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European Journal of Advances in Engineering and Technology, 2017, 4(11): 850-857

Research Article ISSN: 2394 - 658X

Heat Generation and Radiation Effects on MHD Casson Fluid Flow Over a Stretching Surface Through Porous Medium

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ABSTRACT

In this paper, the heat generation and radiation effects on magnetohydrodynamic stagnation-point flow of non-Newtonian Casson fluid flow over a stretching surface through porous medium have been obtained and studied numerically. The governing continuity and momentum equations are converted into a system of non-linear ordinary differential equations by means of similarity transformation. The resulting system of coupled non-linear ordinary differential equations is solved numerically. Numerical results were presented for velocity and temperature profiles for different parameters of the problem. Also the effects of the pertinent parameters on the skin friction, the rate of heat and mass transfer are obtained and discussed numerically.

Keywords: MHD, Heat Generation, Radiation, Porous medium, Casson fluid

INTRODUCTION

There is continuous increasing interest of recent researchers in flow problems of non-Newtonian fluids due to their high applications in industry and engineering. It is well known now that the stretched flow problems of non-Newtonian fluids occur in production of plastic, paper and food materials. Heat transfer involvement has great role in these processes. Various recent researchers are engaged in exploring the heat transfer characteristics in the flow of non-Newtonian fluids over a stretching surface. Mahapatra and Gupta [1] reconsidered the stagnation point flow problem towards a stretching sheet taking different stretching and straining velocities and they observed two different kinds of boundary layer near the sheet depending on the ratio of the stretching and straining constants. Mahapatra and Gupta [2] presented heat transfer in stagnation point flow towards a stretching sheet. Nazar. Amin, *et al* [3] showed the unsteady boundary layer flow in the region of the stagnation point on a stretching sheet. Heat and mass transfer analysis for boundary layer stagnation-point flow towards a heated porous stretching sheet with heat absorption/generation and suction/ blowing by Layek *et al* [4].

Attia [5] initiated the study of on the effectiveness of porosity on stagnation point flow towards a stretching surface with heat generation; Nadeem *et al* [6] studied the HAM solutions for boundary layer flow in the region of the stagnation point towards a stretching sheet. Dual solutions in boundary layer and unsteady stagnation-point flow and mass transfer with chemical reaction past a stretching/shrinking sheet by [7, 8]. Salem and Fathy [9] who analyzed the effects of variable properties on MHD heat and mass transfer flow near a stagnation point towards a stretching sheet in a porous medium with thermal radiation. The detailed discussion on the stagnation-point flow over stretching/shrinking sheet can be found in the works of [10-14].

In the present work, we extend and generalize the work of [15] to include the numerical study of MHD Stagnation-Point Flow of Casson Fluid and Heat Transfer over a Stretching Sheet with Thermal Radiation. The aim of the presented paper is to study the effects of heat generation and radiation on magnetohydrodynamic stagnation-point flow of non-Newtonian Casson fluid flow over a stretching surface through porous medium are used to similarity transformation into a system of nonlinear ordinary differential equations which are solved numerically. Numerical results were presented for velocity and temperature profiles for different parameters of the problem. Also the effects of the pertinent parameters on the skin friction, the rate of heat and mass transfer are also discussed.

MATHEMATICAL ANALYSIS

Consider the steady two-dimensional incompressible flow of a non-Newtonian Casson fluid over a stretching surface through porous medium under the influence of a transversely applied magnetic field B in the y direction with the induced magnetic field being neglected, with the flow being confined in $y > 0$. It is also assumed that the rheological equation of state for an isotropic and incompressible flow of a Casson fluid can be written as [16-17].

$$\tau_{ij} = \begin{cases} \left(\mu_A + \dfrac{p_y}{\sqrt{2\pi}}\right) 2e_{ij}, & \pi > \pi_c \\ \left(\mu_A + \dfrac{p_y}{\sqrt{2\pi_c}}\right) 2e_{ij}, & \pi < \pi_c \end{cases} \tag{1}$$

where μ_A is plastic dynamic viscosity of the non-Newtonian fluid, p_y is the yield stress of fluid, π is the product of the component of deformation rate with itself, namely, $\pi = e_{ij}e_{ij}$, e_{ij} is the (i, j) component of the deformation rate, and π_c is a critical value of π based on non-Newtonian model.

