COMPARATIVE ANALYSIS OF THE OPTIMIZATION CRITERIA OF GREVILLE'S ONE-PARAMETRIC INTERPOLATION KERNEL

Nataša Savić[*] , Zoran Milivojević[**] , Dejan Blagojević [***] , Darko Brodić [****]

Keywords: *Convolution, Interpolation kernel, Optimal value parameter.*

Abstract: First part of this paper describes Greville's one-parametric interpolation kernel and provides its calculated spectral characteristics. Then it provides the calculation of optimal parametric values of Greville's kernel a) by minimizing wiggles of spectral characteristics, b) by minimizing differences of the spectral characteristics in relation to kernel characteristics in the form sin(*x*)/*x*. Furthermore, this paper provides analytical form of the slope of spectral characteristics in boundary area. Second part of this paper describes an experiment in which the assessment of Greville's algorithm efficiency in interpolation of audio signals (G tones played on piano) and speech signals (logatoms spoken in Serbian language) was performed. The results are presented in tables and graphs.

1. INTRODUCTION

Digital processing of image and audio and speech signals, among other things, requires interpolation [1-3]. With systems for working in real time it is required that the algorithm for interpolation is of smaller numerical complexity and that therefore it has higher speed of execution, as well as a high accuracy of interpolation. Polynomial interpolation algorithms require a high degree of interpolation polynomial (n>10). However, an algorithm with this

---

[*] Nataša Savić is with the College of Applied Technical Sciences Niš, Serbia (phone: 381-69-1159001; e-mail natasa.savic@vtsnis.edu.rs).

[**] Zoran Milivojević is with the College of Applied Technical Sciences Niš, Serbia (phone: 381-63-8609206; e-mail: zoran.milivojevic@vtsnis.edu.rs).

[***] Dejan Blagojević is with the College of Applied Technical Sciences Niš, Serbia (phone: 381-69-4555069; e-mail: dejan.blagojevic@vtsnis.edu.rs).

[****] Darko Brodić is with the Faculty of Engineering, Bor, University in Belgrade, Serbia (phone: 381-6- e-mail: dbrodic@tf.bor.ac.rs).

kind of interpolation function is numerically complex and requires longer execution time. Therefore, this kind of interpolation is not suitable for systems working in real time.

In order to reduce numerical complexity, interpolation with polynomials of low degree (n=2, 3) is applied. Based on such low-degree polynomial, an interpolation kernel whose length depends directly on the degree of polynomial is constructed. Therefore, interpolation requires that the kernel moves over the function that should be interpolated. In mathematics, this procedure is described as convolution. Interpolation in which convolution is applied is called convolution interpolation [4-12]. It is known that [7] in band limited signals it is possible to perform an ideal interpolation by applying interpolation kernel in the form of $\sin(x)/x$ whose spectral characteristics is a box function. However, a practical implementation of this kernel is impossible because it requires an infinite length of the interpolation kernel. To overcome this limitation, kernels of small length were constructed which were thus suitable for practical application. The use of this kind of kernels tends to obtain a good approximation of an ideal kernel, that is, of its box characteristics.

Cubic convolution interpolation is most commonly used for working in real time. It is suitable because it uses cubic kernels which represent a compromise between speed of execution and numerical precision. Cubic convolution was mentioned for the first time in Rifman's paper [8], Simon wrote about it in more detail [9], while general form of the kernel first appeared in Bernstein's work [10]. Parametric kernels represent a significant class of cubic interpolation kernels. Starting from the general kernel shape from Bernstein's work, Keys meticulously studies the technique of convolution interpolation and the one-parameter cubic interpolation kernel in his own work [11]. After his work in the literature on parametric cubic convolution, this kernel is called Keys kernel. Greville's one-parametric kernel [3] is also often used for purposes of image processing. By selecting optimal kernel parameter it is possible to increase its interpolation precision. A large number of algorithms for the optimization of kernel parameter was derived. The optimization is carried out in the time and spectral domain. In paper [11] by applying parametric kernel on image reconstruction is suggested, and algorithm for determining optimal value of parameter α_{opt} is shown. Optimal value for implementation in image processing is α=-0.5. In [7] the algorithm for determining α_{opt} for Keys kernel in spectral domain is presented by applying Taylor's series. In paper [12] optimal parameter of Greville's kernel in estimating fundamental frequency of speech signals modelled by SYMPES method is obtained. In [13], kernel parameter of speech signals compressed by MP3 algorithm is determined.

Optimization of 1P Greville's kernel is carried out in this paper. Optimization was performed based on two algorithms: a) by reducing wiggles of spectral characteristics (Algorithm 1) and b) by minimizing error of the spectral characteristics in passband and stopband range with regard to spectral characteristics of an ideal interpolation kernel in the form $\sin(x)/x$ (Algorithm 2). Efficiency of these two algorithms will be tested in two ways: a) by determining the slope of spectral characteristics at the boundary between passband and stopband range (Algorithm 3) [7] and b) by using results of the experiment in which audio and speech signals are interpolated. In [14] shows that the kernel whose characteristic has a greater slope is more similar to box function, i.e. to the characteristic of an ideal $\sin(x)/x$ interpolation kernel, and thus is of higher quality than the kernel whose slope of characteristics is smaller.

By implementation of 1P Greville's kernel, interpolation of audio signals (tones G_1–G_7 of August Förster piano) and speech signals (logatoms in Serbian language) was conducted and optimal values of the parameters α_{opt} were calculated. Logatoms are used as test signals in testing the intelligibility of speech in various conditions of acoustic noise, both with healthy persons and those with the impared hearing [15]. Then a comparative analysis was performed and the effectiveness of Greville's kernel with parameters chosen by different criteria was assessed. Results are presented in tables and graphs.

This paper is organized as follows: Section 2 describes Greville's one-parametric interpolation kernel. Section 3 shows algorithms for optimization of kernel parameters. Results of the experiment and analysis are provided in Section 4. Section 5 is the conclusion.

