	44628.1	125285217
1.01E+08	2863090	1431150	6880574	2376696	1275204	41185.5	115620860
1.02E+08	2900075	1449638	6969457	2407398	1291677	41717.6	117114443
1.06E+08	3002328	1500750	7215192	2492280	1337220	43188.5	121243759
1.09E+08	3102406	1550775	7455698	2575356	1381794	44628.1	125285217
1.07E+08	3045840	1522500	7319760	2528400	1356600	43814.4	123000914
1.08E+08	3057443	1528300	7347645	2538032	1361768	43981.3	123469489
1.08E+08	3057443	1528300	7347645	2538032	1361768	43981.3	123469489
1.08E+08	3057443	1528300	7347645	2538032	1361768	43981.3	123469489
1.09E+08	3102406	1550775	7455698	2575356	1381794	44628.1	125285217
1.09E+08	3089352	1544250	7424328	2564520	1375980	44440.3	124758070
1.11E+08	3147368	1573250	7563752	2612680	1401820	45274.9	127100945
1.12E+08	3169849	1584488	7617779	2631342	1411833	45598.3	128008809
98609280	2802173	1400700	6734179	2326128	1248072	40309.2	113160841
1.09E+08	3102406	1550775	7455698	2575356	1381794	44628.1	125285217
1.07E+08	3045840	1522500	7319760	2528400	1356600	43814.4	123000914
1.11E+08	3147368	1573250	7563752	2612680	1401820	45274.9	127100945
1.06E+08	3002328	1500750	7215192	2492280	1337220	43188.5	121243759

1.08E+08	3079924	1539538	7401672	2556694	1371781	44304.7	124377354
1.09E+08	3102406	1550775	7455698	2575356	1381794	44628.1	125285217
1.06E+08	3002328	1500750	7215192	2492280	1337220	43188.5	121243759
1.02E+08	2900075	1449638	6969457	2407398	1291677	41717.6	117114443
98762400	2806524	1402875	6744636	2329740	1250010	40371.8	113336557
1.08E+08	3079924	1539538	7401672	2556694	1371781	44304.7	124377354

Source: Bottle glass Company

BATCHES/DAY WITH PRICES

1.Sand #925/ton

2.SodaAsh #110,000/ton

3.Limestone #7,700/ton

4.Feldspar #12,500/ton

5.Salt Cake #86,000/ton

6.Solenuim #19,000/kg

7.Cobalt Oxide #2,608/kg

COST OF RAW MATERIALS PER PRODUCTION BATCH.

Sand (Silica sand)

1ton = #925

1batch = 784kg

Cost of sand per batch = 784÷1000 × #925 = #725.2

Limestone

1ton = #7,700

1batch = 224kg

Cost of limestone per production batch = 224÷ 1000 × #7,700 = #1724.8

Soda Ash

1ton = #110,000

1batch = 232kg

Cost of soda ash per batch = 232 ÷ 1000 × #110,000 = #25,520

Feldspar

1ton = #12,500

1batch = 29kg

Cost of Feldspar per production batch = 29 ÷ 1000 × #12,500 = #362.5

Salt cake (Sodium sulphate)

1ton = #86,000

1batch = 7kg

Cost of salt cake per production batch = 7 ÷ 1000 × #86,000 = #602

Selenium

1kg = #19,000

1batch = #17g

Cost of Selenium per production batch = 17 ÷ 1000 × #19,000 = #323

Charcoal Cobalt (Cobalt Oxide)

1kg = #2,608

1batch = 4g

Cost of charcoal cobalt per batch = 4÷ 1000 × #2,608 = #10.432

TOTAL COST OF RAW MATERIALS PER PRODUCTION BATCH

		#
Soda Ash	=	25,520

Sand	=	725.2
Feldspar	=	362.5
Limestone	=	1,742.8
Salt cake (Sodium sulphate)	=	602
Selenium	=	323
Charcoal cobalt	=	10.43
Total cost	=	29,267.932

Monthly production data were collected for three years production. The data were collected and analyzed to observe and to advice the case company on the optimal cost to budget every month for their production the

Analysis and Results

>> Optimal solution FOUND

>> Maximum = 1.2252e+008

*** RESULTS - VARIABLES ***

Variable	Value	Obj. Cost	Reduced Cost
X1	1.14757	1.06765e+008	0
X2	0	3.03393e+006	0.00128057
X3	0	1.51655e+006	0.000806818
X4	0	7.29115e+006	0.00260972
X5	0	2.51852e+006	0.000452979
X6	0	1.3513e+006	8.85583e-005
X7	0	43643.1	1.69593e-005

