

CHAOS THEORY IMPLEMENTATION ON THE LMS ALGORITHM OPTIMIZATION APPLIED ON LINEAR ANTENNA ARRAYS

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Abstract: In this paper, we proposed the chaos based optimization of the LMS algorithm. The goal of this beamforming algorithm optimization using the chaos theory is the adaptation of a linear antenna array radiation diagram to the specific combination of received signals' incident angles, their as much as possible precise estimation and the setting of radiation diagram's depth zero in the directions of the interfering signals. Unlike conventional adaptive algorithms, the proposed chaotic LMS algorithm allows the adjustment of radiation diagrams in the case of a large number of incident signals, desired and interfering signals, and reducing the width and the number of dead zones. The tracking capabilities performance check has been performed on this when the reference signal has noise present.

1. INTRODUCTION

Antenna arrays and their performance, as integral parts of modern communication systems, have a large impact on the efficient operation of the entire communication system. The direction of arrival estimation results, performed by a smart antennae system, are used to adjust the adaptive beamformer weight coefficients with the aim of adjusting the transmission and / or reception of the antenna array's radiation pattern on a specific signal scenario. The successful design of adaptive antennae system depends on the selection and performance of a beamforming algorithm used for adaptation.

To date, there are many adaptive algorithms which are used for the analysis and synthesis of antennae arrays' radiation diagrams. In references [1] and [2] the algorithm for adaptation and synthesis of the radiation pattern is based on a square optimization of the Lagrangian function, which is done by adjusting the phase using the SPQ-a (Sequential Quadratic Programming) algorithm. In [3], to optimize the position of elements in the array

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and to determine the amplitude and phase of the array elements, we used the PSO (Particle Swarm Optimization) algorithm. The chaos based optimization of the Mind Evolutionary Algorithm introduced in [4] defines a fitness function used to adjust the amplitude and phase of the antenna element, whose analysis allows the optimization of this algorithm. The use of GA (Genetic Algorithm) and Levenberg-Marquardt algorithm is presented in [5], [6] and [7].

In this paper we used chaotic LMS algorithm based on the optimization of the LMS algorithm for the synthesis of linear antenna array's radiation diagrams, using chaos theory. LMS algorithm and its modifications achieved good results only in cases of antenna arrays consisting of a large number of elements and under condition that a reference signal is noiseless.

The optimization of the LMS algorithm is performed by adding a chaotic optimization block for the step μ , based on the equations that describe Chua's oscillator [8]. The fitness function defines the precise estimation of the signals' arrival angles, the setting of radiation diagram's deep nulls in the direction of interfering signals, the reduction of the main lobe's width and the reduction of side lobes. The criteria for the chaotic LMS algorithm optimal parameters' selection is the minimum value of a given fitness function.

2. THEORY

Let a uniform linear antenna array be composed of N antenna elements, equally spaced and oriented along a line, at an equal spacing d and let it receive M signals from the desired source $s_m(t)$ arriving at angles $\theta_1, \theta_2, \dots, \theta_M$. The antenna array also receives I signals from the interfering sources $s_i(t)$ arriving at angles $\alpha_1, \alpha_2, \dots, \alpha_I$. The total signal received by antenna array $x(t)$ is the sum of the desired signals $x_m(t)$, interfering signals $x_i(t)$ and white Gaussian noise $n(t)$:

$$x(t) = x_m(t) + x_i(t) + n(t) \quad (1)$$

where:

$$\begin{aligned} x_m(t) &= A_M s_m(t) \\ s_m(t) &= [s_{m1}(t) \quad s_{m2}(t) \quad \dots \quad s_{mM}(t)]^T \\ x_i(t) &= A_I s_i(t) \\ s_i(t) &= [s_{i1}(t) \quad s_{i2}(t) \quad \dots \quad s_{iI}(t)]^T \end{aligned} \quad (2)$$

A_M and A_I are matrices of array steering vectors of the desired and interfering signals. Taking as a reference the first antenna in the antenna array, the matrices are given by the following expression:

$$A_M = \begin{bmatrix} 1 & 1 & \dots & 1 \\ e^{j\psi_1} & e^{j\psi_2} & \dots & e^{j\psi_M} \\ \vdots & \vdots & \dots & \vdots \\ e^{j(N-1)\psi_1} & e^{j(N-1)\psi_2} & \dots & e^{j(N-1)\psi_M} \end{bmatrix}_{N \times M} \quad A_I = \begin{bmatrix} 1 & 1 & \dots & 1 \\ e^{j\xi_1} & e^{j\xi_2} & \dots & e^{j\xi_I} \\ \vdots & \vdots & \dots & \vdots \\ e^{j(N-1)\xi_1} & e^{j(N-1)\xi_2} & \dots & e^{j(N-1)\xi_I} \end{bmatrix}_{N \times I} \quad (3)$$

