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Analytic Solutions of the Schamel-KdV Equation by Using Different Methods: Application to a Dusty Space Plasma

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Shock wave,
Soliton

Abstract: The wave properties in a dusty space plasma consisting of positively and negatively charged dust as well as distributed nonisothermal electrons are investigated by using the exact traveling wave solutions of the Schamel-KdV equation. The analytic solutions are obtained by the different types (G'/G)-expansion methods and direct integration. The nonlinear dynamics of ion-acoustic waves for the various values of phase speed V_p, plasma parameters α, σ, and σ_d, and the source term μ are studied. We have observed different types of waves from the different analytic solutions obtained from the different methods. Consequently, we have found the discontinuity, shock or solitary waves. It is also concluded that these parameters play an important role in the presence of solitary waves inside the plasma. Depending on plasma parameters, the discontinuity wave turns into solitary wave solution for the certain values of the phase speed and plasma parameters. Additionally, exact solutions of the Schamel-KdV equation may also be used to understand the wave types and properties in the different plasma systems.

Farklı Metodlar ile Schamel-KdV denkleminin Analitik Çözümleri: Tozlu Uzay Plazmasına Uygulanması

Anahtar Kelimeler
Schamel-KdV denklemi,
Tozlu uzay plazması,
Şok dalgası,
Soliton

Özet: İçerisinde negatif ve pozitif yüklü tozların yanında dağılmış izotermal olmayan elektronlar barındıran tozlu uzay plazmasındaki dalganın özellikleri, Schamel-KdV denklemlerinin tam ilerleyen dalga çözümleri kullanılarak incelenmiştir. Analitik çözümler, (G'/G)-genişleme methodunun farklı tipleri ve direk integrasyon kullanılarak bulunmuştur. İon-akustik dalgasının lineer olmayan dinamiği, faz hızının V_p, plazma parametreleri α, σ, ve σ_d, ve kaynak terimi μ'nun farklı değerleri için çalışılmıştır. Bunun sonucunda, farklı methodlardan elde edilen farklı analitik çözümler ile farklı türden dalgalar gözlemledik ve süreksiz, şok veya soliton dalgası bulduk. Aynı zamanda, yukarıda verilen parametrelerin plazma içerisinde soliton tipi dalgaların oluşmasında önemli bir rol oynadığı sonucuna ulaşılmıştır. Bu parametrelere bağlı olarak süreksiz dalga plazma parametrelerinin ve faz hızının belli değerleri için soliton tipi bir dalgaya donüşür. Bunlara ek olarak, Schamel-KdV denkleminin tam analitik çözümleri, verilen bir plazmanın özelliklerinin ve dalga tiplerinin anlaşılması için farklı plazma sistemlerine de uygulanabilir.

1. Introduction

Ion-acoustic wave consisting of nonlinear phenomena and appearing in the different plasma systems [1] and fluid mechanics [2] is one of the fundamental problems. A system which consists of the negatively and positively charged dusts and nonisothermally distributed electrons has a highly nonlinear behavior and it is represented with the Schamel-KdV equation which can be written in the following form [3–6]

$$\Phi_t + a\Phi^{\frac{1}{2}}\Phi_x + b\Phi\Phi_x + p\Phi_{xxx} = 0, \qquad (1)$$

where a, b, and p are arbitrary coefficients. Note that we reach to the Schamel equation when $b = 0$ [7, 8] and the KdV equation when $a = 0$ [9, 10]. Due to the various applications of Eqs.1 in the literature, many exact solutions of the Schamel-KdV equation have been obtained [3, 7–9, 11–17].

In this paper, we use three different types of (G'/G)-expansion methods and direct integration to obtain the exact solutions of the Schamel-KdV equation. The first method is so-called the original (G'/G)-expansion method described by [18]. Some applications of this

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method can be seen in [18–20]. Ref.[21] used different form of the (G'/G)-expansion method. Some application of this method to the different nonlinear differential equations can be found in [19, 21]. The third type of the expansion method is called the $(G'/G, 1/G)$-expansion method which considers the generalization of the original (G'/G)-expansion method. As a pioneer work, two-variable $(G'/G, 1/G)$-expansion method was explained and applied to Zakharov equation in [22]. There are several studies related to this generalized method to obtain the traveling wave solutions of the some nonlinear differential equations [10, 20, 23–26]. Hence it would be interesting to obtain the different forms of the traveling wave solutions of the Schamel-KdV equation. These exact solutions may be used to explain the properties of the ion-acoustic waves arised in laboratory and astrophysical plasmas.