2. GREVILLE'S ONE-PARAMETRIC CUBIC CONVOLUTION INTERPOLATION KERNEL

Paper [4] shows convolution interpolation with the implementation of Greville's cubic one-parametric interpolation kernel which he defined as follows:

$$r(x)=\begin{cases}\left(\alpha+\frac{3}{2}\right)|x|^3-\left(\alpha+\frac{5}{2}\right)|x|^2+1 & , \quad 0\le |x|\le 1\\ \frac{1}{2}(\alpha-1)|x|^3-\left(3\alpha-\frac{5}{2}\right)|x|^2+\left(\frac{11}{2}\alpha-4\right)|x|-(3\alpha-2) & , \quad 1<|x|\le 2\\ -\frac{1}{2}\alpha|x|^3+4\alpha|x|^2-\frac{21}{2}\alpha|x|+9\alpha & , \quad 2<|x|\le 3\\ 0 & , \quad 3<|x|\end{cases}, \tag{1}$$

where α is kernel parameter. Figure 1 displays Greville's kernels for several values of α parameter.

Kernel $r(x)$ (eq. 1) can be written in the form of components sum:

$$r(x)=r_0(x)+\alpha r_1(x), \tag{2}$$

where components are defined in the following way:

$$r_0(x)=\begin{cases}\frac{3}{2}|x|^3-\frac{5}{2}|x|^2+1 & , \quad 0<|x|\le 1\\ -\frac{1}{2}|x|^3+\frac{5}{2}|x|^2-4|x|+2 & , \quad 1<|x|\le 2\\ 0 & , \quad |x|>2\end{cases}, \tag{3}$$

and

$$r_1(x)=\begin{cases} |x|^3-|x|^2 & , \quad 0<|x|\le 1 \\ \frac{1}{2}|x|^3-3|x|^2+\frac{11}{2}|x|-3 & , \quad 1<|x|\le 2 \\ -\frac{1}{2}|x|^3+4|x|^2-\frac{21}{2}|x|+9 & , \quad 2<|x|\le 3 \\ 0 & , \quad 3<|x| \end{cases}. \tag{4}$$

Kernel components r_0 and r_1 are displayed in fig. 2.

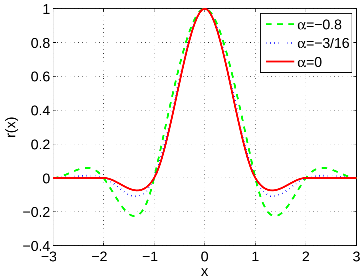


Fig. 1. One-parametric Greville's kernel for different values of α parameter.

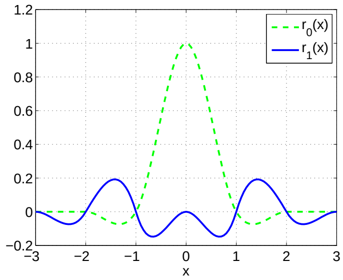


Fig. 2. Components r_0 and r_1 of Greville's one-parametric kernel.

3. OPTIMIZATION OF KERNEL PARAMETERS

Spectral characteristic of an ideal interpolation kernel in the form $\sin(x)/x$ is a box function. The change of α (eq. 2) parameter changes spectral characteristics of the kernel. In this way it is possible to make spectral characteristic similar, to a large extent, to rectangular function, which is derived as Fourier transform of an ideal interpolation kernel of the form $\sin(x)/x$. The process of adjusting α parameters and calculating the optimal value α_{opt} represent optimization of kernel parameters. It is possible to perform optimization according to criteria of characteristics similarity:

a) by minimizing wiggles of spectral characteristics through implementation of Taylor's series expansion (Algorithm 1),

b) by minimizing the difference of spectral characteristics in relation to rectangular characteristics (Algorithm 2), as well as by determining the slope of amplitude characteristics at transition from stopband to passband (Algorithm 3).

Afterwards, a comparative analysis will be performed based on analyzing the slope of spectral characteristics at transition from stopband to passband. For box characteristics, slope of the tangent at transition from stopband to passband has ∞ value. In accordance with this fact, the spectral kernel characteristic with a higher slope is better at approximating box function, that is, the spectral characteristic of an ideal interpolation kernel.

A. Algorithm 1: The optimization based on Taylor's series

By applying Fourier transform (FT) on kernel *r* (eq. 2), spectral characteristic of the kernel is obtained:

$$H(f)=FT(r(x))=FT(r_0(x)+\alpha\, r_1(x))=FT(r_0(x))+\alpha\, FT(r_1(x))=H_0(f)+\alpha H_1(f), \quad (5)$$

where α is kernel parameter, and $H_0(f)$ and $H_1(f)$ are spectral components of the kernel.

Spectral component, $H_0(f)$, is:

$$H_0(f)=\int_{-\infty}^{\infty} r_o(x)e^{-2\pi x f i}dx=\int_{-\infty}^{\infty} r_o(x)\cos(2\pi x\mathrm{f})dx-i\int_{-\infty}^{\infty} r_o(x)\sin(2\pi x\mathrm{f})dx=H_{0R}(f)-i\cdot H_{0I}(f), \quad (6)$$

By substituting (3) in (6), real part, $H_{0R}(f)$, of $H_0(f)$ spectral component, is:

$$\begin{aligned}H_{0R}(f)=&\int_{-2}^{-1}\left(\frac{1}{2}x^3+\frac{5}{2}x^2+4x+2\right)\cos(2\pi x f)dx+\int_{-1}^{0}\left(-\frac{3}{2}x^3-\frac{5}{2}x^2+1\right)\cos(2\pi x f)dx\\&+\int_{0}^{1}\left(\frac{3}{2}x^3-\frac{5}{2}x^2+1\right)\cos(2\pi x f)dx+\int_{1}^{2}\left(-\frac{1}{2}x^3+\frac{5}{2}x^2-4x+2\right)\cos(2\pi x f)dx\end{aligned}, \quad (7)$$

where $H_{0I}(f)$ is imaginary part of the spectral component $H_0(f)$:

$$\begin{aligned}H_{0I}(f)=&\int_{-2}^{-1}\left(\frac{1}{2}x^3+\frac{5}{2}x^2+4x+2\right)\sin(2\pi x f)dx+\int_{-1}^{0}\left(-\frac{3}{2}x^3-\frac{5}{2}x^2+1\right)\sin(2\pi x f)dx\\&\int_{0}^{1}\left(\frac{3}{2}x^3-\frac{5}{2}x^2+1\right)\cos(2\pi x f)dx+\int_{1}^{2}\left(-\frac{1}{2}x^3+\frac{5}{2}x^2-4x+2\right)\sin(2\pi x f)dx\end{aligned}. \quad (8)$$

By applying partial integration and the corresponding trigonometric equations it is calculated that $H_{0I}(f)=0$, so it follows that spectral component (6) $H_0(f)=H_{0R}(f)$ and it is equal to:

$$H_0(f)=\frac{3\sin^4(\pi f)-\pi f\sin^2(\pi f)\cdot\sin(2\pi f)}{\pi^4 f^4}=3\mathrm{Sinc}^4(f)-2\mathrm{Sinc}^2(f)\cdot\mathrm{Sinc}(2f), \quad (9)$$

where $\mathrm{Sinc}(f)=\sin(\pi f)/\pi f$.

