



## Thermo-economic design and optimization of Parallel-plates Counter flow Heat exchanger

Mustafa S. Ahmed and Mostafa H. Sharqawy

Department of Mechanical Engineering, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia

**Abstract** In the present study an exergy-economic criteria that combine the exergy destruction caused by system irreversibilities- due to operation of the heat exchanger on temperature and pressure difference- and the total cost accompanied with this destruction. The prediction of the outlet conditions using EES software for different shape parameter also established, furthermore the optimization has been done to find the optimum heat exchanger size by calculating different aspect ratios defined as  $D_{ratio}$ .

**Keywords** Heat exchanger, Exergy-economic analysis, Thermodynamic design

### Nomenclature

$W$	width of the heat exchanger	Cm
$L$	length of heat exchanger in flow direction	Cm
$D_{ratio}$	thickness ratio	
$\dot{m}$	mass flow rate	kg s <sup>-1</sup>
$P$	Pressure	kPa
$R$	Heat capacity ratio	
$th_m$	thickness of plate	mm
$K_m$	metal conductivity at average temperature	W. m <sup>-1</sup> K <sup>-1</sup>
$th_c$	channel width on cold-side	mm
$th_H$	channel width on hot-side	mm
$N_{ch}$	number of channel pairs	
$h_{av}$	average convective heat transfer coefficient	W. m <sup>-2</sup> K <sup>-1</sup>
$U$	overall heat transfer coefficient	W. m <sup>-2</sup> K <sup>-1</sup>
$T_{av}$	average temperature	K
$Q$	heat capacity of the heat exchanger	kW
$NTU$	No of transfer unit	
$C$	Total annual cost	\$ Year <sup>-1</sup>
$Pr$	Prandtl Number	
$Re$	Reynolds Number	
$St$	Stanton Number	
$I$	Annual capital cost	\$ Year <sup>-1</sup>
$I_0$	Fixed maintenance cost	\$ Year <sup>-1</sup>



$I_F$	Capital cost conserved with the heat transfer area	\$
$NN$	Payback period	Year
$t$	annual operating time	s
$f$	relative lost of exergy	
$J$	Loan rate of interest	
$i_c$	tax rate	
$C_e$	Unit price of exergy	\$ J <sup>-1</sup>
$n_t$	Exergy-economic criteria	\$ J <sup>-1</sup>
$S_{genT}$	Entropy generation due to temperature difference	W k <sup>-1</sup>
$S_{genP}$	Entropy generation due to pressure difference	W k <sup>-1</sup>
<b>Greek symbols</b>		
$P$	Density	kg m <sup>-3</sup>
$\epsilon$	Effectiveness of the Heat exchanger	
<b>Subscripts</b>		
$C$	Cold	
$H$	Hot	
$i, in$	Inlet	
$o, out$	Outlet	

## Introduction

Parallel-plates Counter flow Heat exchanger is a thermal equipment that widely used in many engineering applications to deliver energy between systems, the classical example of this kind that has been used in the cryogenics or refrigeration at very low temperatures, where the power requirement is highly related to the entropy generated in the cold space. The performance improvement of this type of heat exchangers is very important in order to save energy and improving the efficiency.

Engineering thermodynamics methodology is one way to analysis such thermal systems. However the exergy analysis technique and entropy generation minimization are the most used one for nowadays engineering thermodynamic applications.

Although the applicability of a system or process is usually based on various factors such as technical performance, efficiency and the environmental impact, the economics aspects can also play a critical role in the designing of such systems. Hence the Analysis, design and optimization that combine technical disciplines like thermodynamics with the economical aspects often provides acceptable compromises and also emphasizes the applicability of the system.

In the design of Parallel-plates counter flow heat exchangers the goal of the optimization procedures from the economical point of view is to minimize the initial and operating cost while maintaining better heat exchanger effectiveness.

Therefore the employment of exergy-economic criteria to evaluate heat exchanger performance may open the floor for a new approach that can give practical range of operation for better trends when designing and optimizing these thermal applications .

Different design problems of thermal systems based on the thermodynamic optimization techniques [1–4] were used to find the least exergy destroys while taking into account the tradeoff between two or more competing irreversibilities.

An entropy generation minimization has been done for the counter flow heat exchanger that used Cryocooler systems. The detailed analysis presented the optimal CFHX configuration which produces lesser entropy generation and heat leakage, using different CFHX length, width and high parameters [5].