Higher order nonlinear solution of four-component dusty plasma with nonisothermal electron had been studied in [27]. The authors of [27] used the reductive perturbation method to derive the Schamel-KdV equation and investigated the effects of plasma parameters, ratio of ion to electron temperatures, mass and charge ratio, and the ratio of dust to ion temperatures, on solitary wave. They had found the compressive and rarefactive solitons. The effect of electron trapping on the traveling wave solution was examined in [28] solving the Schamel-KdV equation with time-dependent coefficients and they found the traveling wave soliton solutions. The nonlinear behavior of ion-acoustic wave contains lots of detail about the solitary wave solutions in an unmagnetized plasma that is the plasma consisting nonrelativistic drifting ions and relativistic drifting electrons. The Schamel-KdV equation was also derived for this kind of plasma by reductive perturbation method [29]. Analytic solutions of the equation were obtained by using a linearized principle and the slow ion-acoustic monotonic double layers were suggested in [29].

The body of our paper is structured in the following order: Methods and their detailed structures are given in Section 2. In Section 3, we apply the different methods, introduced in Section 2, to the Schamel-KdV equation to find the exact solutions. Different forms of the exact solutions are derived from these methods. The analytic solutions obtained here are used in dusty space plasma in Section 4. The parameter dependencies of the discontinuity, shock waves, and solitons are explained in detail. Finally, we conclude our results in Section 5.

2. Methods

Let

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \ldots) = 0, \tag{2}$$

be a partial differential equation (PDE) where $u = u(x,t)$ is an unknown function. By using the transformation $u(x,t) = u(\eta)$ where $\eta = x - V_p t$ in Eq.2, we obtain an ordinary differential equation (ODE) in the following form:

$$P(u, -V_p u', u', V_p^2 u'', -V_p u'', u'', \ldots) = 0, \tag{3}$$

where $u' = \frac{du}{d\eta}$. In order to obtain the exact solutions of the Schamel-KdV equation given by Eq.1, we will use four different methods which are $(G/'G)$-expansion method [18], different form of the (G'/G)-expansion method [21], $(G'/G, 1/G)$-expansion method [22], and direct integration. The general forms of the solutions will be obtained from these methods directly without making any approximation and we will apply them to dusty space plasma to understand the properties of shock waves and solitons.

3. Exact Solutions of the Schamel-KdV Equation

The Schamel-KdV equation Eq.1 can be converted into following ordinary differential equation by using the transformation $\eta = x - V_p t$, $\Phi = \Phi(\eta)$,

$$p\Phi''' - V_p\Phi' + a\Phi^{\frac{1}{2}}\Phi' + b\Phi\Phi' = 0, \tag{4}$$

where $\Phi' = \frac{d\Phi}{d\eta}$. Integrating Eq.4 once gives

$$p\Phi'' - V_p\Phi + \left(\frac{2a}{3}\right)\Phi^{\frac{3}{2}} + \left(\frac{b}{2}\right)\Phi^2 + c = 0, \tag{5}$$

where c is an arbitrary integration constant. If we change the variable using the definition $\Psi = \Phi^{\frac{1}{2}}$ in Eq.5, we get

$$\Psi\Psi'' + \Psi'^2 - \left(\frac{V_p}{2p}\right)\Psi^2 + \left(\frac{a}{3p}\right)\Psi^3 + \left(\frac{b}{4p}\right)\Psi^4 + \frac{c}{2p} = 0. \tag{6}$$

Now we will present some exact solutions of the Schamel-KdV equation.

3.1. Exact solutions of the Schamel-KdV Equation by using (G'/G)-expansion method

Eq.1 can be solved by using the (G'/G)-expansion method with different transformations [17]. To have a self consistent paper, we also give the solution of Eq.1 by using (G'/G)-expansion method with a transformation $\eta = x - V_p t$. Balancing the terms $\Psi\Psi''$ and Ψ^4 in Eq.6, we get the positive balancing parameter $m = 1$. Thus, we get the following form of the solution:

$$\Psi(\eta) = \alpha_1\left(\frac{G'}{G}\right) + \alpha_0. \tag{7}$$

Substituting Eq.7 and its derivatives into Eq.6, we get a set of algebraic equations. From these algebraic equations, we find α_1, α_0, λ, μ, b, and c as

$$\begin{aligned} \alpha_1 &= \pm\frac{15\sqrt{pV_p}}{2a} \\ \alpha_0 &= \frac{15}{8a}\left(V_p \pm 2\lambda\sqrt{pV_p}\right), \\ \lambda^2 - 4\mu &= \frac{V_p}{4p}, \quad b = -\frac{16a^2}{75V_p}, \quad c = 0. \end{aligned} \tag{8}$$

Substituting these expressions in Eq.7 and the corresponding solution of ODE, then using the transformation $\Psi^2 = \Phi$, we will have the following types of solutions of

the Schamel-KdV equation:

Case I: When $\lambda^2-4\mu>0$

$$\Phi(\eta)=\left[\pm\frac{15\sqrt{pV_p}}{2a}\left(\frac{G'}{G}\right)+\frac{15}{8a}\left(V_p\pm2\lambda\sqrt{pV_p}\right)\right]^2, \quad (9)$$

where

$$\left(\frac{G'}{G}\right)=\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{c_1\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta+c_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta}{c_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta+c_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta}\right)-\frac{\lambda}{2}.$$

Case II: When $\lambda^2-4\mu<0$

$$\Phi(\eta)=\left[\pm\frac{15\sqrt{pV_p}}{2a}\left(\frac{G'}{G}\right)+\frac{15}{8a}\left(V_p\pm2\lambda\sqrt{pV_p}\right)\right]^2, \quad (10)$$

where

$$\left(\frac{G'}{G}\right)=\left(\frac{-c_1\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta+c_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta}{c_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta+c_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta}\right)-\frac{\lambda}{2}.$$

3.2. Exact solutions of the Schamel-KdV Equation by using different form of the (G'/G)-expansion method

In order to have an analytic solution of ODE, first of all, we balance the terms and get the following form of the solution

$$\Psi(\eta)=\alpha_1\left(\frac{G'}{G}\right)+\alpha_0+\alpha_{-1}\left(\frac{G'}{G}\right)^{-1}. \quad (11)$$

Substituting Eq.11 and its derivatives into Eq.6, we find a set of of algebraic equations. The parameters α_1, α_0, α_{-1}, μ, b, and c can be found from the algebraic equations:

Group I:

$$\begin{aligned}&\alpha_1=\pm\frac{15\sqrt{pV_p}}{2a},\quad \alpha_0=\frac{15V_p}{8a},\\&\alpha_{-1}=0,\\&\mu=-\frac{V_p}{16p},\quad b=-\frac{16a^2}{75V_p},\quad c=0.\end{aligned} \quad (12)$$

Group II:

$$\begin{aligned}&\alpha_1=\pm\frac{15\sqrt{pV_p}}{2a},\quad \alpha_0=\frac{15V_p}{8a},\\&\alpha_{-1}=\pm\frac{15V_p\sqrt{pV_p}}{64ap},\\&\mu=\frac{V_p}{32p},\quad b=-\frac{16a^2}{75V_p},\quad c=\frac{225V_p^3}{512a^2}.\end{aligned} \quad (13)$$

Substituting these results into Eq.11 and the corresponding solution of ODE, then using the $\Psi^2=\Phi$, the following types of solutions for the Schamel-KdV Equation will be found:

Case I: When $-\mu>0$
In this case

$$\left(\frac{G'}{G}\right)=\sqrt{-\mu}\left(\frac{c_1\sinh\sqrt{-\mu}\eta+c_2\cosh\sqrt{-\mu}\eta}{c_1\cosh\sqrt{-\mu}\eta+c_2\sinh\sqrt{-\mu}\eta}\right)$$

Group I:

$$\Phi(\eta)=\left[\pm\frac{15\sqrt{pV_p}}{2a}\left(\frac{G'}{G}\right)+\frac{15V_p}{8a}\right]^2. \quad (14)$$

Group II:

$$\Phi(\eta)=\left[\pm\frac{15\sqrt{pV_p}}{2a}\left(\frac{G'}{G}\right)\pm\frac{15V_p\sqrt{pV_p}}{64ap}\left(\frac{G'}{G}\right)^{-1}+\frac{15V_p}{8a}\right]^2. \quad (15)$$

Case II: When $-\mu<0$
In this case

$$\left(\frac{G'}{G}\right)=\sqrt{\mu}\left(\frac{-c_1\sin\sqrt{\mu}\eta+c_2\cos\sqrt{\mu}\eta}{c_1\cos\sqrt{\mu}\eta+c_2\sin\sqrt{\mu}\eta}\right)$$

Group I:

$$\Phi(\eta)=\left[\pm\frac{15\sqrt{pV_p}}{2a}\left(\frac{G'}{G}\right)+\frac{15V_p}{8a}\right]^2. \quad (16)$$

Group II:

$$\Phi(\eta)=\left[\pm\frac{15\sqrt{pV_p}}{2a}\left(\frac{G'}{G}\right)\pm\frac{15V_p\sqrt{pV_p}}{64ap}\cdot\left(\frac{G'}{G}\right)^{-1}+\frac{15V_p}{8a}\right]^2. \quad (17)$$

3.3. Exact solutions of the Schamel-KdV Equation by using $(G'/G,1/G)$-expansion method

The following form of the solution is reached using the balancing result

$$\Psi(\eta)=a_1\phi+a_0+b_1\psi. \quad (18)$$

Substituting Eq.18 and its derivatives into Eq.6, we have a set of algebraic equations. By solving the algebraic equations, we get a_1, a_0, b_1, λ, b, and c.