Spectral component, $H_1(f)$, is:

$$H_1(f)=\int_{-\infty}^{\infty} r_1(x)e^{-2\pi x f i}dx=\int_{-\infty}^{\infty} r_1(x)\cos(2\pi x\mathrm{f})dx-i\int_{-\infty}^{\infty} r_1(x)\sin(2\pi x\mathrm{f})dx=H_{1R}(f)-iH_{1I}(f). \quad (10)$$

By substituting (4) in (10), real part, $H_{1R}(f)$, of $H_1(f)$ spectral component, is:

$$
\begin{aligned}
H_{1R}(f) &= \int_{-3}^{-2}\left(\frac{1}{2}x^3+4x^2+\frac{21}{2}x+9\right)\cos(2\pi xf)dx \\
&+\int_{-2}^{-1}\left(-\frac{1}{2}x^3-3x^2-\frac{11}{2}x-3\right)\cos(2\pi xf)dx \\
&+\int_{-1}^{0}\left(-x^3-x^2\right)\cos(2\pi xf)dx+\int_{0}^{1}\left(x^3-x^2\right)\cos(2\pi xf)dx \\
&+\int_{1}^{2}\left(\frac{1}{2}x^3-3x^2+\frac{11}{2}x-3\right)\cos(2\pi xf)dx+\int_{2}^{3}\left(-\frac{1}{2}x^3+4x^2-\frac{21}{2}x+9\right)\cos(2\pi xf)dx,
\end{aligned}
\tag{11}
$$

where $H_{1I}(f)$ is imaginary part of the spectral component $H_1(f)$:

$$
\begin{aligned}
H_{1I}(f) &= \int_{-3}^{-2}\left(\frac{1}{2}x^3+4x^2+\frac{21}{2}x+9\right)\sin(2\pi xf)dx \\
&+\int_{-2}^{-1}\left(-\frac{1}{2}x^3-3x^2-\frac{11}{2}x-3\right)\sin(2\pi xf)dx \\
&\int_{-1}^{0}\left(-x^3-x^2\right)\sin(2\pi xf)dx+\int_{0}^{1}\left(x^3-x^2\right)\cos(2\pi xf)dx \\
&+\int_{1}^{2}\left(\frac{1}{2}x^3-3x^2+\frac{11}{2}x-3\right)\sin(2\pi xf)dx+\int_{2}^{3}\left(-\frac{1}{2}x^3+4x^2-\frac{21}{2}x+9\right)\sin(2\pi xf)dx.
\end{aligned}
\tag{12}
$$

After calculating the integrals, it follows that imaginary part $H_{1I}(f)=0$, so the spectral component $H_1(f)=H_{1R}(f)$ and it is equal to:

$$
\begin{aligned}
H_1(f) &= \frac{\sin(\pi f)\cdot\sin(2\pi f)\cdot\left(3\sin(\pi f)\cdot\sin(2\pi f)-6\pi f\sin(\pi f)+4\pi f\sin^2(\pi f)\right)}{\pi^4 f^4} \\
&= 4\,\mathrm{Sinc}(2f)\cdot\mathrm{Sinc}(f)\cdot\left(3\,\mathrm{Sinc}(2f)\,\mathrm{Sinc}(f)-3\,\mathrm{Sinc}(f)+2\pi^2 f^2\,\mathrm{Sinc}^3(f)\right)
\end{aligned}
\tag{13}
$$

Fig. 3 shows spectral components H_0 and H_1, of Greville's interpolation kernel.

After Taylor series expansion about a point f=0, the spectral components take the following form:

$$H_{T0}(f)=1-\frac{1}{5}(\pi f)^4+\frac{68}{945}(\pi f)^6-\frac{62}{4725}(\pi f)^8+\cdots, \tag{14}$$

and

$$H_{T1}(f)=-\frac{16}{15}(\pi f)^4+\frac{80}{63}(\pi f)^6-\frac{3056}{4725}(\pi f)^8+\cdots . \tag{15}$$

Based on equations (14) and (15) Taylor's series of spectral characteristics of the kernel are:

$$H_T(f)=1-\frac{1}{5}\left(\frac{16}{3}\alpha+1\right)(\pi f)^4+\frac{4}{63}\left(\frac{17}{15}\alpha+20\right)(\pi f)^6-\frac{2}{4725}(1521\alpha+31)(\pi f)^8\cdots . \tag{16}$$

In paper [7], in order to decrease function wiggles in passband range it is suggested that the coefficient following the second term in the Taylor's series of spectral characteristics $H_T(f)$ should be equal to zero, where from:

$$\frac{16}{3}\alpha+1=0 \quad\Rightarrow\quad \alpha=\alpha_{opt}=-\frac{3}{16}=-0.1875 . \tag{17}$$

B. Algorithm 2: Optimization based on mean square error

The spectral characteristic of the kernel $\sin(x)/x$ is an ideal box function $H_B(f)$ which has value 1 in interval [0-0.5] and value 0 in interval [0.5-1]. As a measure of deviation of spectral characteristic of Greville's one-parametric interpolation kernel in relation to spectral characteristic of an ideal interpolation kernel $\sin(x)/x$ in passband and stopband range, total mean square error is defined:

$$E_T=2\left(\int_0^{0.5}|1-H(f)|^2 df+\int_{0.5}^{1}|H(f)|^2 df\right). \tag{18}$$

After shifting from continuous frequency axis *f* to discrete *k* by dividing segments [0-1] into *M* points, total mean square error is:

$$MSE=\frac{1}{M}\sum_{k=0}^{M-1}|H_B(f_k)-H(f_k)|^2=\frac{1}{M}\sum_{k=0}^{\frac{M-1}{2}}|1-H(f_k)|^2+\frac{1}{M}\sum_{k=\frac{M-1}{2}+1}^{M}|0-H(f_k)|^2 . \tag{19}$$

As spectral characteristic depends on parameter *α*, MSE will also depend on *α*. Optimal value of kernel parameters is obtained by minimizing mean square error. The dependence of MSE(*α*) is presented in fig. 4.