The phase field shift between two array's consecutive elements is given by the following expressions:

$$\psi_m = \frac{2\pi}{\lambda} d \sin \theta_m, m = 1, \dots, M \quad (4)$$

$$\xi_i = \frac{2\pi}{\lambda} d \sin \alpha_i, i = 1, \dots, I \quad (5)$$

The total signal described by the relation (1) is applied to the input of the network for the formation of a radiation pattern.

A. LMS algorithm

The method of least squares (LSM- Least Mean Square) is an adaptive algorithm based on an iterative procedure that makes successive correction of weight coefficients with the aim of minimizing the MSE (Mean Square Error). LMS belongs to the type of algorithms capable for continuous adaptation, where the algorithms adjust weight coefficients while the signal is sampled i.e. for each samples the coefficients are determined for the optimal output.

During each iteration of the LMS algorithm, the weight coefficients are adjusted according to the formula:

$$w(n+1) = w(n) + 2\mu e(n)x(n). \quad (1)$$

Here the $x(n)$ is the input vector for the signal values from the previous sample, while $w(n)$ represent the adaptive algorithm's weight coefficients. The rate of LMS' convergence is proportional to the step μ . If μ it is too little, convergence is slow, and in the case when μ is large, the algorithm may exceed the optimal weight coefficients and become unstable. The optimal value for the LMS algorithm used in the simulations is $\mu = 0.0208$.

The output of the LMS adaptive system is calculated by the relation:

$$y(n) = w^T(n)x(n) \quad (2)$$

While the estimated error is calculated by the following equation:

$$e(n) = d(n) - y(n) \quad (3)$$

B. Chaos theory

Chaos is a form of steady-state that many natural and man-made systems exhibit, under certain conditions. Chaotic behavior is described in [8], 9] and [10]. Searching by chaos represents an alternative to statistical search. The chaotic search is faster, with increased speed of convergence and accuracy of results, as described in [11].

In this paper, the chaotic search use the equations that describe Chua's oscillator, with parameters corresponding to the

double-helical chaotic attractor [8]. Chaotic optimization method is shown in Figure 1.