Case I: $\lambda<0$

$$\begin{aligned}&a_1=\pm\frac{15\sqrt{pV_p}}{4a},\quad a_0=\frac{15V_p}{8a},\\&b_1=\pm\frac{15\sqrt{16\mu^2p^2+\nu V_p^2}}{8a},\\&\lambda=-\frac{V_p}{4p},\quad b=-\frac{16a^2}{75V_p},\quad c=0.\end{aligned} \quad (19)$$

Substituting Eq.19 into Eq.18 we have the solution of Eq.6 and then by using the transformation $\Psi=\Phi^{\frac{1}{2}}$, we get the solution of Eq.1

$$\Phi(\eta)=\left[\pm\frac{15\sqrt{pV_p}}{4a}\phi+\frac{15V_p}{8a}\pm\frac{15\sqrt{16\mu^2p^2+\nu V_p^2}}{8a}\psi\right]^2, \quad (20)$$

where ϕ and ψ are defined and given in [22].

Similarly, substituting Eq.18 and its derivatives into Eq.6, it yields a set of algebraic equations. By solving the algebraic equations we find a_1, a_0, b_1, λ, b, and c.

Case II: $\lambda > 0$

$$\begin{aligned} a_1 &= \pm\frac{15\sqrt{pV_p}}{4a}, \quad a_0 = \frac{15V_p}{8a}, \\ b_1 &= \pm\frac{15\sqrt{16\mu^2p^2 - \nu V_p^2}}{8a}, \\ \lambda &= -\frac{V_p}{4p}, \quad b = -\frac{16a^2}{75V_p}, \quad c = 0. \end{aligned} \tag{21}$$

Substituting Eq.21 into Eq.18 we have the solution of Eq.6 and then by using the transformation $\Psi = \Phi^{\frac{1}{2}}$, we obtain the solution of Eq.1 as

$$\Phi(\eta) = \left[\pm\frac{15\sqrt{pV_p}}{4a}\phi + \frac{15V_p}{8a} \pm \frac{15\sqrt{16\mu^2p^2 - \nu V_p^2}}{8a}\psi\right]^2, \tag{22}$$

where ϕ and ψ are defined in [22].

At last, substituting Eq.18 and its derivatives into Eq.5, we get the algebraic equations and by solving them we find a_1, a_0, b_1, λ, a, V_p and c.

Case III: $\lambda = 0$

$$\begin{aligned} a_1 &= \pm\sqrt{\frac{-3p}{b}}, \quad a_0 = 0, \\ b_1 &= \pm\sqrt{\frac{6c_2\mu p - 3c_1^2 p}{b}}, \\ \lambda &= 0, \quad a = 0, \quad V_p = 0, \quad c = 0. \end{aligned} \tag{23}$$

Substituting Eq.23 into Eq.18 we have the solution of Eq.6 and then by using the transformation $\Psi = \Phi^{\frac{1}{2}}$, the solution of Eq.1 is given as

$$\Phi(\eta) = \left[\pm\sqrt{\frac{-3p}{b}}\phi \pm \sqrt{\frac{6c_2\mu p - 3c_1^2 p}{b}}\psi\right]^2, \tag{24}$$

where ϕ and ψ are defined in [22].

3.4. Exact solutions of the Schamel-KdV Equation by using direct integration

There are many different methods defined in literature to solve the nonlinear differential equation, analytically. We can use these methods to solve the analytic solution of the differential equation but there is also possibility to have the analytic solution of the nonlinear differential equation with direct integration. Here we will use the direct integration to obtain the exact solution of the Schamel-KdV equation [10]. Starting from Eq.5 and recalling c as c_1, we get

$$p\Phi'' - V_p\Phi + \left(\frac{2a}{3}\right)\Phi^{\frac{3}{2}} + \left(\frac{b}{2}\right)\Phi^2 + c_1 = 0, \tag{25}$$

where c_1 is an arbitrary integration constant. Let us multiply Eq.25 by Φ' and then integrate it once with respect to η. We find [10].

