OPTIMAL CHEBYSHEV MULTISECTION MACHING TRANSFORMER DESIGN IN WIPL-D

Arsenije Maliković[*], Milica Ljumović[**] , Dragan Filipović[***] , Budimir Lutovac[****]

Keywords: *transmission line, matching, Chebyshev multisection transformer, WIPL-D, circuit simulation.*

Abstract: A transmission line in which reflected EM waves do not exist in normal working mode is referred to as matched. There are different methods of performing the matching and preventing additional losses or malfunctioning of the devices. In this paper, we shall review the Chebyshev multi-section matching transformer and analytical calculation of an optimal one for a given matching problem. Verification of the results obtained will be performed in WIPL-D Microwave software.

1. INTRODUCTION

The computer aided design of microwave circuits and systems dates from the foundation of the Microwave Theory and Techniques Society and birth of the computer era. In its very beginning, it was accepted with great skepticism and scientists relied more on theoretical methods in circuit design and analysis. Rapid development of the computers and their performances led to possibility of addressing complex geometries, as well as modelling and optimizing large circuits. Hence, real design nowadays includes the use of computer hardware, software and information processing in various forms [1].

A transmission line, terminated with some impedance Z_L, that is different from its characteristic impedance Z_0, will result in a EM wave being reflected from the termination back to the source. This can cause various problems: additional losses, overheating, malfunctioning etc. Therefore, importance of the matching is undoubtable.

[*] Arsenije Maliković is with the Faculty of Electrical Engineering, University of Montenegro, Montenegro (e-mail: arsenijemali@gmail.com).

[**] Milica Ljumović is with the Faculty of Electrical Engineering, University of Montenegro, Montenegro (e-mail: ljumitza@ac.me).

[***] Dragan Filipović is with the Faculty of Electrical Engineering, University of Montenegro, Montenegro (e-mail: draganf@ac.me).

[***] Budimir Lutovac is with the Faculty of Electrical Engineering, University of Montenegro, Montenegro (e-mail: budo@ac.me).

The basis for development of the matching methods with multi-section transformers is a circuit named quarter-wave transformer [2-3]. It represents a transmission line, exactly one quarter of the wavelength (λ) long and terminated with some known impedance Z_L . This circuit is useful for real load impedance matching to the transmission line and demonstration of the properties of the standing waves on the mismatched line. Another feature of this type of circuit is that it can be extended to multi-section designs to provide a broader bandwidth.

Multi-section matching transformer may be constructed as serial connection of N transmission line sections between the feeder line with characteristic impedance Z_0 and the load impedance Z_L .

If we assume that all the sections have the same length, l, then all the local reflection coefficients $\Gamma_0, \Gamma_1, \ldots, \Gamma_N$ will be of the same sign, which makes theory of small reflections applicable to this case [2]. In accordance to that, we can approximate the overall reflection coefficient as:

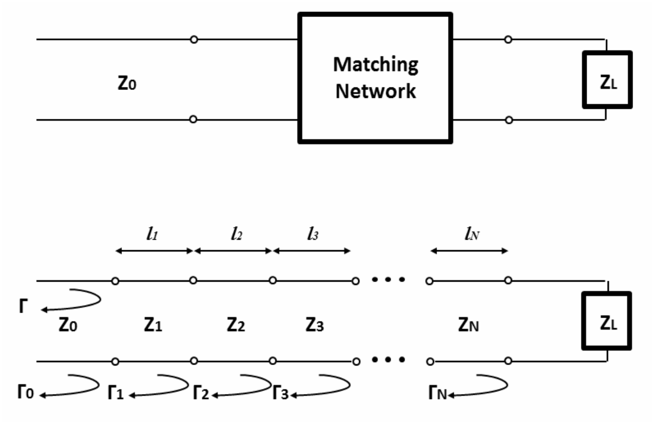

Fig. 1: Multisection matching network; general form with reflection coefficients.

$$\Gamma(\theta) \approx \Gamma_0 + \Gamma_1 e^{-j2\theta} + \Gamma_2 e^{-j4\theta} + \cdots + \Gamma_N e^{-j2N\theta}, \tag{1.1}$$

where $\theta = \beta l = 2\pi / \lambda$. l is electrical length of the sections.

