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PROFICIENT CLASSROOM MANAGEMENT THROUGH FOCUSED MATHEMATIC TEACHING

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Abstract

A not entirely unusual position among teachers is that they believe that they must first establish a peaceful classroom before they can begin to teach the subject. This research, shows how a proficient mathematics teacher teaches his subject and thereby creates a quiet and focused classroom and exerts effective leadership, just by teaching mathematics. The researchers observed a male mathematics teacher for almost half a year, i.e. one semester. The results of research present several patterns that the researchers saw during the observations of his teaching. The teacher showed an interest in each student's mathematical thinking and expressed explicitly how students were expected to learn mathematics. He also directed students' attention to mathematics and established a culture where all solutions were important in the teaching process. In the teaching process, he used multiple representations to motivate students and a lot of supportive expressions that made them feel that they were able to learn mathematics. He worked patiently to establish structures, and there was almost no disruptive behaviour. Students simply did not have time to interfere because they were so engaged in learning mathematics.
Keywords: *classroom management, mathematics teaching, proficient teacher.*

Introduction

A central task for teachers is to lead classroom activities so that students can acquire the knowledge stipulated in national policy documents. Coping with this work requires different abilities (Shulman, 1987). Based on empirical research, Granström (1998, 2006, 2007) suggests one way to describe teachers' management roles in the classrooms. He argues that teachers are responsible for all activities and processes occurring in classroom. This presupposes that teachers are able to efficiently manage at least two important roles associated with classroom management, leadership and teachership. Granström (2007) presents a somewhat simplified picture of the work of teachers which is not comprehensive, but that captures key aspects of teachers' work. Teachership means (a) the knowledge of a field (a subject), and (b) the ability to teach this subject. Leadership is about (a) the knowledge of classroom interaction and group processes, and (b) the ability to manage classroom interaction and group processes. These concepts are neither comprehensive nor categorical, but they provide a sense of the features connected to the concepts and offer the opportunity to investigate teachers' classroom management (Granström, 2007). Granström (2007) constructs a four-field figure (see Figure 1), where he describes teachers who exert good or bad leadership as well as good or bad teachership.

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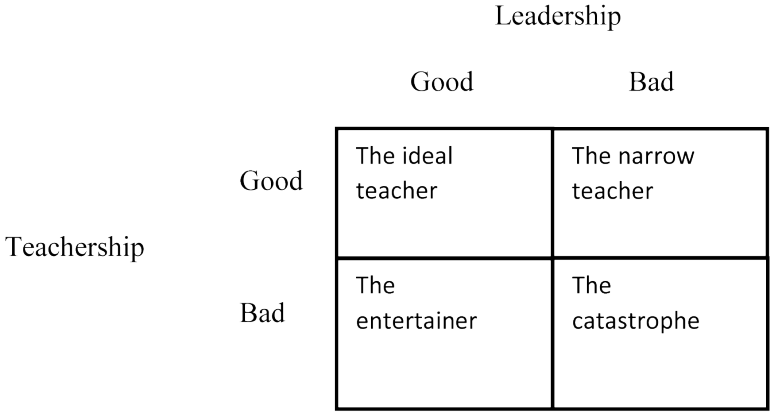

Figure 1: Four different teacher types exerting classroom management in different ways (based on Granström, 2007, p. 18).

As seen in the figure, Granström named the four positions as follows:

- *The ideal teacher* is a teacher who knows how to teach and is highly capable of leading the classroom.
- *The entertainer* is capable of organizing and activating students, but lacks comprehensive knowledge of the subject matter.
- *The narrow specialist* is a knowledgeable teacher who is not skilled in organizing the classroom.
- *The catastrophe* is incapable of maintaining order in the classroom and lacks the knowledge to teach the subject.

Thus, each of the four fields includes aspects of both leadership and teachership. A not entirely unusual thought among teachers is that a teacher must first establish peace and quiet in the classroom before he or she can begin to teach the subject. In this study, we will show how a proficient maths teacher (the ideal teacher in Granström's model, 2007) teaches his subject and thereby creates peace and quiet from the class and exerts effective leadership in the classroom. The following figure (Figure 2) illustrates the shift from exerting leadership in order to teach to exercising teachership and thereby applying effective leadership.

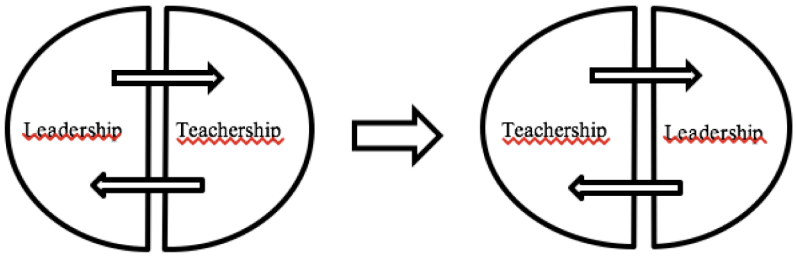

Figure 2: Illustrates the shift from initially exerting leadership to initially exerting teachership.