The parallel-plate heat exchanger in a counter flow configuration has been optimized based on thermodynamic methodology subject to volume constant by Juan et al., [6] the optimization carried out based on the spacing between the two channel and the total heat transfer area between the two sides. Furthermore the entropy generation has been calculated to show the system irreversibility.



Energy efficiencies not always the criteria to assess how the performance of a system approaches near ideality and also not properly describe factors that cause performance to deviate from ideality, several studies based on thermo-economic and exergy-economic criteria has been developed on Heat Exchangers to enhance the overall system performance [7-10], These criteria provide the practical link and can give the full picture to the designers of such systems.

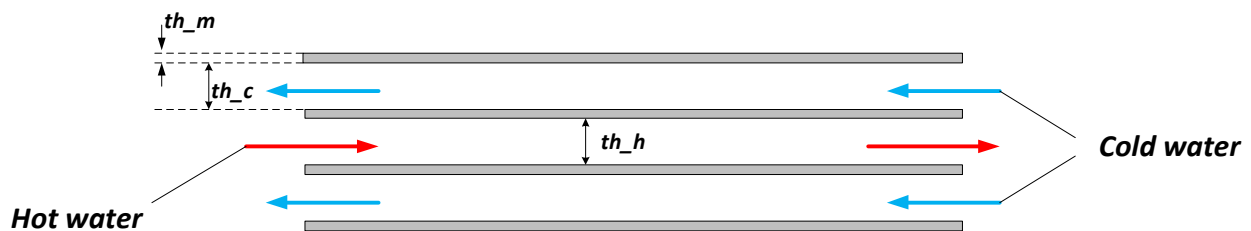
Sahin et al., [11] investigated the single pass counter-flow heat exchanger by using the actual heat transfer rate per unit total cost considering lost exergy and investment costs as an objective function to evaluate the performance. This model assumed that the irreversibilities only due to heat transfer between the hot and cold streams and no irreversibilities such due to pressure drops and flow imbalance.

Yourong et al., [12] developed a model based on a exergy-economic criteria for describing the performance of different heat exchanger configurations. This criteria which are defined as the total costs per unit heat transfer rate has been examined using an illustrative example and finally concluded that the total costs per unit heat transfer rate decreases with the increment of  $R$  for the same flow arrangement.

### Mathematical Model

#### Heat transfer

The plate heat exchanger operating in a counter-flow configuration between cold and hot water streams shown schematically in Figure 1.



**Figure 1:** The parallel-plates structure of the counter-flow heat exchanger.

#### The thermodynamic assumptions for the Heat exchanger:

- Adiabatic rectangular boundary.
- No heat transfer along the longitudinal direction.
- The two streams carry single-phase fluids.

The state equations are obtained from the differential energy balances:

#### Hot stream

$$\frac{dT_H}{dx} = - \frac{2N_{ch}(T_H - T_C)}{m'_H C_H \left[ \frac{1}{h_{HW}} + \frac{th_m}{k_{mW}} + \frac{1}{h_{CW}} \right]}$$

#### Cold stream

$$\frac{dT_C}{dx} = - \frac{2N_{ch}(T_H - T_C)}{m'_C C_C \left[ \frac{1}{h_{HW}} + \frac{th_m}{k_{mW}} + \frac{1}{h_{CW}} \right]}$$

The two differential equations has been treated as coupled equations and solved analytically, moreover to simplify the solution reasonable guess value and bounds for the  $T_{C\ out}$  used to find constant  $C_1$  and  $C_2$  in equations (3-6).



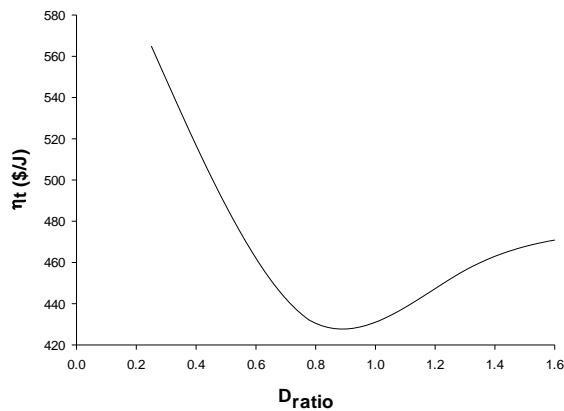


Figure 2: Variation of the exergy-economic criteria with different  $D_{ratio}$  parameter for  $f = 2$

