

Estimation of $R_3 = \Pr(Y_3 > \max(Y_1, Y_2))$ for Exponential Distribution

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Abstract This paper deals with the estimation of the reliability function $R_3 = \Pr(Y_3 > \max(Y_1, Y_2))$, when the random variables Y_1, Y_2 and Y_3 are independent exponential with respective unknown location parameters μ_1, μ_2 and μ_3 . Here we have derived several estimators of R_3 which includes uniformly minimum variance unbiased estimator (UMVUE), Bayes estimator (BE) and generalized Bayes estimator (GBE). Moreover, we have compared these estimators based on their risks under different loss functions.

Keywords: Stress-strength reliability, UMVUE, Bayes estimator, generalized Bayes estimator.

1 Introduction

Stress-strength reliability of a component having strength X subject to a stress Y is defined by $R = \Pr(Y > X)$. Literature review reveals the importance of the estimation of Stress-strength reliability under various lifetime distributions. In this regard mention may be made to Kotz *et al.* (2003), Baklizi (2008a, 2008b), Eryilmaz (2008a, 2008b, 2010, 2011), Krishnamoorthy *et al.* (2007, 2009, 2010), Krishnamoorthy and Lin (2010), Kundu and Raqab (2009), Rezaei *et al.* (2010). Recently, Rao (2012, 2013) studied the estimation of reliability in a multicomponent stress-strength model. Rinco (1983) initiated a study on the estimation of $R_p = \Pr(Y_p > \max(Y_1, Y_2, \dots, Y_{p-1}))$ and provided a particular estimator. Gupta and Gupta (1988) considered a similar problem and provided three other estimators. Unfortunately, some estimators are difficult to derive in general and only the special case of $p=2$ was found to be explicitly solvable. Literature review also reveals that even for the case $p = 3$, no work has yet been done for the case of exponential distribution with different location parameters.

The study of the estimation of stress-strength reliability of a component for exponential lifetime distributions with common scale parameters are important from the theoretical point of view which can be evidenced from Ali *et al.* (2004), Ivshin (1996), Pal *et al.* (2005) and Sengupta (2008).

The paper is organized as follows. In Section 2, we provide the notations and preliminaries. Section 3 considers the estimation aspect of R_3 and Section 4 compares the performances of the estimators through a simulation study.

2 Notation and Preliminaries

In this section, we provide the following notations and preliminaries which will be helpful in developing the estimation of the reliability function $R_3 = \Pr(Y_3 > \max(Y_1, Y_2))$, where Y_3 denotes the strength and Y_1, Y_2 are the stresses. It is assumed that Y_1, Y_2, Y_3 are independent random variables having the following probability density functions.

$$f(y_i) = \exp\{-(y_i - \mu_i)\} ; y_i \geq \mu_i, -\infty < \mu_i < \infty, i = 1, 2, 3. \quad (1)$$

Defining $T = \max(Y_1, Y_2)$ and $\mu = \max(\mu_1, \mu_2)$, we can write

$$R_3 = \Pr(Y_3 > T) = 1 - \int_{\mu}^{\infty} \Pr(Y_3 \leq t | T = t) f(t) dt = 1 - \bar{R}_3 \quad (2)$$

where $f(t)$ denotes the probability density function of T which is stated below

$$f(t) = \sum_{i=1}^2 \exp\{-(t - \mu_i)\} - 2 \exp\{-(2t - \mu_1 - \mu_2)\}, \quad t \geq \mu.$$

In order to get an expression for \bar{R}_3 , we consider the following cases.

Case-1: $\mu \leq \mu_3$

$$\begin{aligned} \bar{R}_3 &= \int_{\mu_3}^{\infty} \Pr(Y_3 \leq t|T = t)f(t)dt \\ &= \frac{1}{2} \sum_{i=1}^2 \exp\{-(\mu_3 - \mu_i)\} - \frac{1}{3} \exp\{-(2\mu_3 - \mu_2 - \mu_1)\} \\ &= R_1(\text{say}) \end{aligned}$$