The governing boundary layer equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 , \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_s \frac{dU_s}{dx} + \nu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}(u - U_s) + \frac{\nu}{\kappa}(u - U_s), \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{Q_0}{\rho c_p}(T - T_\infty), \tag{4}$$

With the appropriate boundary conditions:

$$\begin{aligned} &y = 0: \quad u = U_w = cx, \quad v = 0, \quad T = T_w, \\ &y \to \infty: \quad u = U_s = ax \quad T = T_\infty . \end{aligned} \tag{5}$$

Where c *and* a are the positive constants that represent the characteristic stretching intensity and the free stream strength. ρ, μ *and* C_p are the density, dynamic viscosity and the specific heat at constant pressure, respectively. κ is the permeability of the porous medium, T_w is the constant temperature at the sheet and T_∞ is the free stream temperature assumed to be constant, $U_s = ax$ is the straining velocity of the stagnation-point flow with a (>0) being the straining constant, $\nu = \mu_A / \rho$ is the kinematic fluid viscosity, ρ is the fluid density, $\beta = \mu_A \sqrt{2\pi} / p_y$ is the non-Newtonian or Casson parameter, σ is the electrical conductivity of the fluid, $U_w = cx$ is stretching velocity of the sheet with c (>0) being the stretching constant, k are the effective thermal conductivity, Q_0 is the volumetric heat generation/absorption rate, q_r are the thermal-diffusion rate, concentration susceptibility, fluid mean temperature and the radiative heat flux, respectively. Using the Rosseland approximation (Rashed [18]), the radiative heat flux q_r could be expressed by

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{6}$$

where the σ^* represents the Stefan-Boltzman constant and k^* is the Rosseland mean absorption coefficient.

Assuming that the temperature difference within the flow is sufficiently small such that T^4 could be approached as the linear function of temperature

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{7}$$

The governing partial differential equations (2)-(5) admit similarity solutions for obtaining the dimensionless stream function $f(\eta)$ and temperature $\theta(\eta)$. The relative parameters are introduced as

$$u = cxf^{/}(\eta), \qquad v = -\sqrt{c\nu}\, f(\eta), \qquad \theta(\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}}, \;\; \eta = \sqrt{c/\nu}\, y.$$

The equation of continuity is satisfied automatically. After introducing the similarity transformation, the equations of (3) and (4) can be transformed into a set of following forms in terms with $f(\eta)$ and $\theta(\eta)$, expressed as

$$(1 + 1/\beta) f^{///} + ff^{//} - f^{/2} - (M + S)(f^{/} - A) + A^2 = 0, \tag{8}$$

$$\left(1 + \frac{4}{3} R^*\right)\theta^{//} + P_r f\theta^{/} + P_r Q\theta = 0,$$

(9)

The boundary conditions reduce to:

$$f(0) = 0,\;\; f^{/}(0) = 1,\;\; \theta(0) = 1,$$
$$f^{/}(\infty) = A,\;\; \theta(\infty) = 0. \tag{10}$$

Where $M = \sigma B_0^2 / \rho c$ is the magnetic field parameter, $S = \nu / \kappa c$ is the porous medium parameter, $A = a/c$ is the velocity ratio parameter, $P_r = \mu c_p / k$ is the Prandtl number, $R^* = \frac{1}{R} = 4\sigma^* T_{\infty}^3 / k^* k$ is the radiation parameter and $Q = Q_0 / c\rho c_p$ is the heat generation parameter.