What defines a proficient teacher looks different depending on who defines this and what perspective they adopt when defining skill. Several studies of proficient teachers' present qualities of teachers that can often be related to Granström's (2007) concept of teachership

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and leadership. When teachership is discussed in the literature, the following procedures occur among proficient teachers: (a) the teacher explains well (Beishuizen et al., 2001; Murray, 2001) (b) the teacher offers individualized instruction (Kutnick & Jules, 1993; Murray, 2001; Murphy, Delli & Edwards, 2004) (c) the teacher creates interest (Läänemets, Kalamees-Ruubel, & Sepp, A. 2012;), and (d) they vary their teaching (Kutnick & Jules, 1993; et al., 2012). Leadership involves (a) the teacher being positive, helpful and friendly, and showing concern (Läänemtes, et al., 2012; Murray, 2001), (b) the teacher behaving politely (Murray, 2001), (c) the teacher maintaining relationships (Beishuizen et al., 2001), and (d) the teacher being in control of the classroom (Kutnick & Jules, 1993). In this study, we want to show how effective teachership also results in good leadership. An Australian study shows, however, that proficient teaching of mathematics involves both teachership and leadership (Clark, 1997). Proficient teachers focus on mathematics and discuss mathematics with students (teachership), as well as creating good relationships and showing elation and joy if their students succeed (leadership).

The aim of this research was to describe and analyse how a proficient mathematics teacher taught mathematics and thereby created effective leadership in the classroom based on observation as a method. The following research questions were answered: (a) What patterns can be seen in a proficient mathematics teacher's teaching that also affect classroom management? and, (b) What can we learn about classroom management based on a proficient mathematics teacher's ways of teaching?

The results of research give examples of teaching activities that can help teacher exert effective leadership in mathematics classroom.

Characteristics of Classroom Management

Classroom management could be understood as integrated skills (Nordenbo, Søgaard Larsen, Tifitkçi, Wendt & Østergaard, 2008) which focus on leadership that enables all students to develop academically as well as socially and morally. Good classroom management can be likened to effective classroom management (Brophy, 1988, 2006) in which all students receive customized challenges or opportunities based on their individual circumstances in order to develop. Such an environment optimizes opportunities for all students to learn (Brophy, 2006). Research on classroom management has shown that good classroom management is a result of conscious long-term prevention and patient effort on the part of the teacher (Brophy, 1988, 2006; Evertson & Weinstein, 2006; Nordenbo, Søgaard Larsen, Tifitkçi, Wendt & Østergaard, 2008; Roache & Lewis, 2011; Emmer & Sabornie, 2014). We identified six important factors for good classroom management.

Establishing the School Class as a Social System

The school class as a social system is built on the teacher, the students and their social and cultural backgrounds. Different backgrounds give students different conditions and different expectations regarding school, the teaching and the teachers. Expectations also reflect the way in which the students are socialized. Brophy (1988) emphasizes how important it is for teachers to socialize the students to the social system that the specific educational setting represents. This means exerting an influence on students' attitudes, beliefs and behaviours. It also means expressing how students are expected to behave as well as reinforcing desirable behaviours (Epstein, Atkins, Cullinan, Kutash & Weaver, 2008) when needed. The school class as a social system is based on an ecological approach found in Doyle (2006).

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Creating Respectful Relationships with Each Individual

Teachers who are good classroom managers understand how important they are as leaders. They also understand that their leadership is based on respectful relationships between them and the students as individuals (Lewis, Romi, Katz & Qui, 2008; Muntuoro & Lewis, 2014). They ultimately understand that their leadership is built upon the students accepting to follow them as leaders because they provide the students with opportunities to develop through their teaching. There is thus a link between relationships and achievements (Roache & Lewis, 2011). Classroom management is expressed as a concern for students (Woolfolk Hoy & Weinstein, 2006), as an interest in each student's situation.

Establishing a Good Classroom Climate

Learning environments characterized by respectful relationships are easier to manage. Not just because the students feel involved, but also because such environments are characterized by stimulating learning activities as part of a friendly climate. Such environments feature more support for student learning and a leader who, if necessary, changes the environment in order to establish or maintain a good classroom climate (Epstein, Atkins, Cullinan, Kutash & Weaver, 2008). Brophy (1999) describes such environments as supportive learning communities where teachers, being role models, socialize students in a desirable manner. To lead a class means, based on such reasoning, to create and adjust the classroom climate so that academic, social and moral development is made possible for each student (Evertson & Weinstein, 2006; Wubbels, 2011). Brophy (1999) points out that a good classroom climate, in the sense of a productive context, is marked by a concern that permeates teacher-student interaction as well as student-student interaction.

Establishing Structures, Procedures and Rules

A collaboration that begins with teachers initiating a discussion in order to establish and then uphold a smaller number of rules and procedures has good potential to be effective (Brophy, 1988; 2006). Good leadership means that the teacher introduces, establishes and maintains a small number of rules and procedures as part of a fair system that protects and respects the students (Woolfolk Hoy & Weinstein, 2006). Rules and procedures help the teacher to create standards and thus become a tool for learning and effective leadership (Doyle, 2006). Teachers who, at the beginning of a semester or collaboration, work patiently to establish structures experience less disruptive behaviour, which is related to the established common rules and procedures (Epstein, Atkins, Cullinan, Kutash & Weaver, 2008). Students who feel involved get a sense of community, develop self-awareness and commitment, and perform better (Lewis, Romi, Katz & Qui, 2008).