If we further assume that reflection coefficients are symmetric along the transformer, then overall reflection coefficient can be presented as:

$$\Gamma(\theta) \approx e^{-jN\theta}\left[\Gamma_0\left(e^{jN\theta} + e^{-jN\theta}\right) + \Gamma_1\left(e^{j(N-2)\theta} + e^{-j(N-2)\theta}\right) + \cdots + G(\theta)\right], \tag{1.2}$$

where the last term is:

$$G(\theta)=\begin{cases}\Gamma_{\frac{N-1}{2}}\left(e^{j\theta}-e^{-j\theta}\right) & \text{if } N \text{ is odd}\\ \Gamma_{\frac{N}{2}} & \text{if } N \text{ is even}\end{cases} \tag{1.3}$$

This implies that we may obtain any frequency response given a sufficiently large number of sections with the proper reflection coefficients.

Two most commonly used passband responses are: the binominal (maximally flat) response and Chebyshev (equal-ripple) response [2-3]. In the next chapter, multisection transformer design of equal ripple response will be described.

2. CHEBYSHEV MULTISECTION MATCHING TRANSFORMER

A. Theoretical background

Chebyshev multisection matching transformer can provide larger bandwidths than binominal multisection transformer for a given number of transmission line sections but at the expense of an increased ripple over the passband of the matching network. Chebyshev transformer is designed by equating overall reflection coefficient to Chebyshev polynomials. Using these polynomials we can design matching networks with a reflection coefficient at or below some prescribed level over a wide bandwidth.
Chebyshev polynomials up to the fourth degree are:

$$\begin{aligned} T_0(x) &= 1\\ T_1(x) &= x\\ T_2(x) &= 2x^2-1\\ T_3(x) &= 4x^3-3x\\ T_4(x) &= 8x^4-8x^2+1 \end{aligned} \tag{2.1}$$

Higher order Chebyshev polynomials can be determined by using the recursive formula:

$$T_N(x)=2xT_{N-1}(x)-T_{N-2}(x) \tag{2.2}$$

In order to implement Chebyshev multisection matching transformer, the endpoints of the required passband $(\theta_m,\ \pi-\theta_m)$ must be mapped onto the range where Chebyshev polynomials satisfy the condition:

$$\left|T_N\left(\cos\theta\right)\right|\le 1 \tag{2.3}$$

This mapping is done with the condition:

$$T_N\left(\frac{\cos\theta}{\cos\theta_m}\right)=T_N\left(\sec\theta_m\cos\theta\right) \tag{2.4}$$

Inserting the substitution $x=\sec\theta_m\cos\theta$ in the general form of Chebyshev polynomials, we get the following form:

$$\begin{aligned}
&T_0\left(\sec\theta_m\cos\theta\right)=1\\
&T_1\left(\sec\theta_m\cos\theta\right)=\sec\theta_m\cos\theta\\
&T_2\left(\sec\theta_m\cos\theta\right)=\sec^2\theta_m\left(1+\cos 2\theta\right)-1\\
&T_3\left(\sec\theta_m\cos\theta\right)=\sec^3\theta_m\left(\cos 3\theta+3\cos\theta\right)-3\sec\theta_m\cos\theta\\
&T_4\left(\sec\theta_m\cos\theta\right)=\sec^4\theta_m\left(\cos 4\theta+4\cos 2\theta+3\right)-4\sec^2\theta_m\left(\cos 2\theta+1\right)+1
\end{aligned} \tag{2.5}$$

According to the equations above, overall reflection coefficient of e.g. a Chebyshev 4-section matching transformer will be of the form:

$$\begin{aligned}
\Gamma(\theta)&=Ae^{-j4\theta}T_4\left(\sec\theta_m\cos\theta\right)=\\
&=Ae^{-j4\theta}\left[\sec^4\theta_m\left(\cos 4\theta+4\cos 2\theta+3\right)-4\sec^2\theta_m\left(\cos 2\theta+1\right)+1\right]
\end{aligned} \tag{2.6}$$

while the general form of the 4-section matching transformer is a polynomial obtained from (1.2) and (1.3):

$$\Gamma(\theta)=2e^{-j4\theta}\left[\Gamma_0\left(e^{j4\theta}-e^{-j4\theta}\right)+\Gamma_1\left(e^{j2\theta}-e^{-j2\theta}\right)+\frac{\Gamma_2}{2}\right] \tag{2.7}$$

It is notable that it is sufficient to calculate Γ_0, Γ_1 and Γ_2. Due to the symmetry we have $\Gamma_0=\Gamma_4$, $\Gamma_1=\Gamma_3$.