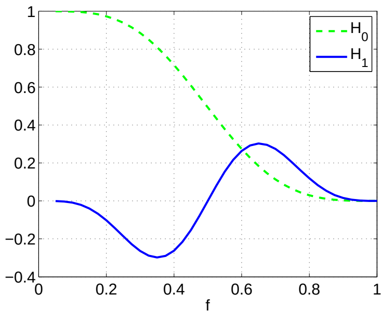


Fig. 3. Spectral components of Greville's interpolation kernel: H_0 and H_1

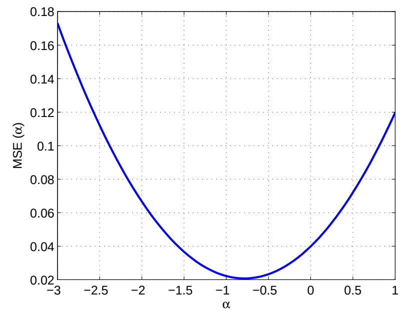


Fig. 4. Dependence of MSE on α parameter.

Optimal value of parameter is obtained based on the position of minimal value MSE(α), α_{opt}=-0.8.

C. Algorithm 3: Algorithm for comparing kernel spectral characteristic based on slope characteristics

The slope of spectral characteristic at a specific point is defined as the slope of tangent $y = k_\alpha x + n$ of spectral characteristic at that point and it represents the tangent of angle made by tangent and abscissa axis. Tangent of the angle is calculated by differentiating spectral characteristics. By differentiating (5), it follows that:

$$k_\alpha = H'(f) = \frac{d(H(f))}{df} = \frac{d(H_0(f) + \alpha H_1(f))}{df} = H'_0(f) + \alpha H'_1(f), \tag{20}$$

where:

$$H'_0 = \frac{1}{f^4\pi^4} \cdot \begin{pmatrix} -2f\pi^2\cos(2f\pi)\sin^2(f\pi) + 6\pi\sin(2f\pi)\sin^2(f\pi) - \\ f\pi^2\sin^2(2f\pi) - \pi\sin^2(f\pi)\sin(2f\pi) \end{pmatrix} - \frac{4}{f^5\pi^4}\left(3\sin^4(f\pi) - f\pi\sin^2(f\pi)\sin(2f\pi)\right), \tag{21}$$

and

$$H'_1 = \frac{1}{f^4\pi^4}\begin{pmatrix} -12f\pi^2\cos(2f\pi)\sin^2(f\pi) + 8f\pi^2\cos(2f\pi)\sin^3(f\pi) - 6f\pi^2\sin^2(2f\pi) - \\ 6\pi\sin^2(f\pi)\sin(2f\pi) + 6f\pi^2\sin(f\pi)\sin^2(2f\pi) + 6\pi\sin^2(f\pi)\sin(4f\pi) + \\ 4\pi\sin(2f\pi)\sin^3(f\pi) + 3\pi\sin^3(2f\pi) \end{pmatrix} - \frac{4}{f^5\pi^4}\left(-6f\pi\sin^2(f\pi)\sin(2f\pi) + 4f\pi\sin^3(f\pi)\sin(2f\pi) + 3\sin^2(f\pi)\sin^2(2f\pi)\right). \tag{22}$$

By analyzing slope, k_α, of spectral characteristic at passband range border, it is possible to perform a comparative analysis of kernels with different kernel parameters [14]. In box function, the slope at passband range border has ∞ value. Based on this fact, spectral characteristics of kernel with a higher slope will be better at aproximating the spectral characteristics of an ideal interpolation kernel, i.e. box function.

4. EXPERIMENTAL RESULTS OF THE ANALYSIS

Theoretically derived values of optimal parameters of Greville's 1P kernel α_{opt}=-3/16 (Algorithm 1) and α_{opt}=-0.8 (Algorithm 2) will be compared to experimentally derived values of kernel parameters.

A. The experiment

Choice of optimal parameter values of Greville's 1P interpolation kernel was performed in interpolation of: a) audio and b) speech signals, which were sampled by different frequencies in process of recording.

Precision of interpolation kernel at estimating signal values is determined experimentally, by analyzing estimation error which represents the difference between true (x_i) and interpolated value ($\hat{x}_i$). The problem of obtaining true value was solved as follows: signal values $\mathbf{x}=\{x_1, x_2,..., x_N\}$, whose values are known and used as true values, are interpolated. Estimation of value of *i*-th component for signal ($\hat{x}_i$) is realized by forming *i*-th block $\mathbf{x}_{Bi}=\{x_{i-5}, x_{i-3}, x_{i-1}, x_{i+1}, x_{i+3}, x_{i+5}\}$ and by using Greville's convolution interpolation kernel. Interpolation error $e = x_i - \hat{x}_i$ where x_i is *i*-th component of signal **x**, is treated as true value, while $\hat{x}_i$ is interpolated value.

Algorithm for determining optimal value of kernel parameters α consists of the following steps:

Input: audio signal **x,** length *N*, kernel parameter $\alpha=\{\alpha_1,..., \alpha_K\}$, *K* - number of parameters, block length *M=2L-1*, kernel *r*, *L* kernel length.
Output: α_{opt} – optimal parameter.