Having Clear Expectations and Motivating the Students

A well-organized school activity is based on regulatory documents such as the Swedish Education Act and the syllabus, and strives to optimize the quality of every lesson (Brophy, 2006). Good classroom management is crucial for students, so they can develop an internal motivation to learn. Most children and students are motivated by (a) interesting tasks, (b) a perceived expectation that they can learn and be successful, and (c) being offered support to develop their skills and abilities (Borich, 1996). Teachers have a crucial role in creating such learning environments (Darling-Hammond & Bransford, 2005). Students' involvement increases if there

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are internal motives to make an effort and succeed in tasks that relate to their lives, experience or future events (Garofalo & Lester, 1985). Expectations are not just about the students. They are also about the teacher's demands and expectations of their own leadership (Lewis, Romi, Katz & Qui, 2008). Good classroom management thus means that the teacher leads and shows what students are expected to do (Brophy, 1988; Nordenbo, Søgaard Larsen, Tifitkçi, Wendt & Østergaard, 2008; Roache & Lewis, 2011).

Establishing Reasonable Disciplinary Interventions

Students, who have not yet acquired social and moral skills and abilities, need continued help and support in order to learn them. Such support is usually provided by a teacher, other students or the whole class. Teachers have an important role for students who violate the norms, values or rules, partly in seeking to understand the students' perspectives (Ziehe, 2000; 2010; 2012) and partly in convincing or, at worst, forcing the students to change their behaviours (Brophy, 1988; Jennings & Greenberg, 2009; Roache & Lewis, 2011). The way teachers discipline matters to students, and teachers consequently need to consider how they treat and support students' academic and social development (Beaman & Wheldall, 2000). If this is done in a productive way, by teachers showing that the students' efforts and achievements are appreciated, the students develop their self-awareness and involvement, which allows them to work better (Beaman & Wheldall, 2000; Lewis, Romi, Katz & Qui, 2008). Teachers also need to explain the rationale behind the discipline they use if they want to stimulate the students' sense of accountability (Lewis, Romi & Roache, 2012).

Methodology of Research

Fieldwork

A hallmark of ethnographic field studies is that they are often described as extensive, at the places or in the practices studied (Delamont, 2006). The idea of fieldwork, to be participating here and now, by "studying at first-hand what people do and say in particular contexts" (Hammersley 2006, 4) still applies. But the traditional notion of fieldwork that takes several years has been challenged by Barth (1994) and perhaps even more by Hannerz (2001), who problematize the basis of Malinowski's (1922) ideas about fieldwork whereby a researcher stayed in one place for a long time and argues for the possibilities offered by shorter fieldwork. The researchers spent half a year following the selected maths teacher as he taught different students in different classrooms at a secondary school. The empirical data was gathered through yo-yo fieldwork (Wulff 2002). As Wulff (2002) suggests, the researchers were stationed in the environment, the classroom where mathematics lessons were conducted, made observations, wrote field notes and gathered as much empirical data as possible for our purpose. They oscillated between being present in the field and days distant from the field to transcribe field notes and reflect on what we had experienced so far. Permission to follow the proficient mathematics teacher had been gained through contact with him. The researchers moved in and out of mathematical practice, week after week for half a year, i.e. one semester (Hannerz 2001). Initially, they sat or walked around during the practical sessions, made open observations and wrote visible field notes. The researchers tried to be open and inquisitive (Geertz 1973) and to write about the interactions between the maths teacher and his students. They tried to register routines and rituals, and to pay attention to critical events. The researchers also made lists of tools, materials and posters or signboards.

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Participant

The aim of the research was to describe and analyse how a teacher of mathematics who is perceived as proficient teaches mathematics. To study how this happens in practice, the researchers obtained access to a classroom where such a teacher appears. One of the researchers had heard through contacts of a teacher who many perceived as proficient. The criteria to be considered were that (a) several groups who were familiar with teaching performed by the teacher would speak well of the person's teaching and (b) that the teacher's students performed a little better than one would expect. In this research, a strategic selection of an appropriate teacher was therefore made. The maths teacher selected was perceived as skilful by his school management, colleagues, students, parents, former students and parents of these children. He had around 40 years of experience working as a mathematics teacher at Swedish secondary schools, and he still enjoyed it.

Analysis

The interpretation of the observation and field notes began at the time of the lessons. The researchers own thoughts were written down in brackets in the observation block. The next step in the process was retyping observation notes using a computer, in connection with the implemented additional interpretations while the notes became richer. Then the researchers read the data to get an overall impression. Sections containing relevant information were highlighted in relation to the purpose of the research. Various portions of the material were compared with each other to find patterns in data and themes. The next step was to describe and conceptualize the patterns that responded to the study's purpose and issues. The patterns were named primarily to reflect the data, which is the most common technique for naming the themes and categories (Merriam, 1998).