If we recall that maximal magnitude of each Chebyshev polynomial should be unity within a passband, the maximal magnitude of the reflection coefficient within a passband satisfies the condition:

$$\left|\Gamma_m\right|=A \tag{2.8}$$

Constant A is determined by the boundary condition when θ approaches 0.

For determining θ_m, the following equation can be used [2]:

$$\sec\theta_m = \cosh\left[\frac{1}{N}\cosh^{-1}\left(\frac{\ln\left(Z_L/Z_0\right)}{2\Gamma_m}\right)\right] \tag{2.9}$$

With known local reflection coefficients, characteristic impedances of the sections can then be computed from the following equation:

$$Z_{n+1} = Z_n e^{2\Gamma_n} \tag{2.10}$$

The fractional bandwidth, obtained for this type of transformer, will be of the form:

$$\frac{\Delta f}{f_0} = 2 - \frac{4\theta_m}{\pi} \tag{2.11}$$

B. Example

Design a Chebyshev matching transformer to match a 300Ω load to a 50Ω line with optimal number of sections, taking the resulting bandwidth into account. The maximal magnitude of the reflection coefficient in the passband is 0.1 .

Using the theory from the previous section, θ_m can be calculated from (2.9). For $N = 4$ and $A = \left|\Gamma_m\right| = 0.1$ we get:

$$\begin{aligned}\sec\theta_m &= \cosh\left[\frac{1}{4}\cosh^{-1}\left(\frac{\ln\left(Z_L/Z_0\right)}{2\Gamma_m}\right)\right] = \\ &= \cosh\left[\frac{1}{4}\cosh^{-1}\left(\frac{\ln\left(300/50\right)}{2\left(0.1\right)}\right)\right] = 1.27 \Rightarrow \theta_m = 38°\end{aligned} \tag{2.12}$$

In order to calculate local reflection coefficients, we combine (2.6) and (2.7):

$$\begin{aligned}&Ae^{-j4\theta}\left[\sec^4\theta_m\left(\cos 4\theta + 4\cos 2\theta + 3\right) - 4\sec^2\theta_m\left(\cos 2\theta + 1\right) + 1\right] = \\ &= 2e^{-j4\theta}\left[\Gamma_0\cos 4\theta + \Gamma_1\cos 2\theta + \frac{\Gamma_2}{2}\right]\end{aligned} \tag{2.13}$$

Local reflection coefficients can be computed from (2.13):

$$2\Gamma_0 = A\sec^4\theta_m \Rightarrow \Gamma_0 = \frac{1}{2}0.1\sec^4\left(38°\right) = 0.131 \tag{2.14}$$

$$\begin{aligned} 2\Gamma_1 &= 4A\left(\sec^4\theta_m - \sec^2\theta_m\right) \Rightarrow \\ &\Rightarrow \Gamma_1 = \frac{1}{2}0.1\left(\sec^4\left(38°\right) - \sec^2\left(38°\right)\right) = 0.2 \end{aligned} \tag{2.15}$$

$$\begin{aligned} \Gamma_2 &= A\left(3\sec^4\theta_m - 4\sec^2\theta_m + 1\right) \Rightarrow \\ &\Rightarrow \Gamma_2 = 0.1\left(3\sec^4\left(38°\right) - 4\sec^2\left(38°\right) + 1\right) = 0.239 \end{aligned} \tag{2.16}$$

The remaining coefficients are found from the symmetry:

$$\Gamma_0 = \Gamma_4,\ \ \Gamma_1 = \Gamma_3 \tag{2.17}$$

Now we can calculate characteristic impedances for each section according to (2.10):

$$\begin{aligned} Z_1 &= Z_0 e^{2\Gamma_0} = 65\,\Omega \\ Z_2 &= Z_1 e^{2\Gamma_1} = 97.1\,\Omega \\ Z_3 &= Z_2 e^{2\Gamma_2} = 156.7\,\Omega \\ Z_4 &= Z_3 e^{2\Gamma_3} = 234.2\,\Omega \end{aligned} \tag{2.18}$$