SKIN-FRICTION COEFFICIENT AND NUSSELT NUMBER

The parameters of engineering interest for the present problem are the local skin-friction coefficient and local Nusselt number which indicate the physical wall shear stress and rate of heat transfer, respectively.
Which are defined as:

$$C_f = \tau_w / \rho u_w^2 \quad \text{and} \quad N_u = x q_w / \alpha (T_w - T_{\infty}), \tag{11}$$

Where (τ_w) is the shear stress or skin friction along the stretching sheet and (q_w) is the heat flux from the sheet and those are defined as

$$\tau_w = \mu_A \left(1 + p_y / \mu_A \sqrt{2\pi}\right)\left(\frac{\partial u}{\partial y}\right)_{y=0} \quad \text{and} \quad q_w = -\alpha \left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{12}$$

Hence, local skin-friction coefficient C_f and the Nusselt number N_u as follows:

$$\sqrt{R_e}\, C_f = (1 + 1/\beta) f^{//}(0) \text{ and } (R_e)^{-1/2} N_u = -\theta^{/}(0). \tag{13}$$

where $R_e = cx^2 / \nu$ is the local Reynolds number.

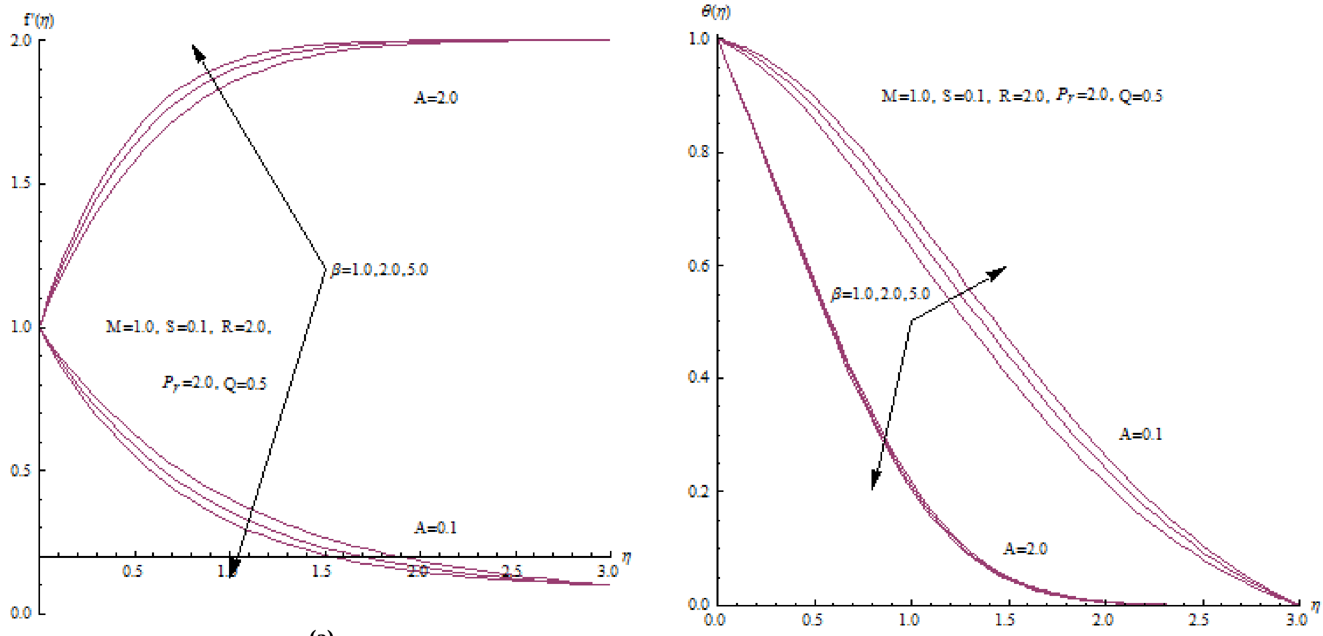
RESULTS AND DISCUSSION

The system of non-linear ordinary differential Eqs. (8)- (9) together with the boundary conditions (10) are solved numerically by using the forth order of Runge-kutta integration accompanied with the shooting method. In order to get a physical insight, the velocity and temperature profiles have been discussed.