Case-2: $\mu > \mu_3$

$$\begin{aligned} \bar{R}_3 &= \int_{\mu}^{\infty} \Pr(Y_3 \leq t|T = t)f(t)dt \\ &= \sum_{i=1}^2 \exp\{-(\mu - \mu_i)\} - \exp\{-(2\mu - \mu_1 - \mu_2)\} - \\ &\quad \frac{1}{2} \exp\{-(2\mu - \mu_1 - \mu_3)\} - \frac{1}{2} \exp\{-(2\mu - \mu_1 - \mu_3)\} \\ &\quad + \frac{2}{3} \exp\{-(3\mu - \mu_1 - \mu_2 - \mu_3)\} \\ &= R_2(\text{say}) \end{aligned}$$

Thus the reliability function which is defined in (2) can be expressed as

$$R_3 = \begin{cases} 1 - R_1 & \text{if } \mu \leq \mu_3 \\ 1 - R_2 & \text{if } \mu > \mu_3 \end{cases}$$

or equivalently,

$$R_3 = (1 - R_1)I(\mu \leq \mu_3) + (1 - R_2)I(\mu > \mu_3) \quad (3)$$

where I denotes the indicator function. In the next section, we will concentrate on the different estimators of the reliability function as defined in (3).

3 Estimation

Let $Y_{i1}, Y_{i2}, \dots, Y_{in_i}$ be a random sample of size n_i from the population of Y_i . Define

$$Y_{i(1)} = \text{Min}(Y_{i1}, Y_{i2}, \dots, Y_{in_i}); \quad i = 1, 2, 3.$$

Throughout this section, these random samples will be used to estimate R_3 .

3.1 Maximum Likelihood Estimator of R_3

It is well known that the MLEs of μ_1 , μ_2 and μ_3 , are $\hat{\mu}_{1M}$, $\hat{\mu}_{2M}$ and $\hat{\mu}_{3M}$, respectively, where $\hat{\mu}_{iM} = Y_{i(1)}$, $i = 1, 2, 3$. Then from (3), MLE of R_3 will be given by

$$\hat{R}_{3M} = (1 - \hat{R}_{1M})I(\hat{\mu}_M \leq \hat{\mu}_{3M}) + (1 - \hat{R}_{2M})I(\hat{\mu}_M > \hat{\mu}_{3M}),$$

where $\widehat{\mu}_M = \max(\widehat{\mu}_{1M}, \widehat{\mu}_{2M})$,

$$\widehat{R}_{1M} = \frac{1}{2} \sum_{i=1}^2 \exp\{-(\widehat{\mu}_{3M} - \widehat{\mu}_{iM})\} - \frac{1}{3} \exp\{-(2\widehat{\mu}_{3M} - \widehat{\mu}_{2M} - \widehat{\mu}_{1M})\},$$

and

$$\begin{aligned} \widehat{R}_{2M} &= \sum_{i=1}^2 \exp\{-(\widehat{\mu}_M - \widehat{\mu}_{iM})\} - \exp\{-(2\widehat{\mu}_M - \widehat{\mu}_{2M} - \widehat{\mu}_{1M})\} \\ &\quad - \frac{1}{2} \exp\{-(2\widehat{\mu}_M - \widehat{\mu}_{1M} - \widehat{\mu}_{3M})\} - \frac{1}{2} \exp\{-(2\widehat{\mu}_M - \widehat{\mu}_{2M} - \widehat{\mu}_{3M})\} \\ &\quad + \frac{2}{3} \exp\{-(3\widehat{\mu}_M - \widehat{\mu}_{1M} - \widehat{\mu}_{2M} - \widehat{\mu}_{3M})\}. \end{aligned}$$

3.2 Uniformly Minimum Variance Unbiased Estimator of R_3

Based on the random sample $Y_{i1}, Y_{i2}, \dots, Y_{in_i}$, let us define

$$U_i = Y_{i(1)}, \quad i = 1, 2, 3.$$

In addition, we set $U_1 = u_1, U_2 = u_2, U_3 = u_3$. We now state and prove the following theorem which will be helpful to derive the UMVUE of R_3 .