Step 1: Selection of *i*-th block $\mathbf{x}_{Bi}$ of length *M=2*L-1* of signal **x** where *i=1...N-L+1.*
Step 2: Estimation $\hat{x}_{(i+L-1)}$ by application of Greville's kernel *r* (over signal components $x_{i+L-1\pm(2n-1)}$ where *n*=1, 2, 3) with parameter α_k where *k*=1 … *K.*
Step 3: Determining error of interpolation $e = x_{(i+L-1)} - \hat{x}_{(i+L-1)}$.
Step 4: Steps 1-3 are repeated until *i=N-L*+1.
Step 5: Calculating mean square error:

$$MSE_{\alpha_k} = \frac{1}{N-L+1}\sum_{i=1}^{N-L+1} |e_i|^2 \; . \tag{23}$$

Step 6: Repetition of steps 1-5 until *k≤K.*
Step 7: Determining α_{opt} based on the range MSE=$\{MSE_{\alpha 1}, ..., MSE_{\alpha K}\}$

$$\alpha_{opt} = \arg\min_{\alpha}(MSE)\,. \quad (24)$$

B. The base

The base consists of:

a) audio signals obtained by recording G tones from seven octaves (G_1–G_7) played on August Förster piano and

b) speech signals obtained by recording speakers who pronounce different logatoms of type: affricates, CCV (Consonant-Consonant-Vovel), CVC (Consonant-Vovel-Consonant), fricatives, laterals, nasals and plosives [16]. Sampling was performed in following frequencies fs={8, 22.05, 44.1} kHz. Recorded material was archived in form of **wav** files.

C. The results

By applying algorithm for determining optimal values of interpolation kernel parameter (Section 4. A) over the signals from the base of audio and speech signals (Section 4. B), MSE (α) (eq 23) was obtained. Optimal values of kernel parameter, α_{opt}, are calculated based on minimum MSE value for each signal (eq. 24).

Table I shows $MSE_{min}(\alpha)$ values for G tones from seven octaves (G_1–G_7).

Table I
$MSE_{min}(\alpha)$ and α_{opt} in interpolation audio signal.

Tone	f_s[Hz]	α_{opt}	MSE
G_1	44.1	-0.2000	$1.8416*10^{-8}$
	22.05	-0.3000	$1.8962*10^{-7}$
	8	-0.3000	$2.9092*10^{-4}$
G_2	44.1	-0.3000	$1.7782*10^{-8}$
	22.05	-0.4000	$8.6402*10^{-7}$
	8	-0.3000	$1.4601*10^{-4}$
G_3	44.1	-0.3000	$3.6858*10^{-8}$
	22.05	-0.3000	$9.4093*10^{-6}$
	8	-0.3000	$2.7414*10^{-4}$
G_4	44.1	-0.4000	$1.3610*10^{-7}$
	22.05	-0.3000	$4.9337*10^{-5}$
	8	-0.5000	0.0012
G_5	44.1	-0.4000	$2.2348*10^{-7}$
	22.05	-0.3000	$2.4520*10^{-5}$
	8	-0.3000	$2.9696*10^{-4}$
G_6	44.1	-0.5000	$1.2164*10^{-6}$
	22.05	-0.3000	$8.9330*10^{-5}$
	8	-0.9000	0.0020
G_7	44.1	-0.4000	$1.3255*10^{-6}$
	22.05	-0.3000	$8.2240*10^{-5}$
	8	0.0000	0.0011

Table II
$MSE_{min}(\alpha)$ and α_{opt} in interpolation speech signal.

Logatom	f_s[Hz]	α_{opt}	MSE
Affricates (Djucu)	44.1	-0.40	$1.1857*10^{-5}$
	22.05	2.00	$5.0133*10^{-4}$
	8	1.70	$7.1272*10^{-4}$
CCV (Pre)	44.1	-0.40	$1.2922*10^{-5}$
	22.05	-0.40	$8.2828*10^{-4}$
	8	3.10	$7.2172*10^{-4}$
CVC (Men)	44.1	0.00	$8.7620*10^{-6}$
	22.05	-0.40	$3.6749*10^{-5}$
	8	1.00	$3.5988*10^{-4}$
Fricatives (Zefo)	44.1	-0.60	$3.0932*10^{-4}$
	22.05	2.70	0.0027
	8	1.00	0.0020
Laterals (Lera)	44.1	-0.30	$1.1764*10^{-5}$
	22.05	0.40	$7.1226*10^{-5}$
	8	0.30	0.0021
Nasals (Noma)	44.1	-0.20	$9.9333*10^{-7}$
	22.05	-0.30	$4.7954*10^{-6}$
	8	-0.40	$5.2955*10^{-5}$
Plosives (Taku)	44.1	0.20	$2.2358*10^{-6}$
	22.05	0.20	$9.6607*10^{-6}$
	8	-0.50	$5.7292*10^{-5}$

Fig. 5 shows the dependence of MSE(α) for piano tone G_2 for a) *f*s=44.1 kHz (fig. 5.a) b) *f*s=22.05 kHz (fig. 5.b) and c) *f*s=8 kHz (fig. 5.c).