$$Z = -2 \cdot \frac{N_{ch}}{\left( \frac{1}{h_{H,avg} \cdot W} + \frac{th_m}{k_m \cdot W} + \frac{1}{h_{C,avg} \cdot W} \right)} \tag{1}$$

$$D_H = \frac{Z}{\dot{m}_H \cdot c_H} \tag{2}$$

$$D_C = \frac{Z}{\dot{m}_C \cdot c_C} \tag{3}$$

The equations solved simultaneously by applying the boundary condition for  $x (0 - L)$  and find the constants values:

$$T_{co} = C_1 + C_2 \exp[(D_H - D_C) \cdot x] \tag{4}$$

$$T_{ho} = \frac{[C_2 \exp[(D_H - D_C) \cdot x] D_H + D_C \cdot C_1]}{D_C} \tag{5}$$

Based on the average conditions  $T_{C_{av}}, T_{H_{av}}$  The iteration procedure accomplished to predict the heat transfer coefficients  $h_{H_{av}}, h_{C_{av}}$  using the EES (Engineering Equations Solver software). Finally the outlet conditions obtained using specified standard dimensions of the heat exchanger. These relations that related to the heat transfer mechanism described below.

$$T_{C_{av}} = \frac{T_{ci} + T_{co}}{2} \tag{6}$$

$$T_{H_{av}} = \frac{T_{hi} + T_{ho}}{2} \tag{7}$$

**Overall heat transfer coefficient**

The coefficient  $U$  calculated by the summation of cold and hot sides resistances using the next equation, in addition to the conduction resistance across the solid wall separating the two streams.

$$\frac{1}{U} = \frac{1}{h_{H_{av}}} + \frac{1}{h_{C_{av}}} + \frac{th_m}{K_m} \tag{8}$$

The effectiveness–NTU relation for the counter flow heat exchanger is as follows:

**No. of transfer units**

$$NTU = \frac{UA}{C_{min}} \quad (9)$$

The Stanton number definition is used to express the total NTU for the two sides [6].

$$\frac{1}{NTU} = \frac{R}{St_h} \left( \frac{th_h}{L} \right) + \frac{1}{St_c} \left( \frac{th_c}{L} \right) \quad (10)$$

Where St number can be calculated using the next correlation for laminar flow between two plates [8]:

$$st = \frac{8.235 Pr}{Re} \quad (11)$$

**Heat exchanger effectiveness**

$$\varepsilon = \frac{1 - \exp(-NTU(1-R))}{1 - R \exp(-NTU(1-R))} \quad (12)$$

Where the heat capacity ratio is given as:

$$R = \frac{C_{min}}{C_{max}} \quad (13)$$

**Exergy-economy**

After the temperature been evaluated based on the average heat transfer coefficients among the plates, the Code has been verified and tested at different parameters such as thickness ratio ( $D_{ratio} = \frac{th_H}{th_C}$ ) in order to find the optimum range. Moreover the exergetic analysis combined with economic criteria has been used to test the performance of the Heat exchanger.

**Model development**

From thermo-economic point of view, the total consumed cost is due to the heat transfer generated and flow resistance (lost of work), then the annual capital costs of the equipment [2] can be represented as :

$$I = ((I_o + I_F) / N) [0.5(i_c + j)(N + 1) + 1 + 0.07N] \quad (14)$$

Then the total annual cost to operate the heat exchanger can be described as:

$$C = C_{\varepsilon} t (T_0 S_{gen}) + I \quad (15)$$

**Entropy generation:**

The main contributor on the exergy destruction analysis is the entropy generation which can be calculated using the next equations:

*Temperature difference Entropy generation ( $S_{genT}$ ):*

$$S_{genT} = c_C \left( \frac{m_C}{N_{ch}} \right) \ln \left( \frac{T_{co}}{T_{ci}} \right) + c_H \left( \frac{m_H}{N_{ch}} \right) \ln \left( \frac{T_{ho}}{T_{hi}} \right) \quad (16)$$

*Pressure difference Entropy generation ( $S_{genP}$ ):*

$$S_{genP} = \left( \frac{m_C}{N_{ch}} \right) \ln \left( \frac{p_{C2}}{p_C} \right) + \left( \frac{m_H}{N_{ch}} \right) \ln \left( \frac{p_{H2}}{p_H} \right) \quad (17)$$



The total entropy generation has been plotted along with the channel width of the heat exchanger for different length (170,135,100 cm) as shown in fig (6). Moreover the pressure drops on both sides along the heat exchanger channel length presented in fig (7).

$$p_{C2} = p_C - \Delta P_C \quad (18)$$

$$p_{H2} = p_H - \Delta P_H \quad (19)$$

The two streams have the same flow path length (L) with thickness  $th_c$  for cold stream and  $th_H$  for the hot stream.