Figs. 1 (a and b) show the effect of Casson parameter on the velocity and the temperature profiles. We found that the velocity profile increases at $A = 2.0$, and decreases at $A = 0.1$ with an increase of Casson parameter. We observe from this figure that the boundary layer thickness increases as β decreases at $A = 0.1$, likewise, this figure depicts that for increasing values of the Casson parameter, it reduces the fluid velocity distribution inside the boundary layer away from the sheet but the reverse is true along the sheet. Physically, with an increase in the non-Newtonian Casson parameter, the fluid yield stress is decreasing causes a production for resistance force which make the fluid velocity decreases. Also, we found that the temperature profile increases at $A = 0.1$, but decreases at $A = 2.0$ with the increase of Casson parameter. On the other hand, the effect of Casson parameter on temperature field in the case of $A = 0.1$is much more pronounced than that of the case of $A = 2.0$.

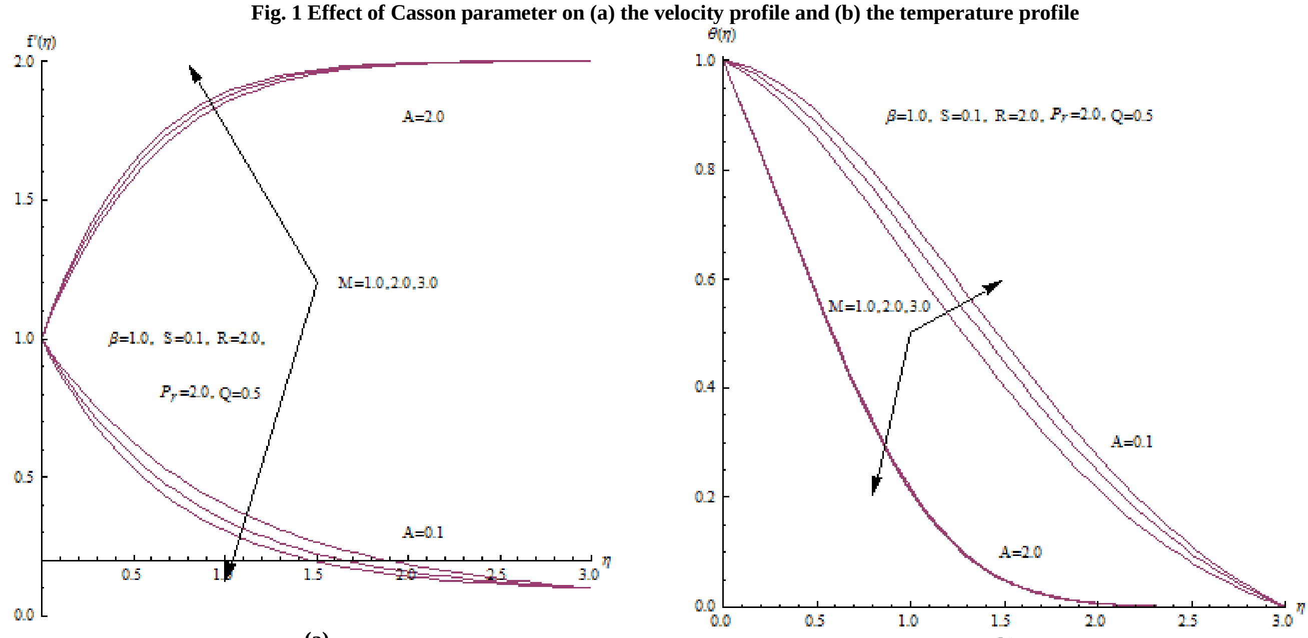
The dimensionless velocity profile for selected values of magnetic parameter $M\,(M=1.0, 2.0, 3.0)$ is plotted in Fig (2) (a). It is apparent that the velocity decreases along the surface with an increase in the magnetic parameter at $A=0.1$. The transverse magnetic field opposes the motion of the fluid and the rate of transport is considerably reduced, this is because with the increase in M, Lorentz force increases and it produces more resistance to the flow. At $A=2.0$, we found that the velocity increases with an increase in the magnetic parameter, and this is because the stretching velocity is greater than the straining velocity. Also, it is found that the temperature distribution along the boundary layer, thermal boundary thickness and the temperature for the stretching surface increase with an increase in the same parameter at $A=0.1$,as we can see from Fig(2)(b).So the temperature inside the thermal boundary layer increases due to excess of heating. Therefore, the magnetic field can be used to control the flow characteristics. Also, it is apparent that there is no effect of M on θ at $A=2.0$.