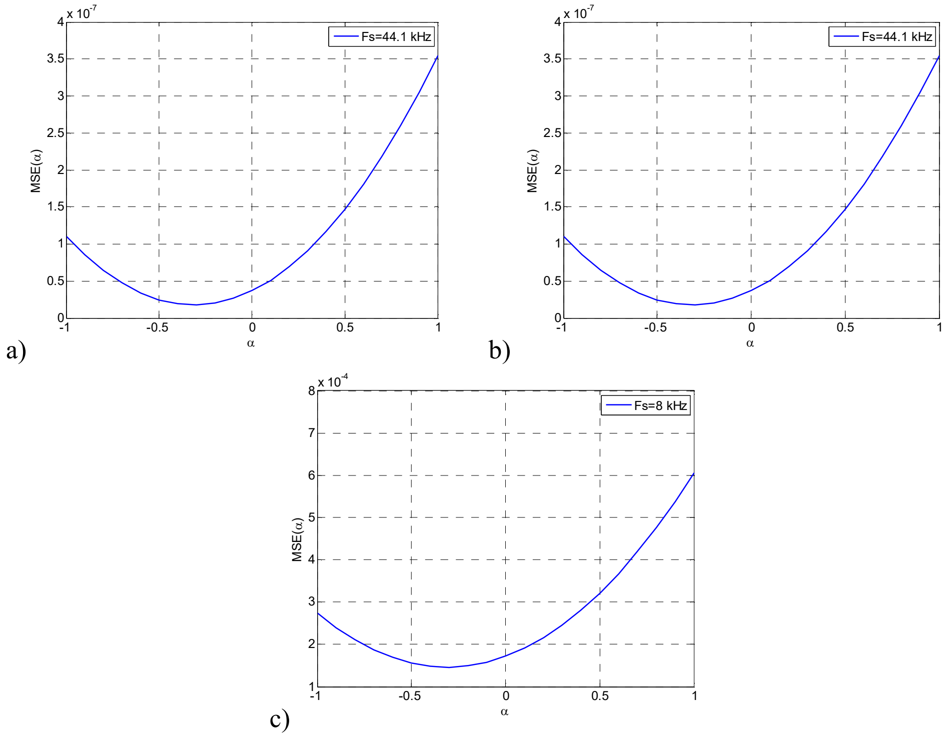


Fig. 5. MSE(*α*) value for interpolation of tone G_2 for sampling frequencies a) *f*s=44.1 kHz, b) *f*s=22.05 kHz and c) *f*s=8 kHz.

Table II shows MSE(α) values for logatoms: Affricates, CCV, CVC, Fricatives, Laterals, Nasals and Plosives.

Figure 6 shows the dependence of MSE(α) for a logatom from Nasal group, the word Noma for a) *f*s=44.1 kHz (fig. 5.a) b) *f*s=22.05 kHz (fig. 5.b) and c) *f*s=8 kHz (fig. 5.c).

Table III shows values of the quotient of tangent direction with amplitude characteristic *H* at the *f*=0.5 for parameter values *α* from Section 3.A (*α*=-3/16) and 3.B (*α*=-0.8), as well as corresponding MSE(*α*) values.

Table III
Values of the quotient of tangent direction (k_α) with amplitude characteristic *H* at point *f*=0.5 for different parameter values of α and MSE(α).

Criterion	*α*	$k_\alpha = \lvert H'(0.5, \alpha) \rvert$	MSE(*α*)
Algorithm 1	-3/16	2.9289	0.0314
Algorithm 2	-0.8	4.9148	0.0204

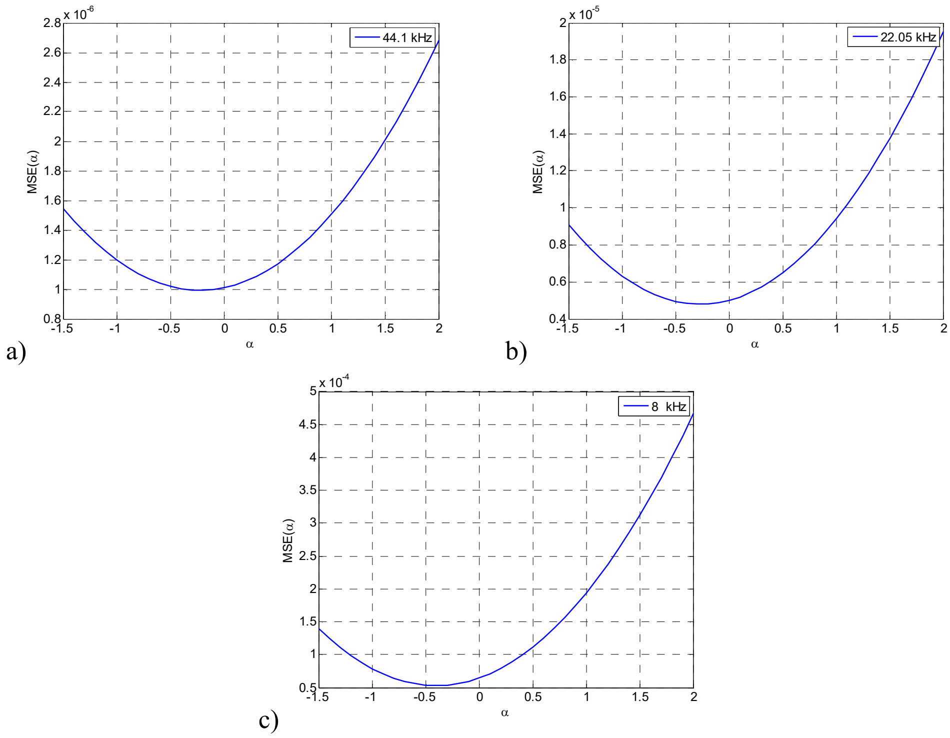
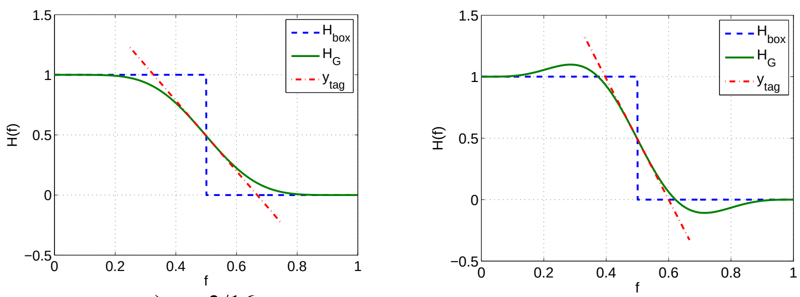


Fig. 6. MSE(α) value in interpolation of the logatom Noma of sampling a) fs=44.1 kHz, b) fs=22.05 kHz and c) fs=8 kHz.

Fig. 7 shows amplitude characteristics of: a) ideal interpolation kernel in the form $\sin(x)/x$ (H_{box}), b) Greville's kernel (H_{G}), and tangent (y_{tag}) characteristics of Greville's kernel at f=0.5 that is for α_{opt} =-3/16 and α_{opt} =-0.8.



a) α=-3/16

b) α=-0.8

Fig. 7. Box function, spectral characteristics and tangent of spectral characteristics of Greville's kernel at transition from stopband to passband for a) α=-3/16 and b) α=-0.8.

