BIOT-SAVART LAW APPLICATION IN WIRELESS POWER TRANSFER – DEPENDENCE OF MAGNETIC FIELD TO ANGLE POSITION

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Abstract*: The magnetic field of a closed loop of conductive wire can be computed due to Biot-Savart law, which analyses the value of the field at an exterior point from the transversal axis. If the measure point is out of the axis then the magnetic field has completely different values. A general stated form of this law can measure the value in any point, in relation to Euclidian distance from the loop.*

1. INTRODUCTION

In electromagnetics, main fact that limits this sector is the magnetic field, a basic component that “runs out” in free space. Since the discovery of the electricity, scientists made numerous researches in magnetic field domain and most of the results are in use today. One of the most important laws that refer to magnetic field induction computation is the Biot-Savart law.

Biot-Savart law was formulated by Jean-Baptiste Biot and Félix Savart around 1820 and is the fundamental law for computing magnetic field. It corresponds to Coulomb's law for calculating the electric field. The Biot-Savart law is an equation describing the magnetic field generated by an electric current as a vector that has magnitude, direction and length [1].

If all the properties of the magnetic field are known, then it is simple to calculate possible energy transfer between devices.

The aim of this work consists in analyzing the computation of magnetic field induction produced by a closed loop of a conducting wire. The general formula is referring to an ideal case when the magnetic induction is calculated in the axis of the loop. In real cases, there are always angular deviations, which determine directly modifications of the magnetic field induction values.

2. MATHEMATICAL ANALYSIS

The magnetic induction is a vector, so its calculation involves to take in consideration the vector properties and Cartesian distribution of its components.

The general expression of the Biot-Savart law is referring to the magnetic field of a conductive wire and it states that variable currents give rise to magnetic field. Take in account a single conductor, passed by the current *I*, the magnetic induction at any point *P* can be calculated summing the contributions $d\vec{B}$ of all the infinitesimal $d\vec{S}$ elements [2]:

$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I \cdot d\vec{S} \times \hat{r}}{r^2} \tag{1}$$

where:

- $d\vec{S}$ is a vector with the magnitude equal to the length of the analyzed segment and with same direction as the current I. In this case, I become a finite elementary current source;
- r is the Euclidian distance from the source to the measuring point;
- $\hat{r}$ is the versor of the corresponding vector [3].

When the conducting wire is coiled around an axis (wire loop) the magnetic induction computation becomes complicated. Major modifications are made on vector $d\vec{S}$. In figure is illustrated the connections between the vectors involved in magnetic induction calculation for a wire loop.

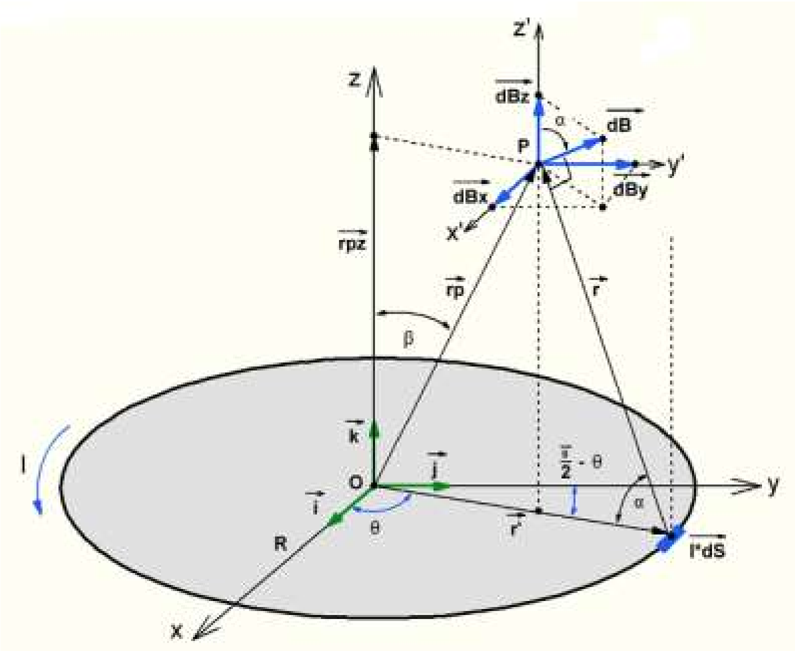

Fig. 1 Explained for magnetic induction calculation for a single wire loop.