Figs. (3)(a and b) show the effects of porous medium parameter S on the velocity and the temperature profiles, we found that the velocity profile decreases at $A=0.1$, but the temperature profile increase at $A=0.1$ with the increases of porous medium parameter. Also, it is apparent that there is no effect of S on f' *and* θ at $A=2.0$.



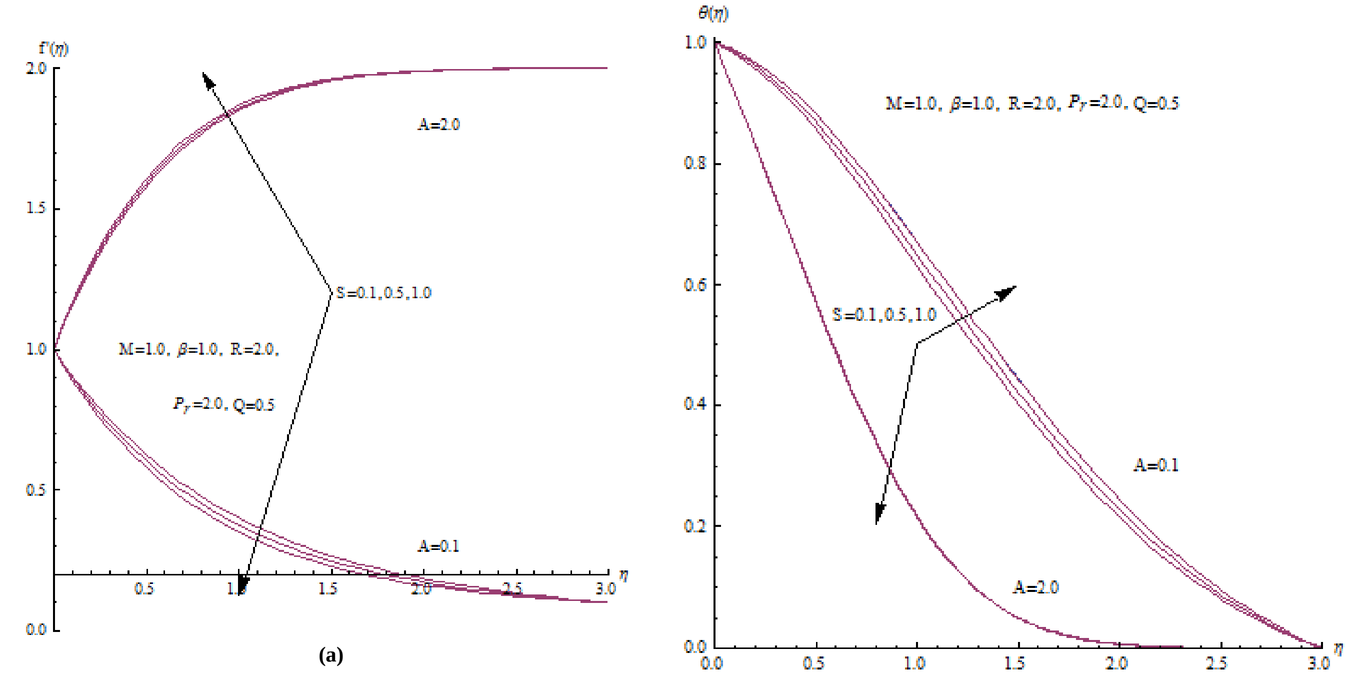
(a) (b)