D. RESULT ANALYSIS

Based on the results of experiment shown in table 1, table 2 and α_{opt} values obtained by optimization based on Taylor's series ($\alpha_{opt}=\alpha_T$=-3/16) (Section 3.A) and optimization based on MSE ($\alpha_{opt}=\alpha_B$=-0.8) (Section 3.B) it can be concluded that:

1) for audio signal (tones G_1–G_7).

a) fs=44.1kHz, $\alpha_{opt\_44.1}\in$ [-0.5÷-0.2] and $\overline{\alpha_{opt\_44.1}}$ = -0.3571;

b) fs=22.0 5 kHz, $\alpha_{opt\_22.05}\in$ [-0.4÷-0.3] and $\overline{\alpha_{opt\_22.05}}$ = -0.3143;

c) fs=8 kHz, $\alpha_{opt}\in$ [-0.9÷0] and $\overline{\alpha_{opt\_8}}$ = -0.3714;

d) error of estimation of optimal parameter, assuming that experimental values are true values, for parameter obtained based on Taylor's series

$\Delta_{asT\_44.1}=\left|\alpha_T-\overline{\alpha_{opt\_44.1}}\right|=\left|-0.1875-(-0.3571)\right|=0.1696$,

$\Delta_{asT\_22.05}=\left|\alpha_T-\overline{\alpha_{opt\_22.05}}\right|=\left|-0.1875-(-0.3143)\right|=0.1268$,

$\Delta_{asT\_8}=\left|\alpha_T-\overline{\alpha_{opt\_8}}\right|=\left|-0.1875-(-0.3714)\right|=0.1839$

is the smallest for f_s=22.05 kHz,

e) error of estimation of optimal parameter, assuming that experimental values are true values, for parameter obtained based on similarity with box function

$\Delta_{asB\_44.1}=\left|\alpha_B-\overline{\alpha_{opt\_44.1}}\right|=\left|-0.8-(-0.3571)\right|=0.4429$,

$\Delta_{asB\_22.05}=\left|\alpha_B-\overline{\alpha_{opt\_22.05}}\right|=\left|-0.8-(-0.3143)\right|=0.4857$,

$\Delta_{asB\_8}=\left|\alpha_B-\overline{\alpha_{opt\_8}}\right|=\left|-0.8-(-0.3714)\right|=0.4286$

is the smallest for f_s=8 kHz,

f) error of estimation of optimal parameter obtained based on criterion 1, $\alpha_{opt}=\alpha_T$=-3/16 is smaller than error of estimation of optimal parameter obtained based on criterion 2, $\alpha_{opt}=\alpha_B$=-0.8 (Δ_{asB_8} / $\Delta_{asT\_22.05}$=0.4286/0.1268=3.38013).

2) For speech signal when pronouncing logatoms

a) fs=44.1 kHz, $\alpha_{opt}\in$ [-0.6÷-0.2] and $\overline{\alpha_{opt\_44.1}}$ = -0.2429;

b) fs=22.05 kHz, $\alpha_{opt}\in$ [-0.4÷2.7] and $\overline{\alpha_{opt\_22.05}}$ = -0.6;

c) fs=8 kHz, $\alpha_{opt}\in$ [-0.5÷3.1] and $\overline{\alpha_{opt\_8}}$ = 0.8857;

d) error of estimation of parameter, assuming that experimental values are true values, for parameter obtained based on Taylor's series expansion

$\Delta_{asT\_44.1}=\left|\alpha_T-\overline{\alpha_{opt\_44.1}}\right|=\left|-0.1875-(-0.2429)\right|=0.0554$,

$\Delta_{asT\_22.05}=\left|\alpha_T-\overline{\alpha_{opt\_22.05}}\right|=\left|-0.1875-(-0.6)\right|=0.7875$,

$\Delta_{asT\_8}=\left|\alpha_T-\overline{\alpha_{opt\_8}}\right|=\left|-0.1875-0.8857\right|=1.0732$ is smallest for f_s=44.1kHz,

e) The error of estimation, assuming that experimental values are true values, for parameter obtained based on similarity with box function

$\Delta_{asB\_44.1} = \left|\alpha_B - \overline{\alpha_{opt\_44.1}}\right| = \left|-0.8-(-0.2429)\right| = 0.5571$,

$\Delta_{asB\_22.05} = \left|\alpha_B - \overline{\alpha_{opt\_22.05}}\right| = \left|-0.8-(-0.6)\right| = 0.2$,

$\Delta_{asB\_8} = \left|\alpha_B - \overline{\alpha_{opt\_8}}\right| = \left|-0.8-0.8857\right| = 1.6857$

is smallest for f_s=22.05 kHz,

f) error of estimation of optimal parameter obtained based on Algorithm 1, $\alpha_{opt}=\alpha_T=-3/16$ is smaller than the error of estimation of optimal parameter obtained based on Algorithm 2, $\alpha_{opt}=\alpha_B=-0.8$ ($\Delta_{asB\_22.05}$ / $\Delta_{asT\_44.1}$=0.2/0.0554=3.6101).

By comparing errors of estimation of parameters, it is concluded that interpolation by a kernel whose parameter was obtained by optimization based on Taylor's expansion (Algorithm 1) is more precise than interpolation by a kernel whose parameter was obtained by optimization based on MSE (Algorithm 2) both on audio and speech signals.

If in the purpose to compare the quality of amplitude characteristic of kernels, Algorithm 3 which analyzes slope of the spectral characteristic at the boundary point of passband range is applied, it can be concluded that kernel with parameter α_{opt}=-0.8 (Algorithm 2) is of higher quality than kernel with parameter α_{opt}=-3/16 (Algorithm 1) because of greater slope of amplitude characteristic of the kernel, since for α_{opt}=-0.8 the slope is │H′(0.5, -0.8)│=4.9148, and for α_{opt}=-3/16 the slope is │H′(0.5, -3/16│=2.9289, so │H′(0.5, -0.8)│> │H′(0.5, -3/16│ and MSE(-0.8) <MSE(-3/16). This conclusion is contrary to the conclusion derived based on experimental results.

This is because it is a fact that increase of the slope of spectral characteristic of Greville's kernel, due to changes of the parameter α, leads to a significant uplift of the amplitude characteristic in the passband range. In this way, amplitude chracteristic of Greville's kernel is increasingly different from box function, except in the narrow part around the border of passband range. A detailed analysis of this effect, as well as of the consequences on precision of interpolation by Greville's kernel will be the subject of further research.