For a given Γ_m and known impedance ratio Z_L/Z_0, useful tables for calculating these characteristic impedances can be found in [2].
The resulting bandwidth, calculated with (2.11) is:

$$\frac{\Delta f}{f_0} = 2 - \frac{4\theta_m}{\pi} = 2 - 4\left(\frac{38°}{180°}\right) = 1.155 \Rightarrow 115\% \tag{2.19}$$

For three sections transformer, $N = 3$, using (2.9) we get:

$$\theta_m = 48.13° \tag{2.20}$$

Overall reflection coefficient of Chebyshev 3-section matching transformer will be of the form:

$$\begin{aligned}\Gamma(\theta) &= Ae^{-j3\theta}T_3\left(\sec\theta_m \cos\theta\right) = \\ &= Ae^{-j3\theta}\left[\sec^3\theta_m\left(\cos 3\theta + 3\cos\theta\right) - 3\sec\theta_m\cos\theta\right]\end{aligned} \tag{2.21}$$

Combining (2.21) with (1.2) and (1.3), using $N = 3$, we get following reflection coefficients:

$$\Gamma_0 = \Gamma_4 = \frac{1}{2}0.1\sec^3\left(48.13°\right) = 0.17 \tag{2.22}$$

$$\Gamma_1 = \Gamma_3 = \frac{3}{2}0.1\left[\sec^3\left(48.13°\right) - \sec\left(48.13°\right)\right] = 0.28 \tag{2.23}$$

Characteristic impedances for each section are:

$$\begin{aligned}Z_1 &= Z_0 e^{2\Gamma_0} = 70\,\Omega \\ Z_2 &= Z_1 e^{2\Gamma_1} = 122.5\,\Omega \\ Z_3 &= Z_2 e^{2\Gamma_2} = 214.3\,\Omega\end{aligned} \tag{2.24}$$

The bandwidth of 3-section matching transformer for our θ_m:

$$\frac{\Delta f}{f_0} = 2 - \frac{4\theta_m}{\pi} = 0.93 \Rightarrow 93\% \tag{2.25}$$

Similar, using $N = 2$ and $N = 1$, we get these results:

For $N = 2$

$$\sec\theta_m = 2.23 \Rightarrow \theta_m = 63.4° \tag{2.26}$$

$$\Gamma_0 = \Gamma_3 = 0.249 \tag{2.27}$$

$$\Gamma_1 = 0.398 \tag{2.28}$$

$$Z_1 = 82.27\,\Omega \tag{2.29}$$

$$Z_2 = 182.37\,\Omega \tag{2.30}$$

$$\frac{\Delta f}{f_0} = 0.59 = 59\% \tag{2.31}$$

For $N = 1$

$$\sec\theta_m = 8.96 \Rightarrow \theta_m = 83.6° \tag{2.32}$$

$$\Gamma_0 = \Gamma_1 = 0.448 \tag{2.33}$$

$$Z_1 = 122.5\,\Omega \tag{2.34}$$

$$\frac{\Delta f}{f_0} = 0.14 = 14\% \tag{2.35}$$

3. IMPLEMENTATION IN WIPL-D SOFTWARE PACKAGE

The multi-section matching transformers, having the parameters obtained in the previous section, will be built and simulated in WIPL-D Microwave software. The results will be presented graphically [3].

WIPL-D is program package for fast and accurate design and simulation of microwave circuits, devices and antennas. Its user friendly interface and built in libraries make it a powerful and easy to use tool within academia and industry. Circuits can be created with built-in or user defined components. One of the advantages of this program package is the possibility of defining the components in four ways: ideal component, analytical component, 3D EM component and data component (when multiport device data is represented by tabular), depending on the requirements. There is a variety of diagrams for circuit simulation results visualization and they include: s-parameters, impedance or admittance parameters, voltages, currents, power. Radiation pattern and near field distribution of surface currents can be represented in 2D or 3D graph. Another feature is possibility of overlapping graphs from different projects for comparison purposes.

The most common applications include design of microwave components: microwave filters, resonators, power dividers, combiners, transistor amplifiers, ferrite components and circuits etc. as well as antennas and scatterers of arbitrary geometry. Apart from component design, WIPL-D can be applied as well to the problems of impedance matching and tuning, which is of the interest in this paper.