Fig. 1 Effect of Casson parameter on (a) the velocity profile and (b) the temperature profile



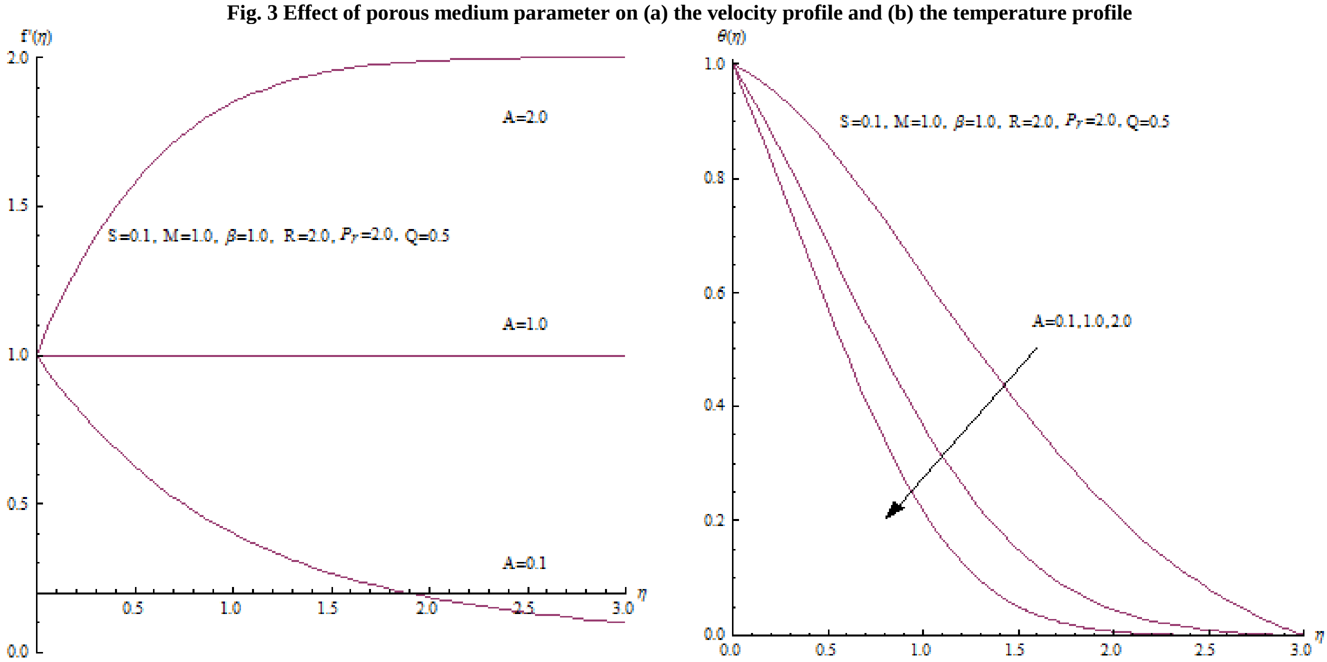
(a) (b)

Fig. 2 Effect of magnetic field parameter on (a) the velocity profile and (b) the temperature profile



(a) **(b)**

Fig. 3 Effect of porous medium parameter on (a) the velocity profile and (b) the temperature profile



(a) **(b)**

Fig. 4 Effect of velocity ratio parameter on (a) the velocity profile and (b) the temperature profile

The velocity and temperature profiles for various values of velocity ratio parameter A are plotted in Fig. (4)(a and b). Depending on the velocity ratio parameter, two different kinds of boundary layers are obtained. In the first kind, the velocity of fluid inside the boundary layer decreases from the surface towards the edge of the layer (for $A<1$) and in the second kind the fluid velocity increases from the surface towards the edge (for $A>1$). Those characters can be seen from velocity profiles in Fig. 4(a). Also, it is important to note that if $A=1\,(a=c)$, that is, the stretching velocity and the straining velocity are equal, and then there is no boundary layer of Casson fluid flow near the sheet.