$$\begin{aligned} (\Phi')^2 = &-\left(\frac{b}{3p}\right)\Phi^3 - \left(\frac{8a}{15p}\right)\Phi^{\frac{5}{2}} + \left(\frac{V_p}{p}\right)\Phi^2 - \\ &\left(\frac{2c_1}{p}\right)\Phi - \left(\frac{2c_2}{p}\right), \end{aligned} \tag{26}$$

where c_2 is an integration constant. Using the new definition of potential $\Psi = \Phi^{\frac{1}{2}}$, Eq.26 becomes

$$\Psi' = \pm \sqrt{-(\frac{b}{12p})\Psi^4 - (\frac{2a}{15p})\Psi^3 + (\frac{V_p}{4p})\Psi^2 - (\frac{c_1}{2p}) - (\frac{c_2}{2p})\Psi^{-2}}. \tag{27}$$

Eq.27 is now an integrable equation if we set all the integration constants to be zero. After the integration of the equation, we have reached the following exact solution of the Schamel-KdV equation [10]

$$\Phi(\eta) = \left[\frac{2A}{Ae^{\mp\sqrt{\frac{V_p}{4p}}\eta} + Be^{\pm\sqrt{\frac{V_p}{4p}}\eta} + C}\right]^2, \tag{28}$$

where $A = 225V_p^2, B = 16a^2 + 75bV_p, C = 120aV_p$ and $\eta = x - V_pt$ [10].

After doing some straightforward calculations, Eq.28 can be written in terms of the hyperbolic and trigonometric functions in case of $\frac{V_p}{4p} > 0$ and $\frac{V_p}{4p} < 0$, respectively

$$\Phi_1(\eta) = \left[\frac{2A}{(A+B)\cosh\sqrt{\frac{V_p}{4p}}\eta \pm (B-A)\sinh\sqrt{\frac{V_p}{4p}}\eta + C}\right]^2, \tag{29}$$

and

$$\Phi_2(\eta) = \left[\frac{2A}{(A+B)\cos\sqrt{\frac{V_p}{4p}}\eta \pm i(B-A)\sin\sqrt{\frac{V_p}{4p}}\eta + C}\right]^2. \tag{30}$$

4. Numerical Investigation of the Wave Type and Exposing the Effect of Plasma Parameters on the Wave Type in the Four-Component Dusty Plasma

Investigating the physical properties and dynamical structure of the dust acoustic wave and dust-ion acoustic wave observed in the physical and astrophysical plasma can be handled by considering the higher order nonlinear terms in plasma equations. The presence of nonisothermally distributed electrons, the positively charge warm dust, and

negatively charged cold dust introduce the Schamel-KdV equation in a four-component dusty plasma [27]. The Schamel-KdV equation is derived by using the reductive perturbation technique [27] and it is generalized by adding higher order terms, given by Eq.1. The coefficients of this equation can be given as

$$\begin{aligned} a &= \frac{3}{2}X_1 \\ b &= X_2X_3 \\ p &= X_2, \end{aligned} \tag{31}$$

where X_1, X_2, and X_3 are expressed in terms of the plasma parameters and some constants. They are given as [27]

$$\begin{aligned} X_1 &= 2y\mu_e\sigma^{\frac{3}{2}}X_2 \\ X_2 &= \frac{\sigma_\alpha^2 V_p^3}{2(\sigma_\alpha^2+\mu_2\alpha)} \\ X_3 &= \mu_i-\mu_e\sigma^2-\frac{3}{V_p^4}\left(1-\frac{\alpha^2\mu_2}{\sigma_\alpha^2}\left(1+\frac{4\alpha\sigma_d}{V_p^2\sigma_\alpha}\right)\right), \end{aligned} \tag{32}$$

where σ_d, σ, V_p, α, and y are the ratio of dust to ion temperature, the ratio of ion to electron temperature, the phase velocity, mass and charge ratio, and constant respectively. $\sigma_\alpha = 1-\frac{3\alpha\sigma_d}{V_p^2}$, $\mu_i = \frac{n_{i0}}{Z_1 n_{10}}$, $\mu_e = \frac{n_{e0}}{Z_1 n_{10}}$, and $\mu_2 = 1+\mu_e-\mu_i$ [27]. Z_1 is the number of electrons staying on negative dust and n_{10} is the negative dust number in an equilibrium condition.

In this section, we use the analytic solutions of the Schamel-KdV equation, found in Section 3, to have a deep understanding of the properties of shock waves and solitons produced in the plasmas. The effects of mass and charge ratio, ratio of dust to ion temperature, phase velocity, ratio of ion to electron temperatures, and constants in the analytic solutions are studied. Examination of the nonlinear properties of the wave in a dusty plasma is an important contribution to the literature for understanding of the propagation of wave inside the plasma and wave type observed in plasma. In order to find out the effect of the phase velocity of the wave on the wave properties, we plot the potential Φ vs. the variable η for various values of the phase speeds seen in Fig.1.