Microwave circuit simulation in WIPL-D consists of the following steps: defining the problem, creating the project and constructing the circuit, saving the project, running the analysis and plotting the results. After the problem is defined and required parameters obtained, we can open a new microwave circuit project by clicking the option New MW Circuit from the File menu of the Main menu bar. First, we define the frequency range in Frequency dialog box from the Edit menu of the Main menu, as shown in Fig. 2. Default reference frequency is set to be calculated as the mean value of start and stop frequencies. However, when needed, there is an option for defining the reference frequency manually.

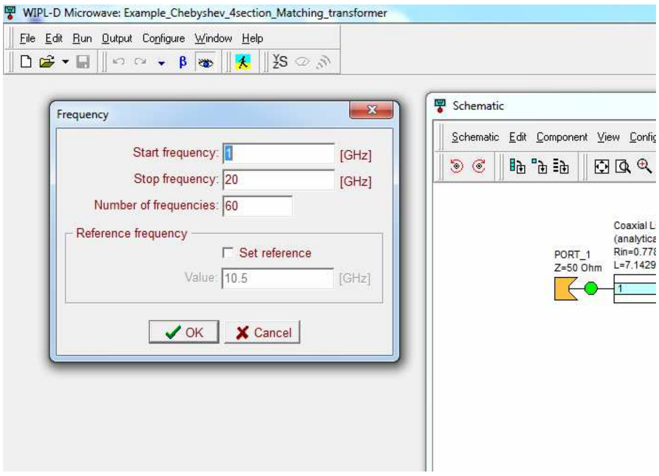

Fig. 2: Dialog box for defining a frequency range in a microwave circuit simulation project.

Now we switch to the *Schematic* window in order to build our circuit. All the components are put in the circuit by drag and drop principle from different palettes. Components and their positions in the circuit can be easily added, removed or adjusted. Resistor was added from the palette Lumped by dragging and dropping it in the Schematic window's blank space. Ports of the component, in this case resistor, are marked with numbers 1 and 2. For Chebyshev multisection matching transformer design we used the Coaxial Line component from the palette Coaxial, as shown in Fig. 3. Each Coaxial line represents one section of the transformer.

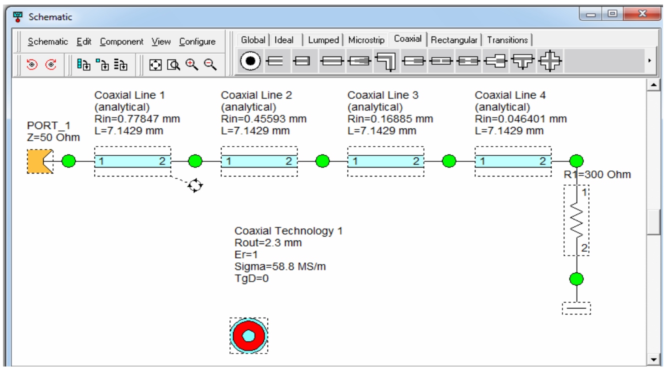

Figure 3: Chebyshev 4-section transformer's scheme with Coaxial palette shown.

Double click on the component opens a dialog box of that component and allows us to set the desired component dimensions, as shown in Fig. 4. In our case, we will set the characteristic impedances for each section and L/λ ratio, which should be 0.25 for a quarter-wave transformer. The circuit is saved by clicking Save MW circuit icon.

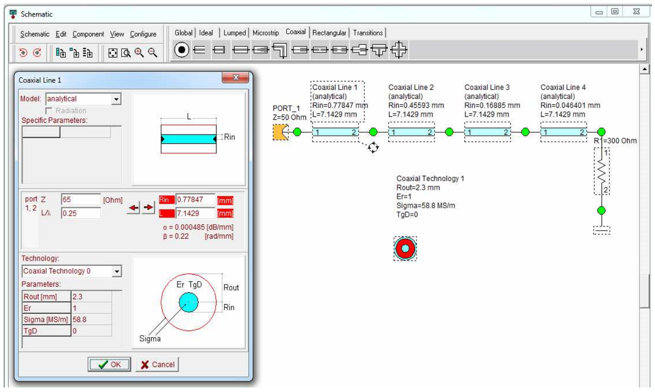

Fig. 4: Dialog box for setting and editing component parameters.