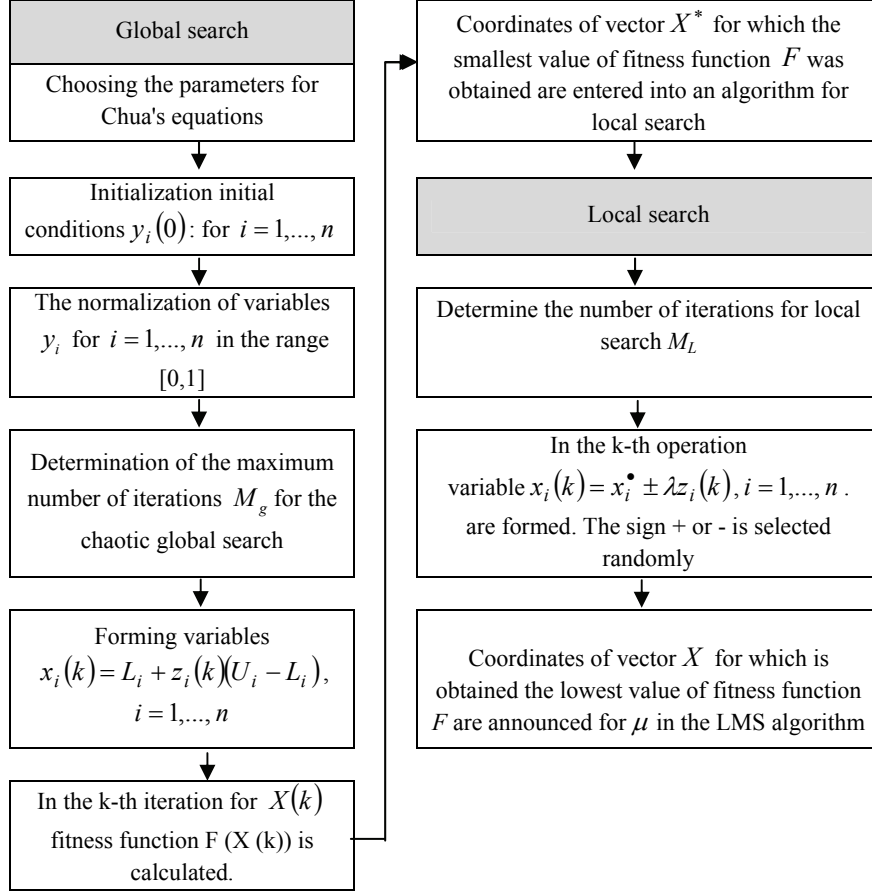


Fig. 1. The chaotic optimization method

C. Fitness function

The fitness function defines the precise estimation of the signals' angles of arrival, deep null setting on the radiation pattern in the direction of the interfering signals, and the reduction of the side lobes and the main lobe's width [4]:

$$\begin{aligned}
 Fitness = & k_1 \frac{|\theta_0 - \theta_{des}|}{180^\circ} + k_2 \sum_{i=1}^N |SLL_{max} - SLL_{des}| \\
 & + k_3 \frac{\theta_{BWFN} - \theta_{BWFN_des}}{180^\circ} + k_4 \sum_{i=1}^N |NULL_{\theta_i} - NULL_{\theta_i_des}|
 \end{aligned} \tag{4}$$

where θ_0 is the steering angle of the main lobe, θ_{des} is the direction of arrival of a desired signal, SLL_{max} is the maximum level of the side lobes, SLL_{des} is the desired maximum level of the side lobes, θ_{BWFN} is the width of the main lobe, θ_{BWFN_des} is the

desired width of the main lobe, $NULL_{\theta_i}$ is the null depth in direction θ_i , while $NULL_{\theta_i, des}$ represents desired null depth in direction θ_i . The following weight coefficients were used in the numerical analysis: $k_1 = 0.3$, $k_2 = 0.9$, $k_3 = 0.5$ and $k_4 = 1.9$. The criteria for the selection of chaotic LMS algorithm's optimal parameters is the minimum value of the given fitness function.

D. Chaotic LMS algorithm

The proposed optimization method adds to the LMS adaptive algorithm a block for the optimization of the step μ which is based on the chaos search. The block diagram of the chaotic LMS algorithm is given in Figure 2.

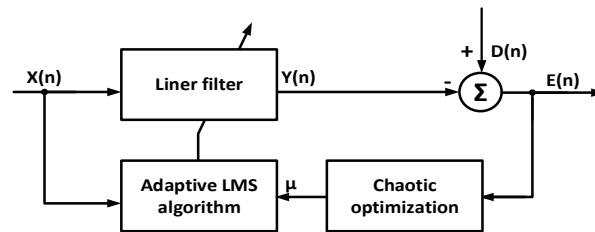


Fig. 2. Block diagram of the chaotic LMS algorithm

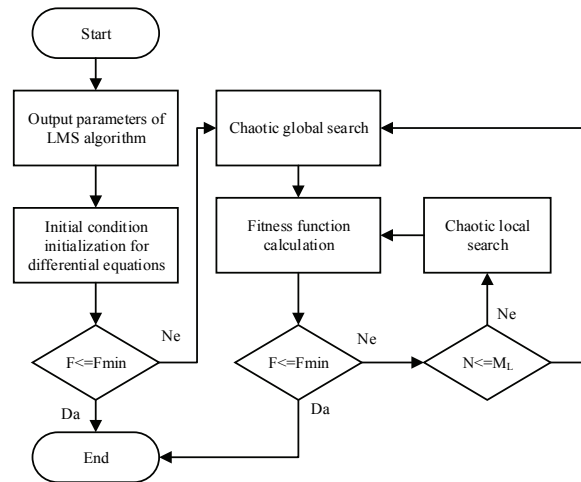


Fig. 3. The algorithm for chaotic optimization of step μ

After the election of the step μ using the LMS algorithm, the value of Fitness function is checked, i.e. it is verified whether the adaptation for a given combination of incident angles is successful. If the value of the fitness function is not satisfactory, the chaotic optimization is started and the search and selection of steps μ is carried out globally, and in case of need also locally.

3. NUMERICAL RESULTS

The signal described by equation (1) is used in all simulations as an input signal. We analyzed the uniform linear antenna array with $N = 10$ and $N = 8$ antennae elements, which are located at the distance $d = \lambda / 2$. The comparative diagrams obtained by the LMS and the chaotic LMS algorithm are given in normalized values.