4.1. Numerical analysis of the wave in the plasma using the analytic solution obtained by the (G'/G)-expansion method

The shock wave is observed from the analytic solution of the Schamel-KdV equation obtained by using the $(\frac{G'}{G})$-expansion method in a four-component dusty plasma. The phase speed gives the rate of wave which is propagating in plasma. As it is seen in Fig.1, the strength of shock increases with decreasing in the phase speed. The decrease in the phase speed causes the abundance of the electron density in the plasma. Therefore the electron Deby radius gets smaller and the weaker electric field due to the space electron is observed in a dusty space plasma [30].

In Fig.2, we study the strength of shock wave in the dusty space plasma taking into account the effects of different plasma parameters such as α, σ, σ_d, and y. The upper left part of Fig.2 shows that the strength of shock wave increases with increasing α but it decreases with increasing σ seen in the upper right part of the same plot. We also analyze the shock properties using the appropriate values of the ratio of dust to ion temperature σ_d and constant y. It is seen in the lower panels of Fig.2 that the strength of shock increases with increasing σ_d but it decreases with increasing constant y.

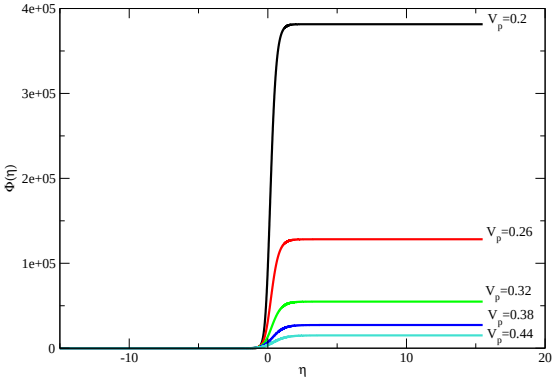

Figure 1. The variation of potential Φ is plotted as a function of η for the different values of phase speed. The other parameters are $\mu_i = 0.8$, $\mu_e = 0.2$, $\sigma = 0.5$, $\sigma_d = 0.001$, $\alpha = 1$, $y = 2.1$, integration constants $c_1 = 1$, $c_2 = 0$, and source $\mu = 1$.

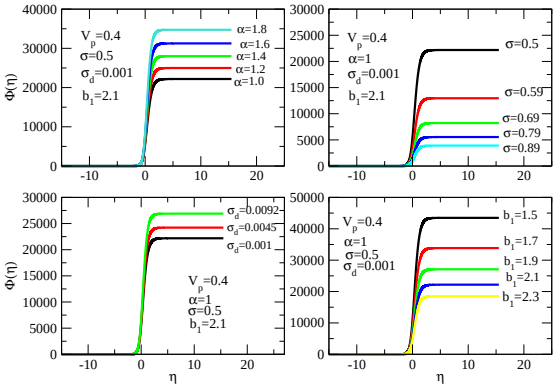

Figure 2. Same as Fig.1 but it is for different values of plasma parameters such as mass and charge ratio α, the ratio of ion to electron temperature σ, the ratio of dust to ion temperature σ_d, and constant y.

4.2. Numerical analysis of the wave in the plasma using the analytic solution obtained by the $(G'/G, 1/G)$-expansion method

In this subsection, we investigate the physical properties and parameter dependencies of the dusty space plasma by using the analytic solution obtained by the $(G'/G, 1/G)$-expansion method in Section 3.3. Here we only focus on the solution Eq.20 which may define solitons in space plasma.

To examine the type of soliton and effect of the phase velocity on it, we plot the potential as a function of η for the various values of phase speeds V_p, seen in Fig.3.

The phase speed varies from 0.1 to 0.46. It is found that we do not observe any soliton or shock wave but we find strong discontinuity wave. The discontinuity locations move toward to lower values of η with increasing phase speed. On the other hand, the location for discontinuity moves to the right (toward to the bigger value of η) when the source term μ gets bigger. It is also noted that all plasma parameters and integration constants have a strong influence on the location of discontinuity but the type of analytic solution has never changed.

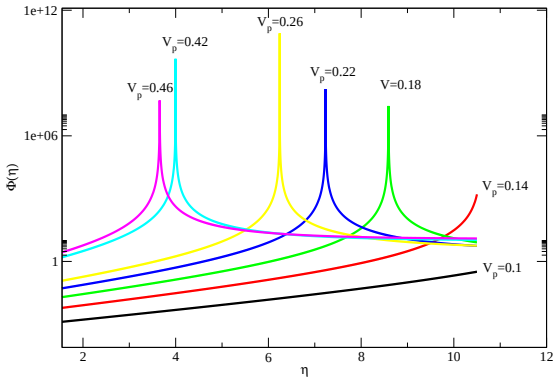

Figure 3. The variation of solution with the phase speed is given for the parameters $\mu_i = 0.8$, $\mu_e = 0.2$, $\sigma = 0.5$, $\sigma_d = 0.001$, $\alpha = 1$, $y = 2.1$, integration constants $c_1 = 1$, $c_2 = 0$, $c = 0$, and source $\mu = 1$.