Simulation is carried out by clicking the Run icon. When finished, results can be displayed by clicking Y,Z,S icon..

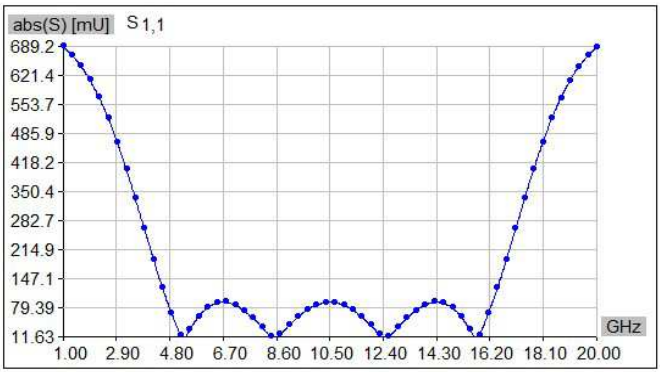

Fig. 5: Reflecrion coefficient (S11 parameter) magnitude.

Considering the requirements and parameters given, we get that the maximal VSWR in the passband should not exceed:

$$\mathrm{VSWR} = \frac{1+\Gamma_m}{1-\Gamma_m} = \frac{1.1}{0.9} = 1.22 \tag{3.1}$$

This corresponds to the results obtained in Fig. 6.

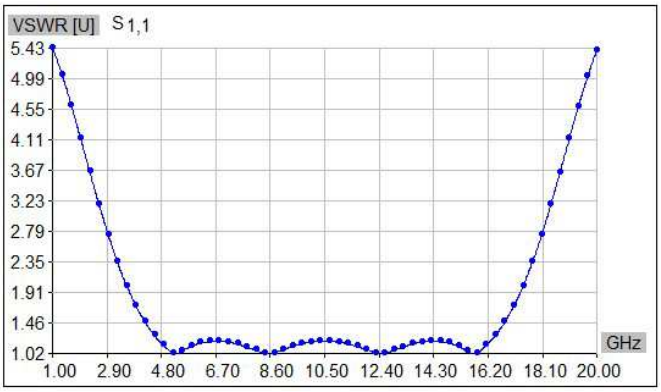

Fig. 6: VSWR versus frequency.

We can also verify our frequency range from the Fig. 6 according to (2.19):

$$\frac{\Delta f}{f_0} = \frac{16.54 - 4.46}{10.5} = 1.15 \tag{3.2}$$

Equations (3.1) and (3.2) confirm the results we obtained analytically.

Similar, schematics in WIPL-D for matching transformers with $N = 1, 2$ and 3 are given in Fig. 7.

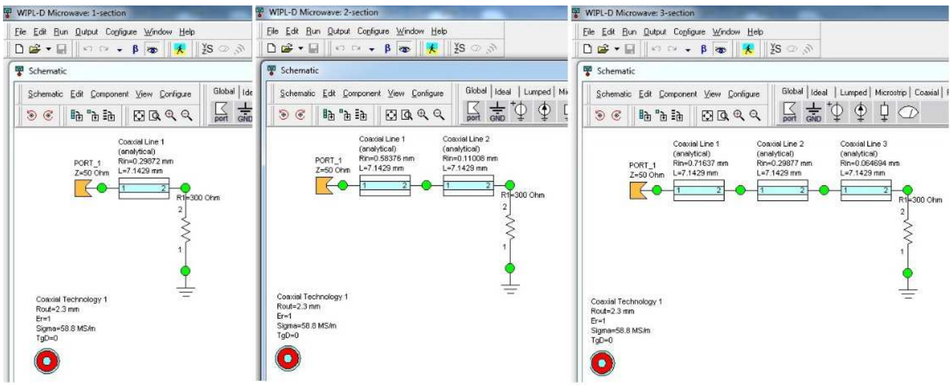

Fig. 7: Chebyshev 1-, 2-, 3-section transformer's schematic.

In WIPL-D we can compere VSWRs of circuits by clicking tab Overlay on YZS form, as shown in Fig. 8. The obtained results are presented in Fig. 9.