$I \cdot d\vec{S}$ is considered the current carrying element and because the loop is a circle, the location is given by the $\vec{r}'$. In Cartesian coordinates (x, y, z), the vector $\vec{r}'$ is the sum of his axis components [3]:

$$\vec{r'} = \vec{r'_x} + \vec{r'_y} = R \cdot \cos\theta \cdot \hat{\imath} + R \cdot \sin\theta \cdot \hat{\jmath} \tag{2}$$

where:

- R is the radius of the circle wire loop;
- θ is the angle with x axis;
- $\hat{\imath}, \hat{\jmath}, \hat{k}$ are the versors of the x axis, y axis, respectively of z axis of the Cartesian coordinates system.

To determine the value of current element, the source element has been derivate in relation with angle of rotation [4]:

$$\begin{aligned} I \cdot d\vec{S} &= I \cdot \left(\frac{d}{d\theta} \vec{r'} \right) \cdot d\theta = I \cdot d\theta \cdot R \cdot \frac{d}{d\theta} \left(\cos\theta \cdot \hat{\imath} + \sin\theta \cdot \hat{\jmath} \right) = \\ &= I \cdot d\theta \cdot R \left(-\sin\theta \cdot \hat{\imath} + \cos\theta \cdot \hat{\jmath} \right) \end{aligned} \tag{3}$$

To measure the value of the magnetic field in a specific point, the point must be located. In *xOyOz* coordinates the point has the coordinates notation *P(x, y, z)* and is characterized by the vector position [3]:

$$\vec{r_P} = x \cdot \hat{\imath} + y \cdot \hat{\jmath} + z \cdot \hat{k} \tag{4}$$

but in the same time, this vector is the sum of his contribution on the axis:

$$\vec{r_P} = \vec{r_{Px}} + \vec{r_{Py}} + \vec{r_{Pz}} \tag{5}$$

So, the position vector can be described from his relation with the contributions from any axis, for example z axis [3]:

$$\vec{r_P} = \frac{\vec{r_{Pz}}}{\cos\beta} = \frac{z}{\cos\beta} \cdot \hat{k} \tag{6}$$

Heaving all components from the vector triangle, the Euclidian distance from the source to the measuring point can be calculated [4]:

$$\vec{r} = \vec{r_P} - \vec{r'} \tag{7}$$

$$\vec{r} = \frac{z}{\cos\beta} \cdot \hat{k} - R \cdot \cos\theta \cdot \hat{i} - R \cdot \sin\theta \cdot \hat{j} \tag{8}$$

and the magnitude becomes:

$$r = |\vec{r}| = \sqrt{\left(\frac{z}{\cos\beta}\right)^2 + (-R \cdot \cos\theta)^2 + (-R \cdot \sin\theta)^2} = \sqrt{R^2 + \frac{z^2}{\cos^2\beta}} \tag{9}$$

If the $\vec{r}$ is the vector position, then it must have a magnitude and a versor:

$$\vec{r} = r \cdot \hat{r} \tag{10}$$

In this case, the versor must be known:

$$\hat{r} = \frac{\vec{r}}{r} \tag{11}$$

The Biot-Savart law states that the field direction is given by the vector product between the carrying element and the versor of the Euclidian distance [5]:

$$d\vec{S} \times \hat{r} = d\vec{S} \times \frac{\vec{r}}{r} \tag{12}$$

$$\begin{aligned} d\vec{S} \times \hat{r} = &\left[(-R \cdot \sin\theta \cdot d\theta) \cdot \hat{i} + (R \cdot \cos\theta \cdot d\theta) \cdot \hat{j}\right] \times \\ &\times \left[(-R \cdot \cos\theta \cdot d\theta) \cdot \hat{i} + (-R \cdot \sin\theta \cdot d\theta) \cdot \hat{j} + \frac{z}{\cos\beta} \cdot \hat{k}\right] \end{aligned} \tag{13}$$

After matrix product of vectors it becomes:

$$d\vec{S} \times \hat{r} = R \cdot \left(\cos\theta \cdot \frac{z}{\cos\beta} \cdot \hat{i} + \sin\theta \cdot \frac{z}{\cos\beta} \cdot \hat{j} + R \cdot \hat{k}\right) \cdot d\theta \tag{14}$$