*** RESULTS - CONSTRAINTS ***

Constraint	Value	RHS	Dual Price
Row1	1.25285e+008	1.25285e+008	0.97793
Row2	1.15504e+008	1.15504e+008	0
Row3	1.23469e+008	1.2347e+008	0
Row4	1.17729e+008	1.1773e+008	0
Row5	1.21654e+008	1.21654e+008	0
Row6	1.22122e+008	1.22123e+008	0
Row7	1.27101e+008	1.27101e+008	0

Row8	1.26193e+008	1.26193e+008	0
Row9	1.22122e+008	1.22123e+008	0
Row10	1.24377e+008	1.24378e+008	0
Row11	1.21244e+008	1.21244e+008	0
Row12	1.25285e+008	1.25286e+008	0
Row13	1.25285e+008	1.25286e+008	0
Row14	1.15621e+008	1.15621e+008	0
Row15	1.17114e+008	1.17115e+008	0
Row16	1.21244e+008	1.21244e+008	0
Row17	1.25285e+008	1.25286e+008	0
Row18	1.23001e+008	1.23001e+008	0
Row19	1.23469e+008	1.2347e+008	0
Row20	1.23469e+008	1.2347e+008	0
Row21	1.23469e+008	1.2347e+008	0
Row22	1.25285e+008	1.25286e+008	0
Row23	1.24758e+008	1.24758e+008	0
Row24	1.27101e+008	1.27101e+008	0
Row25	1.28009e+008	1.28009e+008	0
Row26	1.13161e+008	1.13161e+008	0
Row27	1.25285e+008	1.25286e+008	0
Row28	1.23001e+008	1.23001e+008	0
Row29	1.27101e+008	1.27101e+008	0
Row30	1.21244e+008	1.21244e+008	0
Row31	1.24377e+008	1.24378e+008	0
Row32	1.25285e+008	1.25286e+008	0

Row33	1.21244e+008	1.21244e+008	0
Row34	1.17114e+008	1.17115e+008	0
Row35	1.13337e+008	1.13337e+008	0
Row36	1.24377e+008	1.24378e+008	0

>> Optimal solution FOUND
>> Minimum = 0

*** RESULTS - VARIABLES ***

Variable	Value	Obj. Cost	Reduced Cost
X1	0	1.06765e+008	-1.06765e+008
X2	0	3.03393e+006	-3.03393e+006
X3	0	1.51655e+006	-1.51655e+006
X4	0	7.29115e+006	-7.29115e+006
X5	0	2.51852e+006	-2.51852e+006
X6	0	1.3513e+006	-1.3513e+006
X7	0	43643.1	-43643.1

*** RESULTS - CONSTRAINTS ***

Constraint	Value	RHS	Dual Price
Row1	0	1.25285e+008	0
Row2	0	1.15504e+008	0
Row3	0	1.2347e+008	0
Row4	0	1.1773e+008	0
Row5	0	1.21654e+008	0
Row6	0	1.22123e+008	0
Row7	0	1.27101e+008	0

Row8	0	1.26193e+008	0
Row9	0	1.22123e+008	0
Row10	0	1.24378e+008	0
Row11	0	1.21244e+008	0
Row12	0	1.25286e+008	0
Row13	0	1.25286e+008	0
Row14	0	1.15621e+008	0
Row15	0	1.17115e+008	0
Row16	0	1.21244e+008	0
Row17	0	1.25286e+008	0
Row18	0	1.23001e+008	0
Row19	0	1.2347e+008	0
Row20	0	1.2347e+008	0
Row21	0	1.2347e+008	0
Row22	0	1.25286e+008	0
Row23	0	1.24758e+008	0
Row24	0	1.27101e+008	0
Row25	0	1.28009e+008	0
Row26	0	1.13161e+008	0
Row27	0	1.25286e+008	0
Row28	0	1.23001e+008	0
Row29	0	1.27101e+008	0
Row30	0	1.21244e+008	0
Row31	0	1.24378e+008	0
Row32	0	1.25286e+008	0

Row33	0	1.21244e+008	0		
Row34	0	1.17115e+008	0		
Row35	0	1.13337e+008	0		
Row36	0	1.24378e+008	0		

Discussion and Conclusion

Goal programming software was used analyze and to appraise the cost of production output of the company. The linear programming solver applied as a tool in goal programming was used to model and to optimize the cost thereby reducing wastes that occurs in the production process. From the result, the maximum cost of producing bottle glasses at any given month in any year if wastes were eliminated in the cost of production is 1.2252E0.8. Having found the result, the researcher concludes that in some certain months, there are excess wastes of cost of production and also the stability of the company's economy is at stack. To solve the problem found in the work, the researcher recommend the result to the case company and also recommend that there is a need to resolve the production process and also recommend the use of axiomatic model and its traditional method to solve the problem of production process.

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