Theorem 1. Let $T_1 = \max(Y_{11}, Y_{21})$. Then

(i) for, $u_1 < u_2$, the distribution function of T_1 given (u_1, u_2) will be

$$\begin{aligned} &F(t_1 | (u_1, u_2)) \\ &= \begin{cases} \frac{1}{n_2} [1 - \frac{n_1-1}{n_1} \exp\{-(u_2 - u_1)\}] & \text{if } t_1 = u_2 \\ (1 - \frac{(n_1-1)}{n_1} \exp\{-(t_1 - u_1)\})(1 - \frac{(n_2-1)}{n_2} \exp\{-(t_1 - u_2)\}) & \text{if } t_1 > u_2 \end{cases} \end{aligned}$$

(ii) for, $u_1 > u_2$, the distribution function of T_1 given (u_1, u_2) will be

$$\begin{aligned} &F(t_1 | (u_1, u_2)) \\ &= \begin{cases} \frac{1}{n_1} [1 - \frac{n_2-1}{n_2} \exp\{-(u_1 - u_2)\}] & \text{if } t_1 = u_1 \\ (1 - \frac{(n_1-1)}{n_1} \exp\{-(t_1 - u_1)\})(1 - \frac{(n_2-1)}{n_2} \exp\{-(t_1 - u_2)\}) & \text{if } t_1 > u_1. \end{cases} \end{aligned}$$

Proof. From definition, the distribution function of T_1 given (u_1, u_2) is given by

$$\begin{aligned} F(t_1 | (u_1, u_2)) &= \Pr(T_1 \leq t_1 | (u_1, u_2)) \\ &= \Pr(\max(Y_{11}, Y_{21}) \leq t_1 | (u_1, u_2)) \\ &= \Pr(Y_{11} \leq t_1, Y_{21} \leq t_1 | (u_1, u_2)) \\ &= \Pr(Y_{11} \leq t_1 | u_1) \Pr(Y_{21} \leq t_1 | u_2). \end{aligned}$$

It is to be noted that, for $i = 1, 2, 3$,

$$P(Y_{i1} \leq y_{i1} | u_i) = \begin{cases} 0 & \text{if } y_{i1} < u_i \\ \frac{1}{n_i} & \text{if } y_{i1} = u_i \\ 1 - \frac{n_i-1}{n_i} \exp\{-(y_{i1} - u_i)\} & \text{if } y_{i1} > u_i \end{cases} \quad (4)$$

Now, for $u_1 < u_2$, we have

$$\begin{aligned} F(t_1 | (u_1, u_2)) &= \Pr(Y_{11} \leq t_1 | u_1) \Pr(Y_{21} \leq t_1 | u_2) \\ &= \begin{cases} \Pr(Y_{11} \leq u_2 | u_1) \Pr(Y_{21} \leq u_2 | u_2) & \text{if } t_1 = u_2 \\ \Pr(Y_{11} \leq t_1 | u_1) \Pr(Y_{21} \leq t_1 | u_2) & \text{if } t_1 > u_2 \end{cases}, \end{aligned}$$

and, for $u_1 > u_2$, we have

$$\begin{aligned} F(t_1 | (u_1, u_2)) &= \Pr(Y_{11} \leq t_1 | u_1) \Pr(Y_{21} \leq t_1 | u_2) \\ &= \begin{cases} \Pr(Y_{11} \leq u_1 | u_1) \Pr(Y_{21} \leq u_1 | u_2) & \text{if } t_1 = u_1 \\ \Pr(Y_{11} \leq t_1 | u_1) \Pr(Y_{21} \leq t_1 | u_2) & \text{if } t_1 > u_1 \end{cases} \end{aligned}$$

From (4), the proof of Theorem 1 follows immediately. \square

Let us now define the following:

$$\begin{aligned} I_1 &= \frac{(n_1 - 1)(n_3 + 1)}{2n_1n_3} \exp\{-(u_3 - u_1)\} + \frac{(n_2 - 1)(n_3 + 1)}{2n_2n_3} \exp\{-(u_3 - u_2)\} - \\ &\quad \frac{(n_1 - 1)(n_2 - 1)(n_3 + 2)}{3n_1n_2n_3} \exp\{-(2u_3 - u_2 - u_1)\} \\ I_2 &= 1 - \frac{(n_3 - 1)(n_2 + 1)}{2n_2n_3} \exp\{-(u_2 - u_3)\} + \frac{(n_3 - 1)(n_1 - 1)(n_2 + 2)}{6n_1n_2n_3} \times \\ &\quad \exp\{-(2u_2 - u_1 - u_3)\}, \\ I_3 &= 1 - \frac{(n_3 - 1)(n_1 + 1)}{2n_1n_3} \exp\{-(u_1 - u_3)\} + \frac{(n_3 - 1)(n_2 - 1)(n_1 + 2)}{6n_1n_2n_3} \times \\ &\quad \exp\{-(2u_1 - u_2 - u_3)\}, \end{aligned}$$

and $U_m = \max(U_1, U_2, U_3)$. The following theorem will then provide the UMVUE of R_3 .