Ethical Considerations

Based on the ethical principles of humanities and social sciences (CODEX, 2012), the following considerations were made in the implementation of the research. Both written and verbal information about the research project were given to the teacher, student, parent or guardian. In addition, the school management was also informed. It is imperative that the individual participates voluntarily in the research project, and that they can cancel their participation without any consequences. Consent for participation has thus been given by the teacher without any pressure, either from the project or from the immediate surroundings. Students and their parents or guardians have also given consent for the researcher to participate in classes and to describe and analyse teaching. All collected empirical data from field notes has been handled, processed, and reported so that all the individuals involved are anonymous. The teacher has had access to draft reports and thus the opportunity to comment on the content and interpretation. On these occasions, the teacher has often been in agreement with the researcher about his interpretations. In some cases, the teacher helped with clarifications. In the described research project, it has been clarified to the involved teacher that the research material will be used only for research purposes.

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Results of Research

Mathematics-intensive Classroom

In Leif's classroom, both he and the students were orientated towards mathematics, which resulted in a mathematics-intensive classroom.

Even before the researchers entered the classroom for the first lesson, it was obvious that Leif was a teacher with good contact with the students, both those he taught and those he just met in the corridor. Leif greeted the students that came his way. He said "Hello!" and always followed his greeting with the student's name. "Hello Calle" or "Hello Lisa". It became clear that he knew the students' names. When he opened the door and the students entered the classroom, the students went directly and sat down in their places. Once all the students had sat down, the teacher began his mathematical reasoning. As researchers we sat at the back of the classroom and were completely astonished.

The students and Leif talked about mathematics, they used mathematical words, they solved problems, and they discussed different solutions: why a solution was better than another, why a task was difficult, and why a task was easy. Leif constantly gave encouraging comments, and listened to students' ideas and solutions. He showed a genuine interest in their thoughts. It became clear that Leif also integrated a lot of teaching artefacts that he had invented and conceptualized. The students worked with Leif's tasks, and they used Leif's magic bar, his magic squares and his other artefacts. We had ended up in Leif's mathematics country. The entire educational atmosphere was characterized by serenity and an interest in students' wellbeing in general and in their mathematics learning in particular.

When the students began practising using the mathematics book, Leif noted what time the lesson would end on the board and what task they would do during the lesson. The framework for the lesson was set, and the students and Leif could continue to test their newfound insights individually. Leif could test his acquired insights into students' thinking – important knowledge that helped him to teach at a reasonable level when he walked around the classroom and held individual discussions with his students.

Understanding of Students' Understanding

A recurring action in Leif's teaching is when he tries to understand what students know and how they comprehend content. These actions occurred in different arenas. An arena was defined as a context where meeting between Leif, students and mathematics occurred. Leif's mathematics teaching was carried out in four different arenas: (a) whole-class teaching at individual level, (b) whole-class teaching at group level, (c) individual practising and (d) testing.

Whole-class teaching at individual level

Whole-class teaching at individual level took place in at least two different ways: (a) an introducing way and (b) a reasoning way. The introducing way meant that Leif presented a new concept or procedure. He asked questions that helped him to understand what and how students understood the content that was presented. Different ways of thinking were discussed, whereby students became aware of the advantages and disadvantages of different ways of thinking about mathematics. Whole-class teaching was also carried out in a reasoning way. This meant that Leif presented a problem that the students solved individually. After all the students had tried to solve the problem, a whole-class discussion was conducted. Once again Leif reflected on different ways of thinking about the problem and finally summarized what the class had found out.

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In his summary, he pointed out the difficulties the students had managed to overcome and what they should pay attention to when working with such a kind of problem.

Whole-class teaching at group level

Whole-class teaching at group level meant that students solved problem in different groups thorough discussion. Leif and the students then discussed how the different groups of students had solved the tasks. Each group's solution was tested against solutions from other groups. Leif steered the discussion and asked questions, exposing students' thinking, reasoned with groups and summarized the results of the discussions.

Individual practising

Individual practising meant that students were given the opportunity to practise their abilities to solve tasks that were related to what had been discussed in the class. During individual practising, Leif walked around the classroom and discussed individual problems and supported students' work.

Testing

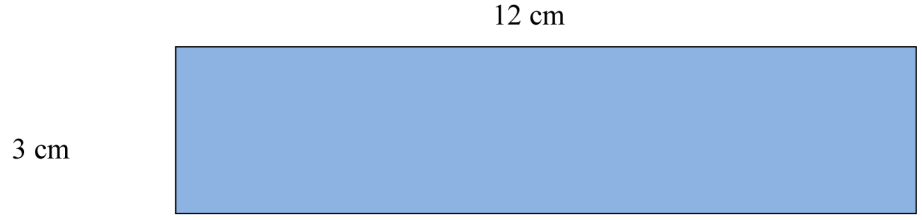
When Leif and the students were in the test arena, students conducted the test that the teacher later gave them feedback on. The feedback was always individual. It was not just about presenting how many points the student received; Leif listened to how students discussed the tasks that had been difficult. While listening, Leif tried to help students in their ways of thinking about these tasks. Tasks that the students found tricky were discussed in depth. Students were thus given an individual review of the task that detailed concepts and methods. They then practised solving similar tasks to those they had experienced problems with.

The intention to identify how students were thinking and how they understood a specific content was significant to Leif's work in all arenas.