Heaving all the components, their values must be replaced in the basic formula to find out the contribution of a single length element for the magnetic field [2]:

$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I \cdot d\vec{S} \times \hat{r}}{r^2} = \frac{\mu_0 \cdot I}{4\pi \cdot r^2} \cdot \frac{d\vec{S} \times \vec{r}}{r} = \\ = \frac{\mu_0 \cdot I \cdot R}{4\pi \cdot (R^2 + z^2)^{3/2}} \cdot \left(\cos\theta \cdot \frac{z}{\cos\beta} \cdot \hat{i} + \sin\theta \cdot \frac{z}{\cos\beta} \cdot \hat{j} + R \cdot \hat{k} \right) \cdot d\theta \tag{15}$$

3. RESULTS

From the expression of the field $d\vec{B}$, the contributions from the *xOyOz* axis represented by the three versors $\hat{i}$, $\hat{j}$, $\hat{k}$ can be observed.

To find the contribution of the current element on all the length of the circle, the field expression must be integrated over the length in relation to the angle of rotation.

$$\vec{B} = \frac{\mu_0 \cdot I \cdot R}{4\pi \cdot (R^2 + z^2)^{3/2}} \cdot \int_0^{2\pi} \left(\cos\theta \cdot \frac{z}{\cos\beta} \cdot \hat{i} + \sin\theta \cdot \frac{z}{\cos\beta} \cdot \hat{j} + R \cdot \hat{k} \right) \cdot d\theta \tag{16}$$

After expanding this relation on separate axis components, it can bee seen the reason why the field has values only on transversal axis.

$$B_x = \frac{\mu_0 \cdot I \cdot R}{4\pi \cdot (R^2 + z^2)^{3/2}} \cdot \int_0^{2\pi} \cos\theta \cdot \frac{z}{\cos\beta} \cdot d\theta = \frac{\mu_0 \cdot I \cdot R}{4\pi \cdot r^3} \cdot \frac{z}{\cos\beta} \cdot \sin\theta \bigg|_0^{2\pi} = 0 \tag{17}$$

$$B_y = \frac{\mu_0 \cdot I \cdot R}{4\pi \cdot (R^2 + z^2)^{3/2}} \cdot \int_0^{2\pi} \sin\theta \cdot \frac{z}{\cos\beta} \cdot d\theta = -\frac{\mu_0 \cdot I \cdot R}{4\pi \cdot r^3} \cdot \frac{z}{\cos\beta} \cdot \cos \bigg|_0^{2\pi} = 0 \tag{18}$$

$$B_z = \frac{\mu_0 \cdot I \cdot R}{4\pi \cdot (R^2 + z^2)^{3/2}} \cdot \int_0^{2\pi} R \cdot d\theta = \frac{\mu_0 \cdot I \cdot R^2}{4\pi \cdot r^3} \cdot \theta \bigg|_0^{2\pi} = \frac{\mu_0 \cdot I \cdot R^2}{2 \cdot \left(R^2 + \frac{z^2}{\cos^2\beta} \right)^{\frac{3}{2}}} \tag{19}$$

4. CONCLUSIONS

It is clear that the magnetic field is null on the *Ox* and *Oy* axis because of the dependence on the angle θ. The only component that has value, B_z, has also a very important property: it's value decreases exponential in relation to Euclidian distance at the point of measure.

If the point of measure is not on the central axis of the loop, the magnitude of the field is measured on the correspondent axis, *Oz*. In this case, the *cosβ* that appears increases the Euclidian distance, which causes a substantial drop of field.

Another important aspect is that, if the angle between $\vec{r}$ and $\vec{r'}$ reaches π/2 radians (the point of measure is perpendicular to the wire, not on the center of circle) the field value drops to 0:

$$d\vec{B} = \cos\alpha \cdot d\overrightarrow{B_z} \tag{20}$$

As closer the point of measure is to the center of the loop, the value of the field increases and gets the maximum potential at *z=0*.

All expressions presented help develop a better understanding of what happens with the magnetic field outside any encircled areas and the dependence on the angles of vector orientation.

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