Theorem 2. Based on the random samples $Y_{i1}, Y_{i2}, \dots, Y_{in_i}$, $i = 1, 2, 3$ and setting $U_i = u_i$, $i = 1, 2, 3$, $U_m = u_m$, the UMVUE of R_3 will be given by

$$\widehat{R}_{3U} = 1 - \widehat{R}_{3U},$$

where

$$\widehat{R}_{3u} = \begin{cases} I_1 & \text{for } u_m = u_3 \\ I_2 & \text{for } u_m = u_2 \\ I_3 & \text{for } u_m = u_1. \end{cases}$$

Proof. Let us define

$$I = \begin{cases} 1 & \text{if } Y_{31} > \max(Y_{11}, Y_{21}) \\ 0 & \text{otherwise} \end{cases}$$

Then, due to the fact that (U_1, U_2, U_3) is a complete sufficient statistic for (μ_1, μ_2, μ_3) and I is an unbiased estimator of R_3 , $E[I | (u_1, u_2, u_3)]$ will be the UMVUE of R_3 .

Therefore,

$$\begin{aligned} \widehat{R}_{3U} &= E[I | (u_1, u_2, u_3)] = \Pr(Y_{31} > T_1 | (u_1, u_2, u_3)) \\ &= 1 - \Pr(Y_{31} \leq T_1 | (u_1, u_2, u_3)) \\ &= 1 - \widehat{R}_{3U}. \end{aligned}$$

Thus it suffices to find an explicit expression for \widehat{R}_{3U} . We employ Theorem 1 to get an expression of \widehat{R}_{3U} . Now, for the case $u_m = u_3$, we have

$$\begin{aligned} \widehat{R}_{3U} &= \Pr(Y_{31} \leq T_1 | (u_1, u_2, u_3)) = \int \Pr(Y_{31} \leq t_1 | u_3, T_1 = t_1) f(t_1 | (u_1, u_2)) dt_1 \\ &= \int_{u_3}^{\infty} \Pr(Y_{31} \leq t_1 | u_3) f(t_1 | (u_1, u_2)) dt_1 \\ &= \frac{(n_1 - 1)(n_3 + 1)}{2n_1n_3} \exp\{-(u_3 - u_1)\} + \frac{(n_2 - 1)(n_3 + 1)}{2n_2n_3} \exp\{-(u_3 - u_2)\} - \\ &\quad \frac{(n_1 - 1)(n_2 - 1)(n_3 + 2)}{3n_1n_2n_3} \exp\{-(2u_3 - u_2 - u_1)\} = I_1 \end{aligned}$$

For the case $u_m = u_2$, we have

$$\begin{aligned}\widehat{R}_{3U} &= \Pr(Y_{31} \leq T_1 | (u_1, u_2, u_3)) = \int \Pr(Y_{31} \leq t_1 | u_3, T_1 = t_1) f(t_1 | (u_1, u_2)) dt_1 \\ &= \Pr(Y_{31} \leq t_1 | u_3) \Pr(T_1 = t_1 | u_1, t_1 = u_2) + \\ &\quad \int_{u_2}^{\infty} \Pr(Y_{31} \leq t_1 | u_3) f(t_1 | (u_1, u_2)) dt_1 \\ &= 1 - \frac{(n_3 - 1)(n_2 + 1)}{2n_2n_3} \exp\{-(u_2 - u_3)\} + \\ &\quad \frac{(n_3 - 1)(n_1 - 1)(n_2 + 2)}{6n_1n_2n_3} \exp\{-(2u_2 - u_1 - u_3)\} = I_2.\end{aligned}$$

Finally, for the case $u_m = u_1$, we have

$$\begin{aligned}\widehat{R}_{3U} &= \Pr(Y_{31} \leq T_1 | (u_1, u_2, u_3)) = \int \Pr(Y_{31} \leq t_1 | u_3, T_1 = t_1) f(t_1 | (u_1, u_2)) dt_1 \\ &= \Pr(Y_{31} \leq t_1 | u_3) \Pr(T_1 = t_1 | t_1 = u_1, u_2) + \\ &\quad \int_{u_1}^{\infty} \Pr(Y_{31} \leq t_1 | u_3) f(t_1 | (u_1, u_2)) dt_1 \\ &= 1 - \frac{(n_3 - 1)(n_1 + 1)}{2n_1n_3} \exp\{-(u_1 - u_3)\} + \\ &\quad \frac{(n_3 - 1)(n_2 - 1)(n_1 + 2)}{6n_1n_2n_3} \exp\{-(2u_1 - u_2 - u_3)\} = I_3.\end{aligned}$$