5. CONCLUSION

This paper presents algorithms for calculating optimal parameter values of Greville's interpolation kernel. Optimal values were determined by analysis of spectral characteristics of the kernel, i.e. by minimizing the wiggle of spectral characteristics (Algorithm 1, $\alpha_{opt}=\alpha_T=-3/16$) and by minimizing the difference of spectral characteristics in relation to ideal box characteristics (Algorithm 2, $\alpha_{opt}=\alpha_B=-0.8$).

Based on the experimental results of interpolation of audio signals (G tones from seven octaves of August Forster piano) and speech signals (logatoms in Serbian language) experimental optimal values of parameter were obtained: $\overline{\alpha_{opt\_44.1}}$ = -0.3571, $\overline{\alpha_{opt\_22.05}}$ = -0.3143, $\overline{\alpha_{opt\_8}}$ = -0.3714 with audio and speech signals $\overline{\alpha_{opt\_44.1}}$ = -0.2429, $\overline{\alpha_{opt\_22.05}}$ = -0.6, $\overline{\alpha_{opt\_8}}$ = 0.8857 . Taking into consideration the difference between optimum parameters obtained by using algorithm 1 and algorithm 2, efficiency of estimation of interpolation values was calculated. It has been concluded that: Greville's kernel with parameter derived by algorithm 1, is more precise than Greville's kernel with parameter derived by algorithm 2.

Estimation of efficiency of the interpolation of kernel with different parameters was conducted by Algorithm 3, in which the slopes of spectral characteristics were analyzed at the boundary of passband range: for α_{opt} =-3/16 (Algorithm 1) is │H´(0.5, -3/16│=2.9289 and b) α_{opt} =-0.8 (Algorithm 2) │H´(0.5, -0.8)│=4.9148. It is concluded that because │H´(0.5, -0.8)│>│H´(0.5, -3/16│ Greville's kernel with parameter α_{opt} =-0.8 is better. This conclusion is in contrast with the conclusion obtained experimentally. The reason for the imprecision of algorithm 3 which analyzes the slope of spectral characteristics is that similarity with box function is analyzed in a small range,at the transition from stopband to passband (f=0.5). Along with the increase of the slope the compatibility in the transitional range increases, while deviation in the remaining part of characteristic increases drastically. By changing α parameter opposite tendencies appear in Algorithms 1 and 2 in relation to Algorithm 3. By adjusting α parameter in order to increase the slope, wiggles increase as well as deviations of spectral characteristics in relation to box function. Calculating optimal values of the parameter along with simultaneous analysis of effects of the slope, wiggles and the error of deviation from the box characteristics will be the following activities of the author.

REFERENCES

[1] N. A. Dodgson, "Quadratic interpolation for image resampling", *IEEE Trans.Image Processing,* Vol. 6, no. 9, pp. 1322-1326, Sept. 1997.

[2] J. A. Parker, R. V. Kenyon, and D. E. Troxel, "Comparison of interpolation methods for image resampling", *IEEE Trans. Med. Imag.,*vol. *2*, pp. 31-39, 1983.

[3] T. N. E. Greville, "The General Theory of Osculatory Interpolation", *Trans. of the Actuarial Society of America,* Vol. 45, pp. 202-265, 1944.

[4] E. Meijering, M. Unser, "A Note on Cubic Convolution Interpolation", *IEEE Transactions on Image Processing,* vol. 12, no. 4, pp. 447-479, April 2003.

[5] H. S. Pang, S. J. Baek, K. M. Sung, "Improved Fundamental Frequency Estimation Using Parametric Cubic Convolution", *IEICE Trans. Fundamentals,* vol. E83-A, no. 12, pp. 2747-2750, Dec. 2000.

[6] S. E. Reicherbach, "Two-Dimensional Cubic Convolution ", *IEEE Trans. Image Processing,* vol. 12, no. 8, pp. 857-865, Aug. 2003.

[7] K. S. Park, R. A. Schowengerdt, "Image reconstruction by parametric cubic convolution", *Computer Vision, Graphics & Image Process.,* vol. 23, pp. 258-272, 1983.

[8] S. S. Rifman, "Digital rectification of ERTS multispectral imagery", in *Proc Symp. Significant Results Obtained From the Earth Resources Tehnology Satellite-1* ,vol. 1, sec. B pp. 1131-1142, 1973.

[9] K. W. Simon, "Digital image reconstruction and resampling for geometric manipulation", in *Symp. On Machine Prosess. Of Remotely Sensed Data, IEEE Pre* pp. 3A-1-3 A-11, 1975.

[10] R. Bernstein, "Digital image processing of earth observation sensor data", *IBM J. Res. Devel.* vol. 20, no. 1. pp. 40-57, 1976.

[11] R. G. Keys, "Cubic convolution interpolation for digital image processing", *IEEE Trans. Acout. Speech*, & *Signal Processing,* vol. ASSP-29, pp. 1153-1160, Dec. 1981.

[12] Z. Milivojević, M. Mirković, "Estimation of the fundamental frequency of the speech signal modeled by the SYMPES method", *Int. J. Electron. Commun. (AEÜ)* 63 pp. 200 – 208, 2009.
[13] Z. Milivojević, D. Brodić, "Estimation of the Fundamental Frequency of the Speech Signal Compressed by MP3 Algorithm" , *Archives of acoustics,* vol. 38, no. 3, pp. 363–373, 2013.
[14] E. Mejering. K. Zuiderveld, M. Viegever, "Image Reconstruction by Convolution with Simetrical Piecewise nth-Order Polynomial Kernels", *IEEE Transactions on Image Processing,* vol. 8, no. 2, pp. 192-201, Feb. 1999.
[15] J. Kocinski, E. Ozimek, "Speech Recognition in an Enclosure with a Long Reverberation Time", *Archives of acoustics,* vol. 41, no. 2, pp. 255–264, 2016.
[16] Lj. Subotić, D. Sredojević, I. Bjelaković, "Fonetika i fonologija: ortoepska i ortografska norma standardnog srpskog jezika", *Filozofski fakultet*, Novi Sad.