Inspiring and motivating mathematics teaching

Another important pattern in Leif's teaching was that he always tried to inspire and motivate the students in different ways. The following instruction sequence exemplifies how Leif worked with representation in order to inspire and motivate the students. Through illustrations and expressions like "Isn't this exciting", Leif tried to support the students' learning. The teaching episode in question drew attention to fractions.

Leif: Now draw a rectangle with sides measuring 12 cm and 3 cm.



Leif: Now I want you to draw lines to divide it up into thirds.

Following Leif's instructions, the students divided up their rectangles into thirds in several different ways. The most common ways are illustrated below.

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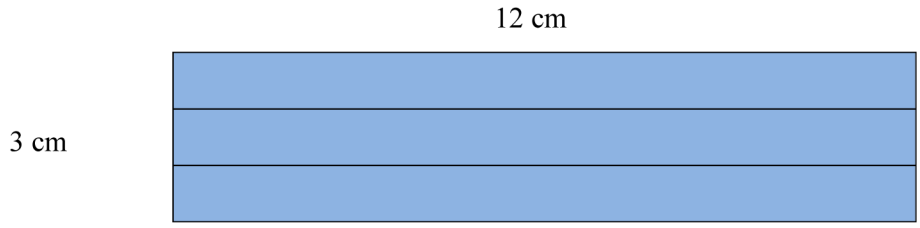
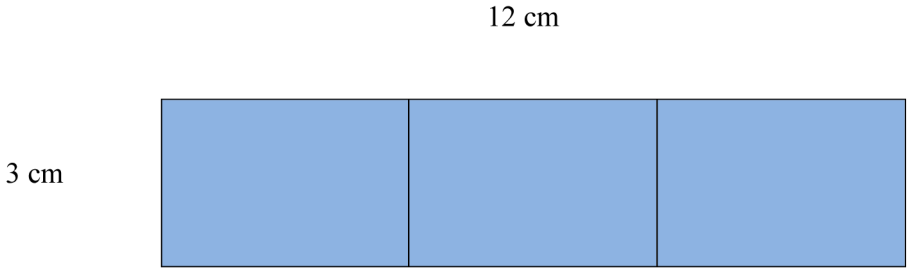
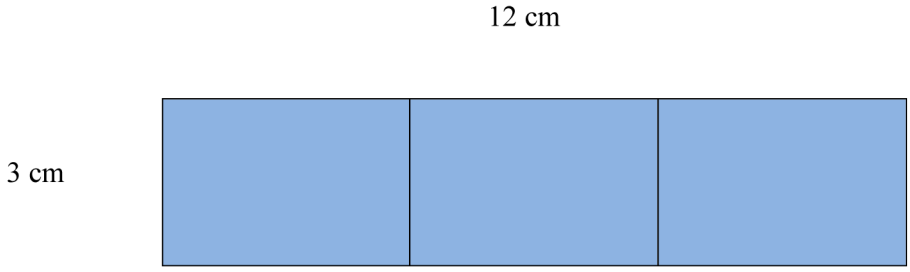


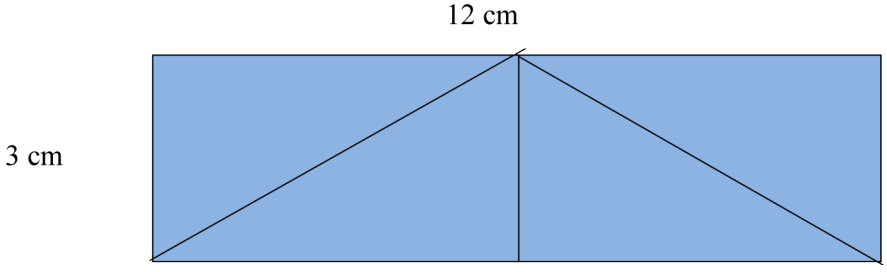
Leif: Now I want you to draw a picture where you divide the rectangle up into thirds but in a different way.
Students who designed the above figure then drew the following figure.



Leif walked around the classroom and watched how the students drew figures. This gave him the chance to make an assessment of how students represented thirds. When looking at how all the students had solved the task, the class carried on with a similar procedure, this time drawing quarters. The first representation of the quarters basically looked the same for all students.



Leif then asked the students to draw another representation of quarters. This task caused some students major difficulties. The following examples are the most common.



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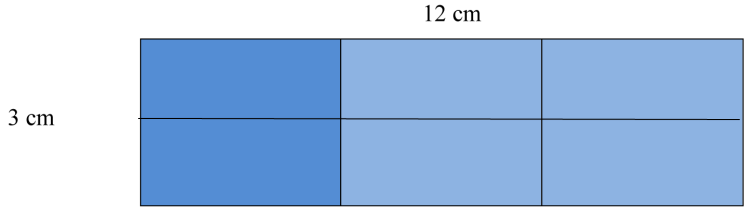
The students had so far not received any instruction on how to divide a figure into parts. To understand how the students were thinking, Leif asked them questions:

Leif: What should you do to find out the size of a quarter?

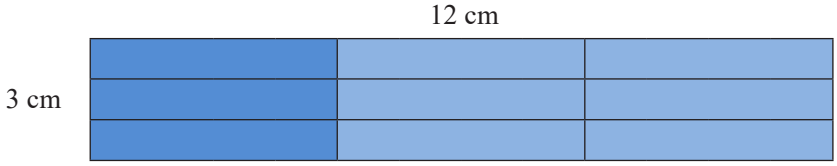
Student: You share all the small squares with the part you want to divide into.