Hence proved. □

3.3 Bayes Estimator

In this section we calculate Bayes estimator and generalized Bayes estimator of R_3 by plug-in method. For Bayes estimator, we consider the following independent priors

$$f(\mu_i) = \exp\{-(\mu_i - \xi_i)\} ; \mu_i > \xi_i, -\infty < \xi_i < \infty, i = 1, 2, 3. \quad (5)$$

Theorem 3. *Based on the independent priors stated in (5), the Bayes estimator of R_3 will be given by*

$$\widehat{R}_{3BE} = (1 - \widehat{R}_{1BE})I(\widehat{\mu}_{BE} \leq \widehat{\mu}_{3BE}) + (1 - \widehat{R}_{2BE})I(\widehat{\mu}_{BE} > \widehat{\mu}_{3BE}),$$

where $\widehat{\mu}_{iBE}, i = 1, 2, 3$ and $\widehat{\mu}_{BE}$ are the Bayes estimators of $\mu_i, i = 1, 2, 3$ and μ respectively.

Proof. Based on the independent priors stated in (5), we have

$$f(u_i | \mu_i) = n_i \exp\{-n_i(u_i - \mu_i)\} ; u_i > \mu_i, -\infty < \mu_i < \infty, i = 1, 2, 3.$$

After some algebraic simplification, we get the posterior distribution of μ_i given u_i

$$f(\mu_i | u_i) = \frac{(n_i - 1) \exp\{(n_i - 1)\mu_i\}}{[\exp\{(n_i - 1)u_i\} - \exp\{(n_i - 1)\xi_i\}]} ; \xi_i < \mu_i < u_i, i = 1, 2, 3.$$

Under squared error loss, the Bayes estimator of μ_i ($i = 1, 2, 3$) will be

$$\widehat{\mu}_{iBE} = \frac{u_i \exp\{(n_i - 1)u_i\} - \xi_i \exp\{(n_i - 1)\xi_i\}}{[\exp\{(n_i - 1)u_i\} - \exp\{(n_i - 1)\xi_i\}]} - \frac{1}{(n_i - 1)} \quad (6)$$

Plugging $\hat{\mu}_{iBE}$ ($i = 1, 2, 3$), as describe in (6), in the expressions of R_1 , R_2 and μ respectively, we get the following estimators

$$\begin{aligned} \hat{R}_{1BE} &= \frac{1}{2} \sum_{i=1}^2 \exp\{-(\hat{\mu}_{3BE} - \hat{\mu}_{iBE})\} - \frac{1}{3} \exp\{-(2\hat{\mu}_{3BE} - \hat{\mu}_{2BE} - \hat{\mu}_{1BE})\} \\ \hat{R}_{2BE} &= \sum_{i=1}^2 \exp\{-(\hat{\mu}_{BE} - \hat{\mu}_{iBE})\} - \exp\{-(2\hat{\mu}_{BE} - \hat{\mu}_{2BE} - \hat{\mu}_{1BE})\} \\ &\quad - \frac{1}{2} \exp\{-(2\hat{\mu}_{BE} - \hat{\mu}_{1BE} - \hat{\mu}_{3BE})\} - \frac{1}{2} \exp\{-(2\hat{\mu}_{BE} - \hat{\mu}_{2BE} - \hat{\mu}_{3BE})\} \\ &\quad + \frac{2}{3} \exp\{-(3\hat{\mu}_{BE} - \hat{\mu}_{1BE} - \hat{\mu}_{2BE} - \hat{\mu}_{3BE})\} \end{aligned}$$

and $\hat{\mu}_{BE} = \max(\hat{\mu}_{1BE}, \hat{\mu}_{2BE})$.

This completes the proof of Theorem 3. □

For deriving the generalized Bayes estimator of R_3 , we consider the following improper prior for each $\mu_i, i = 1, 2, 3$.

$$f(\mu_i) = 1 ; -\infty < \mu_i < \infty, i = 1, 2, 3 \tag{7}$$

Now, we have the following theorem which provides generalized Bayes estimator of R_3 .