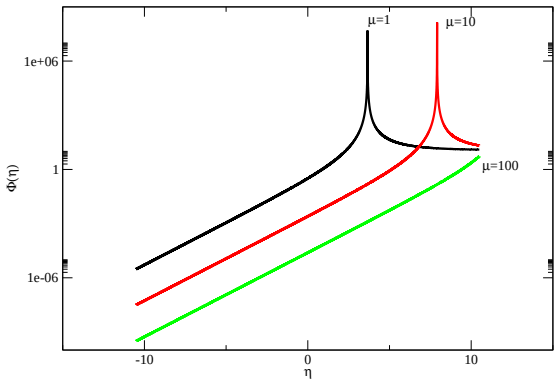

Figure 4. Same as Fig.3 but it is for different values of source terms which appear in the analytic solution of the Schamel-KdV equation with the fixed phase speed $V_p = 0.46$.

4.3. Numerical analysis of the wave in the plasma using the analytic solution obtained by the direct integration

Considering the negatively charged cold dust and warmed adiabatic positively charged dust in the space plasma gives more nonlinear behavior of system. Here the nonlinear variation of the dusty space plasma is analyzed to have a deep understanding of the dust-acoustic solitary wave by using the analytic solution of the Schamel-KdV equation obtained from the direct integration. The solitary type solutions are obtained from the analytic solution and they are given in Fig.5 for the various values of phase speeds with the fixed parameters $\mu_i = 0.8$, $\mu_e = 0.2$, $\sigma = 0.5$, $\sigma_d = 0.001$, $\alpha = 1$, and $y = 2.1$. We illustrate the effect of the phase speed on the soliton amplitude and width and it is found that increasing the phase speed cause to decrease in the amplitude of soliton and it also affects the location of maximum amplitude of the soliton. It slightly shifts to the left with the increasing phase speed.

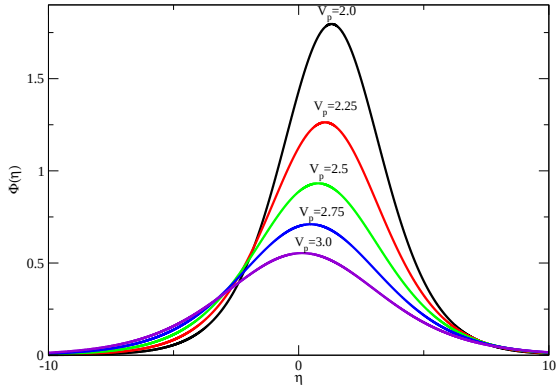

Figure 5. The ion-acoustic solitary potential Φ versus η is plotted for the various values of the phase speeds ($V_p \geq 2$) with the fixed plasma parameters $\mu_i = 0.8$, $\mu_e = 0.2$, $\sigma = 0.5$, $\sigma_d = 0.001$, $\alpha = 1$, and $y = 2.1$.

Based upon the solitons represented by the analytic solution of the Schamel-KdV equation obtained by using the direct integration, we investigate the effects of the phase speed (when $V_p < 2$) and plasma parameters which are the ratio of dust to ion temperature σ_d, the mass and charge ratio α, and the ratio of ion to electron temperature σ. Fig.6 depicts the variation of electrostatic potential as a function of η for cold, adiabatic, and isothermal dusty space plasma. It is clear from the figure that the plasma with supersonic phase speed ($V_p \geq 1.235$) has solitary wave solution but it creates some discontinuities when $V_p < 1.235$.

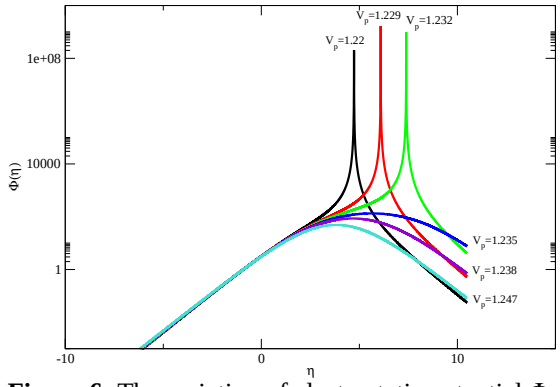

Figure 6. The variation of electrostatic potential Φ with phase velocities is given for the fixed values of plasma parameters and integration constant. Here $\mu_i = 0.8$, $\mu_e = 0.2$, $\sigma = 0.5$, $\sigma_d = 0.001$, $\alpha = 1$, $y = 2.1$, and $c = 1$.