Leif looks questioningly at the student and then asks: Are you thinking like this. If we are to split our figure into quarters, we could think like this. How many small squares are there? 3*12 = 36 pieces of squares. We divide that number into four piles, 36/4=9. Then we know that each quarter contains 9 squares. Neat.

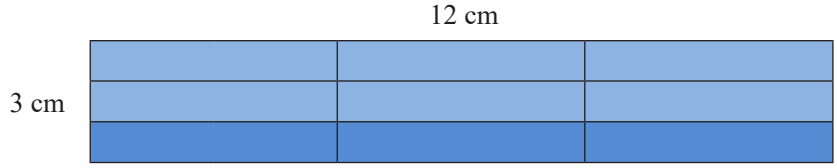
By working with several different examples, the students faced different challenges that gave them opportunities to discover patterns. Leif continued a discussion on the relationship between different fractions and wondered if someone could show what kind of connection there was between thirds and sixths. Some students drew the figure below and Leif drew it on the board and said: "Well, so two sixths is a third, exciting!"



Leif then asked the students: "What is the relationship between thirds and ninths?" The students drew and showed Leif what they thought.



Leif said: "Well, so three ninths are required to make one third, fun. Now comes the next mission for you, twelfths and thirds." Students drew their figures and presented them to Leif as he walked around the classroom.



Once again, the students had been given the opportunity to see that there is a relationship between different fractions. They used the mathematics they had already mastered to make their calculations. After the activity, the students had been given an image, a representation of different fractions and their relationships. These examples illustrate Leif's professional knowledge of how mathematical content can be represented. At the same time, the examples show how Leif perceived learning. Instead of him drawing the figures on the blackboard and showing what he wants the students to learn, he lets them – under supervision – draw and seek the pattern that he was aiming for.

A recurring pattern in Leif's teaching was his attempt to inspire students to think in pictures while he motivated them towards self-discovery, investigation and confirmation of their

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ways of thinking and solving problems. He also used his language to inspire and motivate the students. He constantly used phrases such as "fun" and "exciting".

Supporting Affective Aspects

A fourth pattern that emerges from the observations of Leif's teaching was his supportive behaviour towards affective aspects. There were at least two things that Leif did to affect and develop positive relationships towards school mathematics: (a) create interest, and (b) work with the students' confidence in their own ability.

Create interest

When Leif introduced new content, he often connected the mathematics to everyday events in order to motivate students. The students thereby had the opportunity to understand the benefits of the mathematics they were going to learn. Leif also used images to concretize and illustrate various mathematical phenomena. Everyday events and pictures seemed to affect students' activity in the lessons, which could be an indication of increased interest.

Leif also used language to make mathematics seem fun. All lessons featured a variety of positive expressions so that the students would feel that there was something interesting going on. One example was when various groups had presented their solution to a problem, but Leif seemed unsatisfied with the solutions and said: *"Wouldn't it be fun to see what I think now?"*

Or when a student had demonstrated that he or she had mastered something, he said: "That's good, I suppose you are really happy about knowing that?" By talking to the student in this way, he caught their attention and made the student aware that they actually felt happy that they had managed mathematics.

Work with the students' confidence

Students' thinking about their own mathematical ability has a great impact on their mathematical performance. Success and failure when learning mathematics affects their self-confidence. Leif paid attention to how the students talked about their ability. The students were not allowed to say that they could not do maths or that they were bad at maths.

> Student: *I can't!*
>
> Leif: *Why do you say that? You have demonstrated that you understand how to divide, and you have shown that you can round off.*
>
> Student: *Mmm…*
>
> Leif: *Then I want you to stop saying that you can't. You can do lots of things. Trust yourself.*

Students must succeed in schoolwork to gain self-confidence, but to be successful they should have a certain degree of self-confidence. Leif gave a lot of positive responses, and they were never redundant.

> Leif: *You seem to understand, that's good. This means that you know things, it's good, really good.*

He also involved himself in students' learning. He frequently used expressions such as "We can do this together" and "I will help you, and you and I must be able to solve this". Involving himself in the learning processes reduced students' anxiety towards mathematics in general and learning mathematics.

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Socio-Mathematical Norms and Meta-Cognitive Aspects

A recurring pattern of teaching was that Leif accepted different solutions and ways of thinking in the mathematics classroom. Normative understanding of what counts as an acceptable mathematical explanation or justification are socio-mathematical norms. In Leif's classroom, diversity of thinking was allowed and discussed in order to learn which method was easiest or most problematic, relationships between concepts and why a task was or was not easy. By discussing these aspects, the students developed their meta-knowledge of mathematics.

Knowledge that one can think differently and be creative in mathematics

The structure of Leif's teaching gave the students several opportunities to acquire knowledge about how mathematical operations can be conducted while showing how other students think. Leif encouraged the various creative solutions and students' explanations of their solutions. Students were given an understanding that mathematics was not always something that needs to be treated in a certain way. If none of the students could present a solution that Leif thought was the best, he said: *"Now you have to listen to how I solve the problem. Wouldn't it be exciting to hear that, Moa?"* (Moa gave the impression of not listening so Leif took the chance to catch her attention and bring her back to the teaching.)