Figure 4 shows the radiation pattern when the desired signal is coming at an angle of 20° and interfering at an angle of -50° .

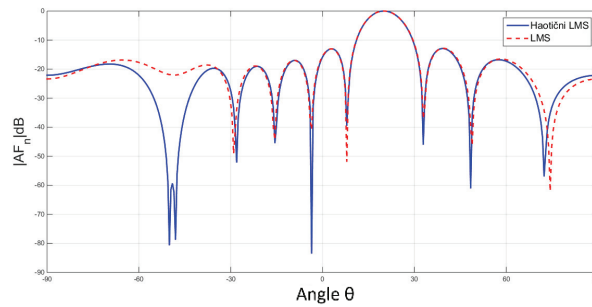


Fig. 4 Radiation patterns when desired signal is arriving at an angle of 20° and interfering at an angle of -50°

We can see on Figure 4 that the chaotic LMS algorithm has high accuracy and deep null (-80dB), which cannot be said for the radiation pattern obtained using the LMS algorithm.

Figure 5 shows the radiation pattern obtained using Chaotic LMS and LMS algorithm when more desired signals arrive to the antenna system.

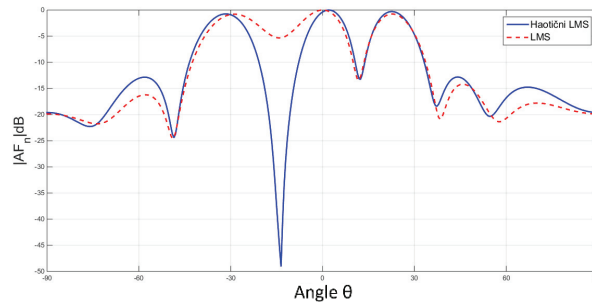


Fig. 5 Radiation patterns when desired signals are arriving at angles -32° , 2° and 23° , and interfering at an angle of -14°

Based on the results shown in Figure 5, it can be concluded that the chaotic LMS algorithm accurately directs the main lobe in the direction of the desired signal and sets a deep null (-50dB) in the direction of interfering signal. LMS algorithm fails to perform adaptations of radiation diagram.

Figure 6 shows the value of nulls in to radiation diagram when the desired signal arrives at an angle of 20° while the interfering signals arrive at angles from -90° to $+90^\circ$. The number of array elements used in the simulations is 10.

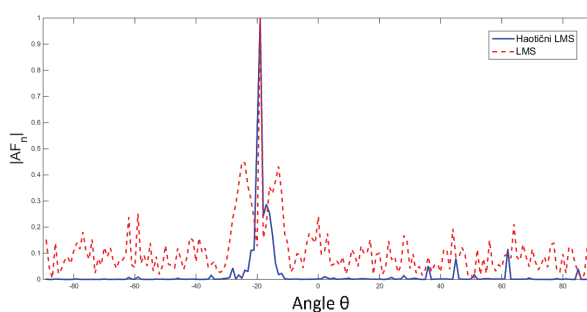


Fig. 6. Null values in the radiation diagram when the desired signal arrives at an angle of -20° and interfering signals at angles from -90° to $+90^\circ$

Figure 7 presents the coefficient μ values for different interfering signal arrival angles obtained by applying chaotic LMS algorithm.

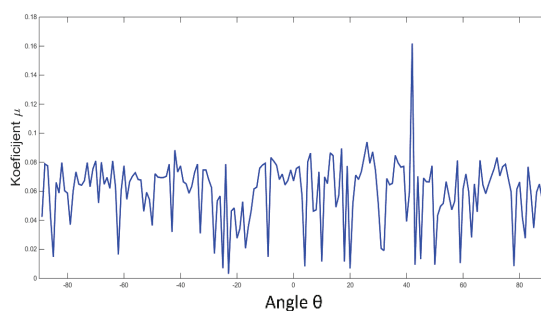


Fig. 7 The coefficient μ when the interfering signals arrive at angles from -90° to $+90^\circ$

Based on the results shown in Figures 6 and 7 it can be seen that the chaotic LMS algorithm for each combination of incident angles achieves larger deep null values in the directions of interfering signals. The aim of interfering signals' arrival angles tracking performance analysis is to establish dead zones existence and to reduce them using the proposed method. From Figure 6, we can see the existence of dead zones close to the incident angle of the desired signal. ($\pm 10^\circ$ from the desired signal and from -60°). Unlike the LMS algorithm that has a wide dead zone near the desired signal's angle of arrival, Chaotic LMS algorithm significantly reduces these dead zones, and near the angle of -60° there are none. To verify the tracking capabilities of the proposed algorithm, we calculated the mean null depth values for different levels of noise on the signal (the reference signal is noised with white Gaussian noise), when the interference signal's angle of incidence changes from -90° to $+90^\circ$. Simulation results are shown in Table 1.