Theorem 4. *Based on the independent priors stated in (7), the generalized Bayes estimator of R_3 will be given by*

$$\hat{R}_{3GBE} = (1 - \hat{R}_{1GBE})I(\hat{\mu}_{GBE} \leq \hat{\mu}_{3GBE}) + (1 - \hat{R}_{2GBE})I(\hat{\mu}_{GBE} > \hat{\mu}_{3GBE}),$$

where $\hat{\mu}_{iGBE}, i = 1, 2, 3$ and $\hat{\mu}_{GBE}$ are the generalized Bayes estimators of $\mu_i, i = 1, 2, 3$ and μ respectively.

Proof. Based on the prior given in (7), the proof of Theorem 4 will follow along the line of the proof of Theorem 3. Here it is to be noted that the the generalized Bayes estimator of μ_i ($i = 1, 2, 3$) will be

$$\hat{\mu}_{iGBE} = u_i - \frac{1}{n_i}. \tag{8}$$

□

Based on the above findings, we have the following notable remark.

Remark 1. *For $i = 1, 2, 3$, if we take the limit $n_i \rightarrow \infty$, then generalized Bayes estimator of R_3 will be limit of the Bayes estimator.*

4 Simulation Study

In this section we compare the estimators of the reliability function (R_3) through the risk values with respect to the following loss functions

$$\text{Log Square Error Loss : } L_1(\hat{R}_3, R_3) = \left(\frac{\log \hat{R}_3 - \log R_3}{\log R_3}\right)^2,$$

$$\text{Linex Loss : } L_2(\hat{R}_3, R_3) = b(e^{a(\hat{R}_3 - R_3)} - a(\hat{R}_3 - R_3) - 1) ; a \in \mathcal{R}, b > 0, \text{ and}$$

$$\text{Square Error Loss : } L_3(\hat{R}_3, R_3) = (\hat{R}_3 - R_3)^2.$$

It is to be noted that it is difficult to get any closed form analytic solutions of the risk values for comparison of the different estimators. So that, for comparing the estimators we have used Monte Carlo simulation method with $N = 5000$ replications. To evaluate Bayes estimator (BE) we have taken $\xi_1 = \xi_2 = \xi_3 = 1$. In the linex loss function we set $a = 1, b = 2$. On the basis of simulation study, we can

draw the following conclusion. Table 1-2 show the risk value of the estimators under log square error loss function. The following points are clear from these Tables. If we take 0.5 as the base value for μ_1 and increase it successively by 0.5 until it reaches 2, the risk values decrease for different choices of μ_2 and μ_3 (0.5, 1, 1.5, 2). It is observed that when $n_1 = n_2 = n_3 = 5$ the maximum likelihood estimator, Bayes estimator and generalized Bayes estimator are equivalent ($MLE \equiv BE \equiv GBE$) and these estimators are better than the uniformly minimum variance unbiased estimator. For unequal sample sizes we can arrange the performance of the estimators as $BE \equiv GBE \geq UMVUE \geq MLE$. It is obvious that the risk values will decrease with the increase in the sample size. It is observed that the performance of the estimators are quite similar when comparison is made on the basis of the risk values under linex loss function and it is clear from Table 3-4.

Table 1. The risk values of different estimators under **log square error loss function** for the choices $n_1 = 5$, $n_2 = 5$, $n_3 = 5$, $\mu_1, \mu_2, \mu_3 = 0.5, 1, 1.5, 2$.

μ_1	μ_2	μ_3	MLE	GBE	BE	UMVUE
0.5000	0.5000	0.5000	0.0504	0.0504	0.0504	0.0569
	1.0000	1.0000	0.0646	0.0646	0.0646	0.0749
	1.5000	1.5000	0.0762	0.0762	0.0762	0.0913
	2.0000	2.0000	0.0956	0.0956	0.0956	0.1152
1.0000	0.5000	0.5000	0.0340	0.0340	0.0340	0.0363
	1.0000	1.0000	0.0507	0.0507	0.0507	0.0573
	1.5000	1.5000	0.0621	0.0621	0.0621	0.0724
	2.0000	2.0000	0.0822	0.0822	0.0822	0.0977
1.5000	0.5000	0.5000	0.0220	0.0220	0.0220	0.0228
	1.0000	1.0000	0.0340	0.0340	0.0340	0.0363
	1.5000	1.5000	0.0504	0.0504	0.0504	0.0569
	2.0000	2.0000	0.0646	0.0646	0.0646	0.0749
2.0000	0.5000	0.5000	0.0148	0.0148	0.0148	0.0152
	1.0000	1.0000	0.0220	0.0220	0.0220	0.0228
	1.5000	1.5000	0.0340	0.0340	0.0340	0.0363
	2.0000	2.0000	0.0507	0.0507	0.0507	0.0573

Table 2. The risk values of different estimators under **log square error loss function** for the choices $n_1 = 5$, $n_2 = 10$, $n_3 = 10$, $\mu_1, \mu_2, \mu_3 = 0.5, 1, 1.5, 2$.