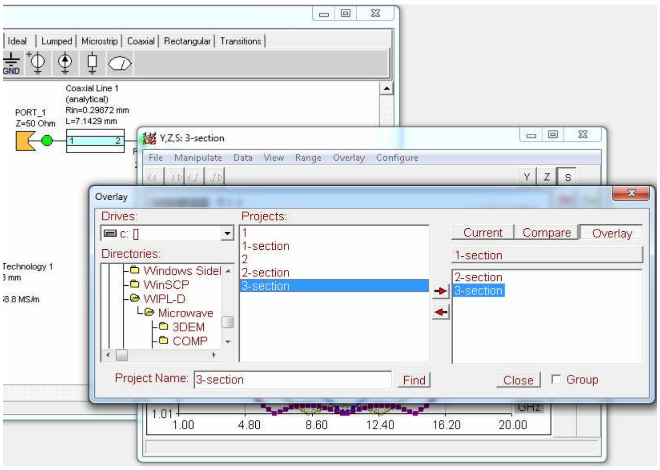

Fig. 8: Dialog box for setting comparison.

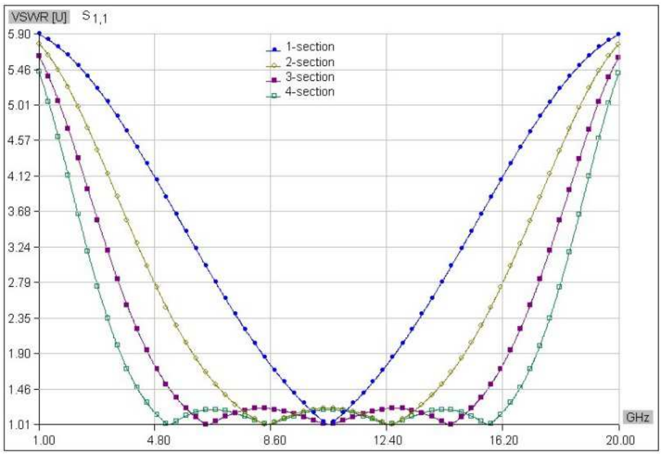

Fig. 9: VSWR comparison for Chebyshev 1-, 2-, 3- and 4-section transformer.

From Fig. 9. we can calculate frequency ranges using maximal VSWR obtained in (3.1)

$$\text{For } N=1 \ \frac{\Delta f}{f_0}=\frac{15.5-5.7}{10.5}=0.933=93.3\% \tag{3.3}$$

$$\text{For } N=2 \ \frac{\Delta f}{f_0}=\frac{13.6-7.4}{10.5}=0.59=59\% \tag{3.4}$$

$$\text{For } N=3 \ \frac{\Delta f}{f_0}=\frac{11.3-97}{10.5}=0.139=13.9\% \tag{3.5}$$

These results match the analytical ones from Chapter 2, as expected. 1- and 2-section transformer provides a narrowband matching transformer, which is not desirable for this type of application. On the other hand, 3- and 4-section transformers exhibit the same ripple size but with a great difference regarding the bandwidth.

This proves once again that this problem type can be solved faster and with desirable accuracy within WIPL-D software. In addition to that, optimal version can be also faster determined from the graphical results, overlayed.

4. CONCLUSION

In this paper we described design of the optimal Chebyshev multisection matching transformer for a given matching problem and its implementation in WIPL-D Microwave software package. The results of a multisections transformer design, obtained analytically were tested and confirmed by circuit simulation in WIPL-D. Their graphical representations are shown in chapter 3. We demonstrated that increasing number of transmission line sections provide larger bandwidth but at the expense of an increased ripple over the passband of the matching network. Thus, the use of WIPL-D software package provides a possibility of an easier and faster solving this type of matching problems and circuit design with high accuracy. Based on the results from Fig. 9 we can conclude that 3-section transformer is the optimal one because it represents the compromise between the passband ripple and desired bandwidth.

REFERENCES

[1] M. B. Steer, J.W. Bandler, C.M. Snowden: "Computer aided design of RF and Microwave Circuits and systems", *IEEE Transactions on Microwave theory and techniques,Vol. 50, No. 3, pp:996-1005*, March 2002
[2] D. M. Pozar, *Microwave engineering*, New York: Wiley,2012
[3] R.E. Collin, *Foundations for Microwave Engineering*, Wiley, 1992.
[4] WIPL-D Microwave, Lite V 1.0, Distributed by: Artech House, 2005.