Table 1.
Mean values of null depth for different levels of noise on reference signal

	No noise	SNR=3dB	SNR=5 dB	SNR=7 dB	SNR=10 dB	SNR=15 dB
LMS	-22.09	-15.26	-16.90	-17.05	-19.67	-21.15
Chaotic LMS	-58.82	-31.65	-31.75	-34.79	-36.49	-38.56

Based on the results shown in Table 1, we can draw conclusion that the chaotic LMS algorithm, even on noised reference signals, sets a very deep null on the radiation diagram in the direction of interfering signals. Based on the foregoing, it can be concluded that the applied algorithm has very good features for tracking and suppression of interfering signals.

The modern trend in the design of antenna arrays is reducing the number of antennas and reducing the radiation in unwanted directions. To perform a performance analysis of the Chaotic LMS algorithm, simulations were performed on antenna arrays with a small number of antennas. Figures 8 and 9 show the results obtained using chaotic LMS algorithm applied to antenna arrays with $N = 5$ elements.

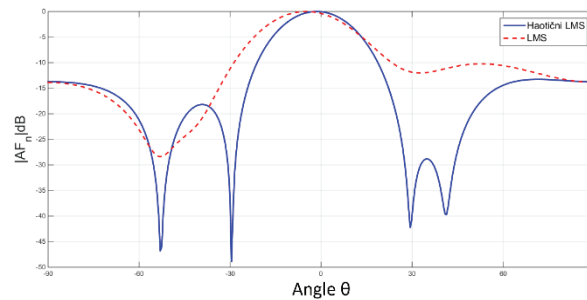


Fig. 8 Radiation patterns when desired signal is arriving at an angle of 0° , and interfering at angles of -30° and 40°

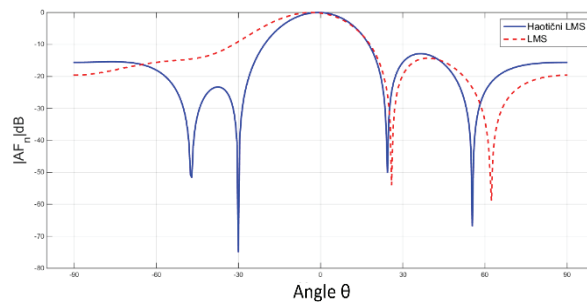


Fig. 9 Radiation patterns when desired signal is arriving at an angle of 0° , and interfering at an of angle of -30°

Based on the results shown in Figures 8 and 9 it can be seen that the LMS algorithm in case of small number of antennae array can make adaptations and set nulls on the radiation pattern. The chaotic LMS algorithm, although applied to an array with a small number of antennas, successfully optimizes the radiation pattern and gives deep nulls in the directions of interfering signals.

4. CONCLUSION

In this paper, for the linear antenna array's radiation diagrams synthesis, we used a chaotic LMS algorithm based on LMS algorithm optimization using chaos theory. LMS algorithm optimization is performed by adding it a block for the step μ chaotic optimization. The criteria for the optimal step μ selection in the chaotic LMS algorithm is the minimum value of a fitness function, which defines the following requirements: setting deep nulls on the radiation diagram in the interfering signals' direction, precisely directing the main lobe in the desired signal's direction, reducing the main lobe width and reducing the side lobe.

We analyzed the uniform linear antenna array and the resulting radiation patterns are given in normalized values. In all analyzed cases chaotic LMS algorithm has successfully adapted the radiation pattern to the particular scenario of the incoming desired and interfering signals.

We simulated the use of chaotic LMS algorithm in the case of small number of antennas array, where the classical LMS algorithm does not give satisfactory results. It is shown that the chaotic LMS algorithm successfully optimizes the radiation pattern and gives the deep nulls in the directions of interfering signals.

Also, unlike conventional adaptive algorithms, the chaotic LMS algorithm enables the synthesis of radiation diagrams in the case of a large number incident signals, desired and interfering signals, and reducing the width and the number of dead zones.

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