After all the solutions had been presented, they discussed which solutions would be best and why. Leif argued for his solution; it suited him, and it was the easiest and most effective way to solve the problem in his experience. Students, for their part, would sometimes argue that their solution suited them better. If their model was generic and could be built upon mathematical ideas, Leif let them use it and gave encouraging comments about their proposed solution. If the solution was not general he tried to reason with the students, so they realized the shortcomings of their thinking and what would happen if they worked with more advanced and complex problems. In this way, Leif made the students pay attention to both different creative solutions and the most efficient and mathematically correct solution.

Knowledge of the relationships between the different components as they work in mathematics

Leif always tried to link new content to earlier experiences so that the student can understand what context the content was related to. If students could not connect the mathematics to prior knowledge, Leif went back a step to a concrete representation. The figure below illustrates how Leif worked with new content. New content was connected to prior abstract knowledge. If the student had difficulties understanding the abstract line of reasoning, Leif incorporated concrete representations like pictures and everyday events in his teaching.

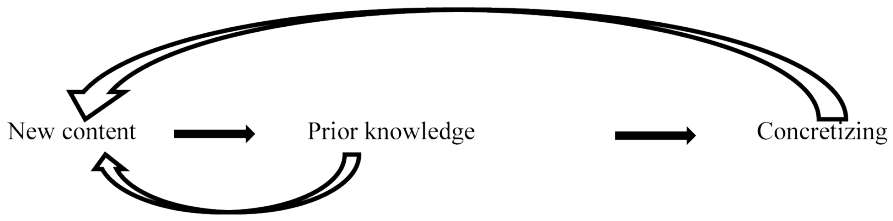

Figure 3: An illustration of how new content is introduced and put in its context.

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The starting point was to work with mathematical abstractions, moving between spoken and written symbols and ending up in a mathematical discourse. In this way, Leif gave the students opportunities to develop their meta-cognitive thinking of school mathematics; they gained knowledge *about* mathematics not just skills *in* mathematics.

Knowledge of what makes the task easy

Another important pattern that arose was Leif's discussion of why a certain task was easier than a task that looked similar. For example, when Leif asked the students to conduct a so-called Leif task, where he presented tasks that contained difficulties that were important to master.

1. 6.2 + 3.9 =
2. 6.5 - 3.2 =
3. 17.2 - 6.4=

The above example shows that task one and two are relatively simple tasks. By reasoning why task two is easier than task one, students were given the opportunity to develop their knowledge about mathematics, not just in mathematics. Leif could have drawn attention to what made task one difficult, but instead he chose to focus on what made the second task easier. This might have had a psychological impact on students' later work on similar tasks. No task was difficult, however was one task a little easier.

Discussion

Managing the Social System of the Mathematics Classroom

Leif has created a social system that physically consists of the mathematics classroom and his students. In the social system that constitutes his classroom, mathematics is in the foreground and it is mathematics that students should be engaged in during the lesson. Each time a student falls outside the framework, and goes off-task by not participating in the lesson, Leif instructs partly in words, but also with body language, how he wants them to work with mathematics. By constantly asking whether students are off-task, if they have finished with the task, he hints and reminds them about the mathematical content and what should be done. He shows in a productive way what students are expected to do (Brophy, 1988; Nordenbo, Søgaard Larsen, Tifitkçi, Wendt & Østergård, 2008; Roache & Lewis, 2011). Through his transparency, Leif influences students' attitudes and beliefs about what should be done in mathematics lessons, and how to work. It also means that he puts into words how students are expected to behave and reinforces desirable behaviours (Epstein, Atkins, Cullinan, Kutash & Weaver, 2008). Leif shows that he is aware of things on many different levels and that he deals with many things that happen simultaneously. In a classroom, changes occur all the time while teaching is in progress. Leif deals with potential interference by being attentive and having a genuine interest in trying to understand the students. He stops and asks questions publicly to the class about how they think. In the classroom, he has created a constructive environment that allows him to think because the students know historically that he needs to get a chance to understand how they understand in order to help them in the best way (Doyle, 2006). Leif makes sure that everyone knows what to do and supports them in their work with the subject. Leif's way of working reduces the student's opportunities for disruptive behaviour in any significant way. Leif focuses on maths and makes it clear that he expects the students to work with maths and nothing else.

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Managing Care and Creating Healthy Relationships around Mathematics

Teachers' skilful leadership is the basis for a positive, helpful and friendly atmosphere (Murray, 2001; Laanemets et al., 2012) and respectful relationships between them and their students (Lewis, Romi, Xing & Katz, 2005; Nordenbo, Søgaard Larsen, Tifitkçi, Wendt & Østergaard, 2008). Such leaders are aware of the importance of students agreeing to follow them as leaders. Again, it is Leif's interest in listening and trying to understand his students' problems with mathematics and his desire to help that allows students to accept him as the skilful leader he is (Murray, 2001; Laanemets et al., 2012). Working with students' learning in several arenas through varied teaching (Laanemets et al., 2012) gives Leif many opportunities to listen to the students' thinking, which is the basis for the positive and productive relationship between him and the students. This information also means that he exercises good teachership (Granström, 2007) and, if necessary, changes students' statements (Murray, 2001; Beishuizen et al., 2001) in order to make them understand (Epstein, Atkins, Cullinan, Kutash & Weaver, 2008). He creates a supportive learning community where he, as a role model, socializes students in a desirable manner (Brophy, 1999; Evertson & Weinstein 2006; Wubbels, 2011). Thus, his concern for students' mathematics learning (Woolfolk Hoy & Weinstein, 2006; Brophy, 1999) and his interest in the individual student's learning are an important factor in terms of creating healthy relations. Through skilful teaching he exercises good leadership.