μ_1	μ_2	μ_3	MLE	GBE	BE	UMVUE
0.5000	0.5000	0.5000	0.0276	0.0225	0.0225	0.0239
	1.0000	1.0000	0.0241	0.0214	0.0214	0.0226
	1.5000	1.5000	0.0247	0.0236	0.0236	0.0254
	2.0000	2.0000	0.0268	0.0262	0.0262	0.0286
1.0000	0.5000	0.5000	0.0215	0.0178	0.0178	0.0183
	1.0000	1.0000	0.0260	0.0211	0.0211	0.0225
	1.5000	1.5000	0.0255	0.0224	0.0224	0.0237
	2.0000	2.0000	0.0250	0.0237	0.0237	0.0255
1.5000	0.5000	0.5000	0.0148	0.0124	0.0124	0.0126
	1.0000	1.0000	0.0215	0.0178	0.0178	0.0183
	1.5000	1.5000	0.0260	0.0211	0.0211	0.0225
	2.0000	2.0000	0.0255	0.0224	0.0224	0.0237
2.0000	0.5000	0.5000	0.0103	0.0086	0.0086	0.0087
	1.0000	1.0000	0.0148	0.0124	0.0124	0.0126
	1.5000	1.5000	0.0215	0.0178	0.0178	0.0183
	2.0000	2.0000	0.0260	0.0211	0.0211	0.0225

Table 3. The risk values of different estimators under **linex loss function** for the choices $n_1 = 5, n_2 = 5, n_3 = 5, \mu_1, \mu_2, \mu_3 = 0.5, 1, 1.5, 2$.

μ_1	μ_2	μ_3	MLE	GBE	BE	UMVUE
0.5000	0.5000	0.5000	0.0071	0.0071	0.0071	0.0081
	1.0000	1.0000	0.0083	0.0083	0.0083	0.0097
	1.5000	1.5000	0.0089	0.0089	0.0089	0.0107
	2.0000	2.0000	0.0103	0.0103	0.0103	0.0124
1.0000	0.5000	0.5000	0.0050	0.0050	0.0050	0.0052
	1.0000	1.0000	0.0071	0.0071	0.0071	0.0081
	1.5000	1.5000	0.0079	0.0079	0.0079	0.0092
	2.0000	2.0000	0.0094	0.0094	0.0094	0.0112
1.5000	0.5000	0.5000	0.0026	0.0026	0.0026	0.0024
	1.0000	1.0000	0.0050	0.0050	0.0050	0.0052
	1.5000	1.5000	0.0071	0.0071	0.0071	0.0081
	2.0000	2.0000	0.0083	0.0083	0.0083	0.0097
2.0000	0.5000	0.5000	0.0011	0.0011	0.0011	0.0010
	1.0000	1.0000	0.0026	0.0026	0.0026	0.0024
	1.5000	1.5000	0.0050	0.0050	0.0050	0.0052
	2.0000	2.0000	0.0071	0.0071	0.0071	0.0081

Table 4. The risk values of different estimators under **linex loss function** for the choices $n_1 = 5, n_2 = 10, n_3 = 10, \mu_1, \mu_2, \mu_3 = 0.5, 1, 1.5, 2$