### Exergy-economic criteria

The objective is to minimize the total cost accompanied with exergy destruction and the total capital cost which is been related to the heat capacity rate , the Exergy-economic criteria ( $\eta_t$ ) represent this relation and will be the factor to be minimized by varying the geometrical aspect ratios of the system.

$$\eta_t = \frac{C_e(T_0 S_{gen}) + \frac{I}{t}}{Q} \quad (20)$$

The relative lost of exergy ( $f$ ) can be assumed to relate the cost of exergy destruction and the total annual cost as follows:

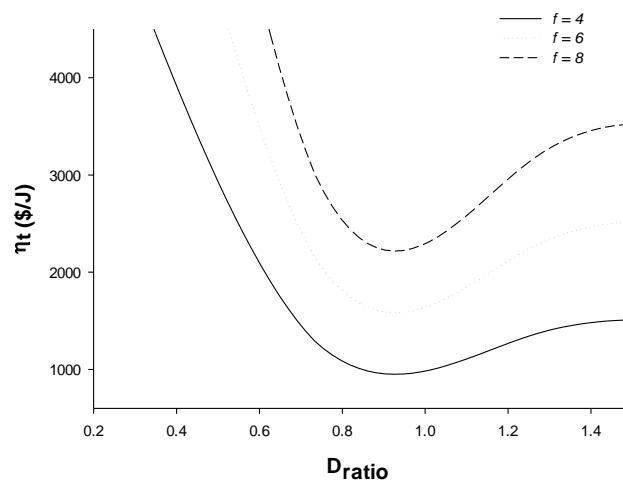
$$f = \frac{C_e + I}{I} \quad (21)$$

Finally:

$$\eta_t = \frac{(I.f - I).(T_0.S_{gen}) + \frac{I}{t}}{Q} \quad (22)$$

### Results and Discussion

The detailed analysis has been done based on a real data of a Gasket Heat exchanger Type ALFA LAVAL that operate on temperature difference of  $11.1^\circ\text{C}$  for child water cooling system. Moreover a parametric analysis carried out for the seek of optimum performance.



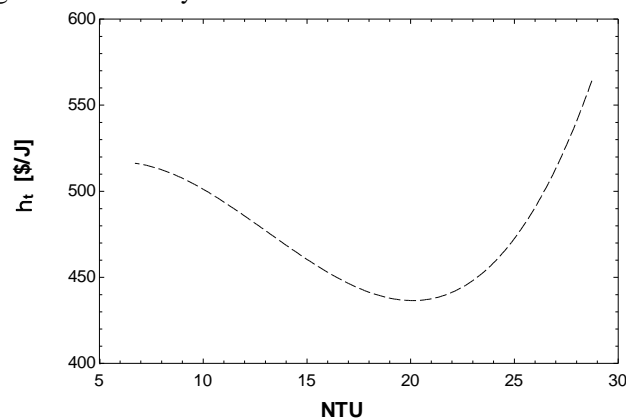
**Figure 3:** Variation of the exergy-economic criteria for various  $D_{ratio}$  parameters with different relative lost exergy cost parameter  $f$ .



**Table 1:** Plate Heat exchanger (ALFA LAVAL) data

$T_{ci} = 1.1^\circ\text{C}$	$W = 63\text{ cm}$
$T_{hi} = 12.2^\circ\text{C}$	$L = 135\text{ cm}$
$p_C = 44\text{ kPa}$	$N_{ch} = 100$
$p_H = 68\text{ kPa}$	$I = 16,650\text{ \$/Year}$
$m_c^\circ = 33.6\text{ kg/s}$	$t = 2.6 \times 10^7\text{ s}$
$m_H^\circ = 43.2\text{ kg/s}$	
$th_C = 3\text{ mm}$	
$th_H = 3\text{ mm}$	
$th_m = 0.6\text{ mm}$	

The exergy-economic criteria ( $\eta_t$ ) used to assess the economical aspects for this type of plate Heat exchanger which present how we can achieve lesser amount of cost accompanied with the total investments and destruction cost due to the difference in temperature and pressure. The lowest value of ( $\eta_t$ ) indicates the lower cost while high amount of heat transfer rate gained from the system.