Following the different plasma parameters, we seek a soli-

tary wave in dusty space plasma. The different values of the ratio of dust to ion temperature σ_d, the mass and charge ratio α, and the ratio of ion to electron temperature σ play an important role to introduce soliton in plasma, seen in Figs.7 and 8. It is important to notice that we have the critical values for these plasma parameters to observe the soliton in dusty plasma. These critical values are $\sigma_d > 0.51$, $\alpha > 1.2$, and $\sigma > 2.0$. Under these parameters it seems plausible that one can produce the soliton in the space plasma using the analytic solution of the Schamel-KdV equation obtained from the direct integration.

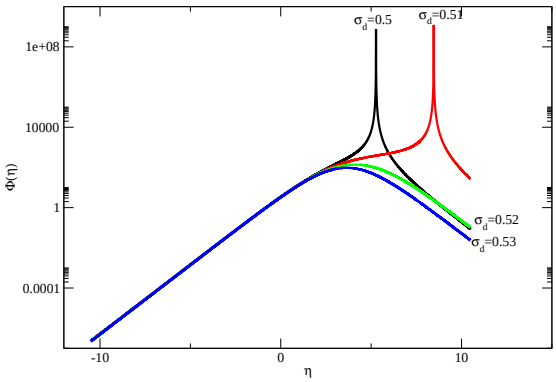

Figure 7. The same as Fig.6 but it is for the different values of σ_d and $V_p = 1.1$.

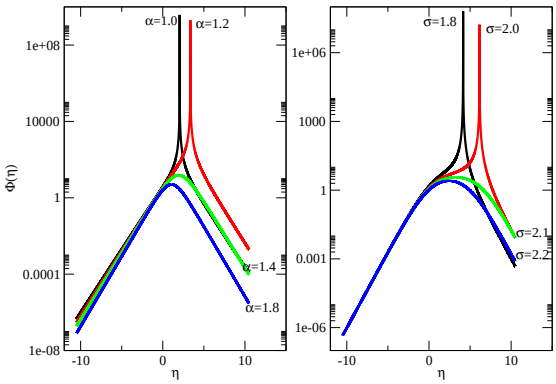

Figure 8. The same as Fig.6 but the left panel is for the different values of α with $\sigma = 0.5$ and the right panel is for the different values of σ with $\alpha = 1$. The phase speed in both cases is $V_p = 1.1$.

5. Conclusion

In this paper, first of all, we have obtained the analytic solutions of the Schamel-KdV equation using four different methods, (G'/G)-expansion method, different form of (G'/G)-expansion method, $(G'/G, 1/G)$-expansion method, and direct integration. And then, we have investigated the types of the solitary waves and the effect of plasma parameters and integration constants on the solitary waves in the four-component dusty space plasma using three different methods (G'/G)-expansion method, $(G'/G, 1/G)$-expansion method, and direct integration. We have numerically examined the types of solitons and their dynamical responds to the different plasma parameters and integration constants. It is found that the solitary wave is only observed from the analytic solution obtained by the direct integration. While the shock waves (rarefactive solitons) are detected from the analytic solutions obtained by the (G'/G)-expansion method and different form of (G'/G)-expansion method, we have also found some discontinuity waves from the analytic solution of the Schamel-KdV obtained by $(G'/G, 1/G)$-expansion method. $(G'/G, 1/G)$-expansion method does not give any solitary wave solution for the Schamel-KdV equation.

In addition to the above findings, we have also studied the properties of shock waves, solitary waves, and discontinuity waves in the presence of variation of plasma parameters and integration constants. It is seen that the plasma parameters α, σ, σ_d, and y, and integration constants c_1, c_2, c, and μ not only modify the strength of shock wave (rarefactive soliton) and discontinuity but they also have a huge impact on the amplitudes of solitons observed in the dusty space plasma.

Finally, it is interesting to point out that the solitary wave solutions can only be found from the analytic solution obtained by the direct integration. These solitons arise under some critical conditions of the plasma parameters and phase speed. The effects of plasma parameters and phase speed are shown to significantly change the type of solution. The discontinuity wave turns into the solitary wave solution when the phase speed is $V_p \geq 1.235$, the ratio of dust to ion temperature is $\sigma_d > 0.51$, the mass and charge ratio is $\alpha > 1.2$, and the ratio of ion to electron temperature is $\sigma > 2.0$ with the fixed values of the other plasma parameters and integration constants.

In conclusion, it is important to know the different types of analytic solutions of the Schamel-KdV equation, which includes higher order nonlinear terms. The analytic solutions can be applied to discover the properties of the shock and solitary waves observed in laboratory and astrophysical dusty plasmas.

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