μ_1	μ_2	μ_3	MLE	GBE	BE	UMVUE
0.5000	0.5000	0.5000	0.0029	0.0028	0.0028	0.0030
	1.0000	1.0000	0.0029	0.0028	0.0028	0.0030
	1.5000	1.5000	0.0030	0.0030	0.0030	0.0033
	2.0000	2.0000	0.0032	0.0032	0.0032	0.0035
1.0000	0.5000	0.5000	0.0019	0.0020	0.0020	0.0019
	1.0000	1.0000	0.0028	0.0027	0.0027	0.0029
	1.5000	1.5000	0.0030	0.0029	0.0029	0.0031
	2.0000	2.0000	0.0030	0.0030	0.0030	0.0033
1.5000	0.5000	0.5000	0.0009	0.0010	0.0010	0.0009
	1.0000	1.0000	0.0019	0.0020	0.0020	0.0019
	1.5000	1.5000	0.0028	0.0027	0.0027	0.0029
	2.0000	2.0000	0.0030	0.0029	0.0029	0.0031
2.0000	0.5000	0.5000	0.0004	0.0004	0.0004	0.0004
	1.0000	1.0000	0.0009	0.0010	0.0010	0.0009
	1.5000	1.5000	0.0019	0.0020	0.0020	0.0019
	2.0000	2.0000	0.0028	0.0027	0.0027	0.0029

In the Table 5-6 we have compared the risk value of the estimators under square error loss function. It is noticed that when $n_1 = n_2 = n_3 = 5$ then $MLE \equiv GBE$ and their risk values are very close for different sample sizes. According to this loss function BE is better than $UMVUE$ and $UMVUE$ is better than MLE and GBE . It is also observed that the Bayes estimator gives the best performance among all estimators.

Table 5. The risk values of different estimators under **square error loss function** for the choices $n_1 = 5$, $n_2 = 5$, $n_3 = 5$, $\mu_1, \mu_2, \mu_3 = 0.5, 1, 1.5, 2$.

μ_1	μ_1	μ_3	MLE	GBE	BE	UMVUE
0.5000	0.5000	0.5000	0.0069	0.0069	0.0015	0.0078
	1.0000	1.0000	0.0082	0.0082	0.0048	0.0095
	1.5000	1.5000	0.0089	0.0089	0.0073	0.0106
	2.0000	2.0000	0.0102	0.0102	0.0094	0.0123
1.0000	0.5000	0.5000	0.0048	0.0048	0.0042	0.0050
	1.0000	1.0000	0.0069	0.0069	0.0038	0.0078
	1.5000	1.5000	0.0078	0.0078	0.0062	0.0091
	2.0000	2.0000	0.0093	0.0093	0.0087	0.0111
1.5000	0.5000	0.5000	0.0024	0.0024	0.0054	0.0024
	1.0000	1.0000	0.0048	0.0048	0.0039	0.0050
	1.5000	1.5000	0.0069	0.0069	0.0058	0.0078
	2.0000	2.0000	0.0082	0.0082	0.0077	0.0095
2.0000	0.5000	0.5000	0.0010	0.0010	0.0033	0.0010
	1.0000	1.0000	0.0024	0.0024	0.0027	0.0024
	1.5000	1.5000	0.0048	0.0048	0.0044	0.0050
	2.0000	2.0000	0.0069	0.0069	0.0066	0.0078

Table 6. The risk values of different estimators under **square error loss function** for the choices $n_1 = 5$, $n_2 = 5$, $n_3 = 5$, $\mu_1, \mu_2, \mu_3 = 0.5, 1, 1.5, 2$.

μ_1	μ_2	μ_3	MLE	GBE	BE	UMVUE
0.5000	0.5000	0.5000	0.0029	0.0028	0.0003	0.0030
	1.0000	1.0000	0.0029	0.0028	0.0027	0.0030
	1.5000	1.5000	0.0030	0.0030	0.0032	0.0033
	2.0000	2.0000	0.0032	0.0032	0.0033	0.0035
1.0000	0.5000	0.5000	0.0019	0.0019	0.0035	0.0019
	1.0000	1.0000	0.0028	0.0026	0.0015	0.0028
	1.5000	1.5000	0.0030	0.0029	0.0026	0.0031
	2.0000	2.0000	0.0030	0.0030	0.0029	0.0032
1.5000	0.5000	0.5000	0.0009	0.0010	0.0051	0.0009
	1.0000	1.0000	0.0019	0.0019	0.0017	0.0019
	1.5000	1.5000	0.0028	0.0026	0.0024	0.0028
	2.0000	2.0000	0.0030	0.0029	0.0028	0.0031
2.0000	0.5000	0.5000	0.0004	0.0004	0.0033	0.0004
	1.0000	1.0000	0.0009	0.0010	0.0012	0.0009
	1.5000	1.5000	0.0019	0.0019	0.0020	0.0019
	2.0000	2.0000	0.0028	0.0026	0.0026	0.0028

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