Managing Motivation and Students' Confidence in their Own Ability

The student's interest in mathematics in general and the student's attitude to mathematics in particular is largely dependent on the teacher's actions (Darling-Hammond & Bransford, 2005; Montuoro & Lewis, 2014). Leif motivates students by representing mathematical phenomena in images, allowing students to solve tasks in different ways. Each solution is important because it provides a basis for discussion. Students are given the opportunity to feel that they have mastered the content, that they are part of a larger mathematical community, and that they have the chance to study mathematics. Thus, Leif's leadership fulfils many of the requirements necessary to support the motivation to study. Another strategy is Leif's recurring expression of how fun and pleasurable mathematics is, and how exciting it is to solve problems and get the answers. He thereby creates interest (Laanemets et al., 2012; Kutnick & Jules, 1993) and exemplifies good teachership (Granström, 2007). Students' thinking about their own mathematical ability has been shown to be important in terms of the ability to solve mathematical problems (Garofalo & Lester, 1985). By constantly trying to get students to experience that they can succeed, he helps them to develop a positive relationship towards mathematics (Borich, 1996; Clark, 1997). One common feature of Leif's teaching is the tremendous effort he puts into supporting each student's feelings for mathematics in order to develop their confidence in their own ability. Another aspect that appears in his teaching is the feedback given to students that may be significant in relation to the students' confidence in their own ability. Leif runs through each lesson and test individually with each student (cf. Brophy, 1996; Borich, 1996). The above testifies how Leif exercises teachership by individualizing (Murray, 2001; Murphy, Delli & Edwards, 2004; Kutnick & Jules, 1993) his mathematics teaching.

Managing Structures, Procedures and Rules

Leif shows skilled leadership by being in control in the classroom (Kutnick & Jules, 1993) and being clear about what applies in the classroom. He writes on the board what to do and how long the work should take, and sets the framework for the lesson. A general rule that

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creates peace and quiet in his classroom is the focus on mathematics. Such a rule gives a clear orientation for the students about what to do. It is also the basis for a fair system that protects and respects students (Woolfolk Hoy & Weinstein, 2006 Brophy, 1988; 2006; Wubbels, 2011). Leif works patiently to establish these structures, and disruptive behaviour rarely or never occurred in the classroom. Students simply did not have time to interfere because they were involved in Leif's mathematics education (Epstein, Atkins, Cullinan, Kutash & Weaver, 2008). Rules and procedures helped Leif to create standards and thus became a tool for learning and effective leadership (Doyle, 2006). In this case, we could talk about socio-mathematical norms, or in other words the fact that Leif is clear about what is considered appropriate mathematical solutions and justifications. He always started with the student's prior knowledge and followed up by arriving at the most appropriate method based on a specific criterion.

Table 1. Classroom management as a construct of leadership and teachership.

Classroom management	Establishing a social system	Establishing respectful relationships	Establishing structures, procedures and rules	Having clear expectations and motivating students
Leadership	Express how students are expected to behave in the classroom.	Show concern for each student's situations.	Establish and maintain a small number of rules and procedures.	Use interesting tasks, and offer students support so that they can be successful.
Teachership	Express how students are expected to learn mathematics.	Show interest in each student's mathematical thinking.	Establish a culture where all solutions are important in the teaching process. Direct attention to mathematics.	Use multiple representations to motivate students. Use supportive expressions that they are able to learn mathematics.

The table above summarizes Leif´s classroom management. It shows how a proficient maths' teacher teaches his subject and thereby exerts effective leadership in the classroom.

Conclusions

A proficient teacher such as Leif carries his teachership in and about the mathematics that students should learn. He wants to understand how students understand the content, so he can meet them and support them in their quest to acquire mathematical knowledge. He directs attention to their needs and, based on the knowledge he has acquired through education and experience, he chooses appropriate strategies as a proficient classroom manager to help individual students in the best way. He creates opportunities that give him the chance to discover how students think, and he optimizes his ability to see where each student is in their thinking. At the same time, he is reminded that there are things that are obvious to him but problematic for students. Working in the previously presented venues has probably helped Leif to acquire unique skills, skills that make him a successful teacher, perhaps without thinking about it. This research illustrates how a skilled mathematics teacher works, focusing on teaching and learning mathematics and thereby benefiting from his leadership. The researchers in the introduction argue that a not entirely unusual position among teachers is that a teacher must first establish a quiet classroom before he or she can begin to teach the subject. The research shows different patterns that occur when a mathematics teacher exerts proficient leadership in the classroom just by teaching his subject.

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