**Figure 4:** Variation of the exergy-economic criteria for various  $NTU$ 

A general optimum performance can be observed from fig (3). showing the different thickness ratio  $D_{ratio}$  varies from (0.25 – 1.5) with the given heat exchanger width and length. Fig (3) shows the variation of ( $\eta_t$ ) with different values of the relative lost exergy cost parameter within the optimum  $D_{ratio}$  range (0.8 – 1) the value of the ( $\eta_t$ ) is about 950 \$/J for  $f$  value of 4, and about 1600 \$/J at  $f = 6$ . Also at  $f = 8$  within the optimum range the ( $\eta_t$ ) value is approximately 2200 \$/J.



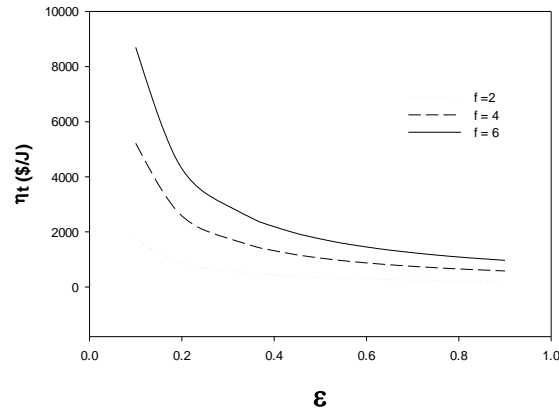


Figure 5: Variation of the exergy-economic criteria for various heat exchanger effectiveness

Finally the summary of the thermal performance for optimized  $D_{ratio}$  range given in fig (4) and the NTU corresponding to the minimum  $\eta_t$  value (440 \$/J for  $f = 2$ ) is estimated about 20 .moreover from the plot in fig (5) as the value of the relative lost exergy increases the exergy-economic criteria increase for various effectiveness , which means that the cost due to exergy destruction become relatively high when the  $f$  value increased.

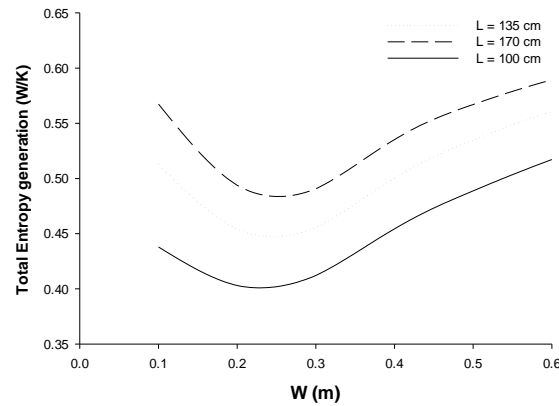


Figure 6: The total entropy generation along with the channel width

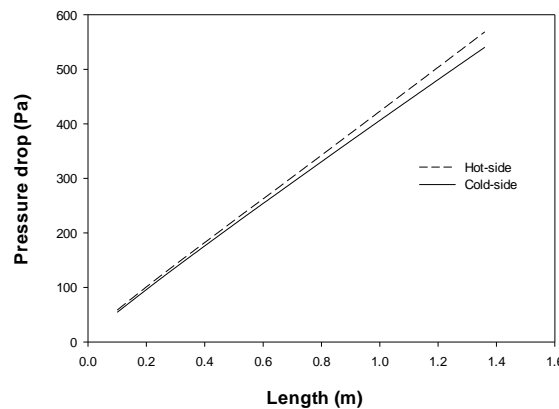


Figure 7: The pressure drop variation with the channel length for cold and hot sides.



### Conclusions

The exergy-economic criteria presents an efficient optimization technique for including the capital costs and the amount of heat transfer rate while calculating the irreversibility loss cost in the Heat exchanger. The design parameter  $D_{ratio}$  play a key role to predict the outlet conditions then the total performance of the heat exchanger. Also it provides a good link to the exergy-economic criteria that leads to the Global thermo economic performance. The detailed analysis presented here may offer a real design analysis and performance investigation for this kind of Liquid-to-Liquid Heat exchanger that operate normally in the district cooling systems to provide Buildings with a child water.

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