It is observed in Fig. (5) that increasing thermal radiation parameter $R^* = 1/R = (1.0, 0.5, 0.2)$ increases temperature profiles, this is due to the fact that as more heat is generated within the fluid, the fluid temperature increases leading to a sharp inclination of the temperature gradient between the surface and the fluid. In Fig. (6), it is clear that the temperature profile decrease with an increasing of Prandtl number, at $A = 0.1$ *and* $A = 2.0$. Also, by comparison these results with the results of Krishnendu [15] we found that they are similar at $A = 0.1$ *and* $A = 2.0$, which are excellent and agreement. Fig.(7) shows that, the temperature profile increases with an increasing in heat generation parameter at $A = 0.1$ *and* $A = 2.0$ .This is because when heat is absorbed the buoyancy forces and thermal diffusivity of the fluid increase, which accelerate the flow rate and thereby rise to an increase in the temperature profile of the flow.

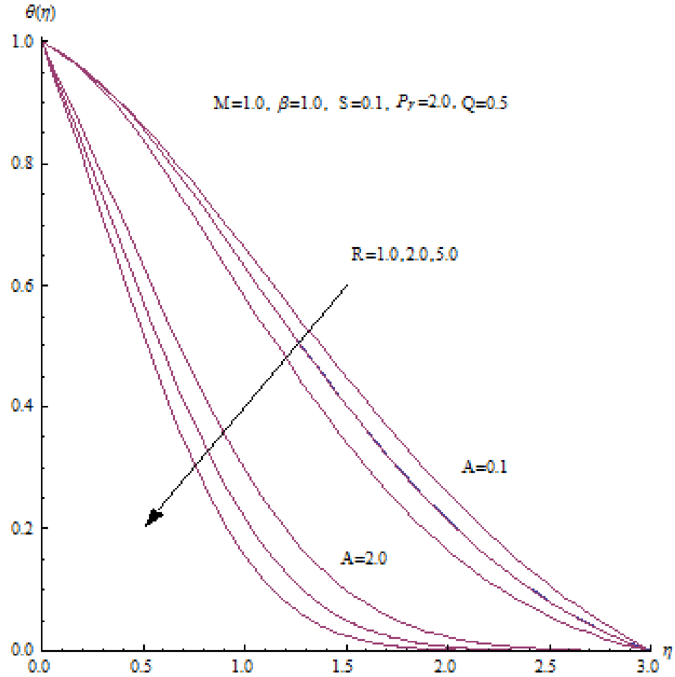


Fig. 5 Effect of radiation parameter on the temperature profile

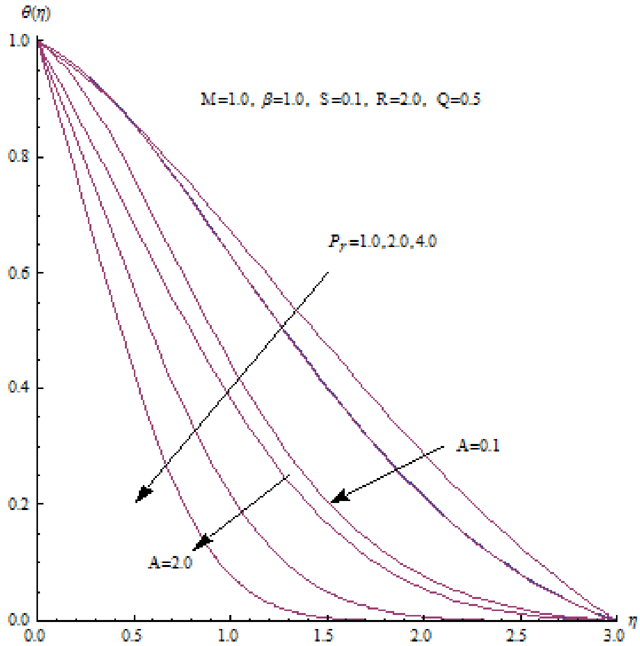


Fig. 6. Effect of Prandtl number parameter on the temperature profile

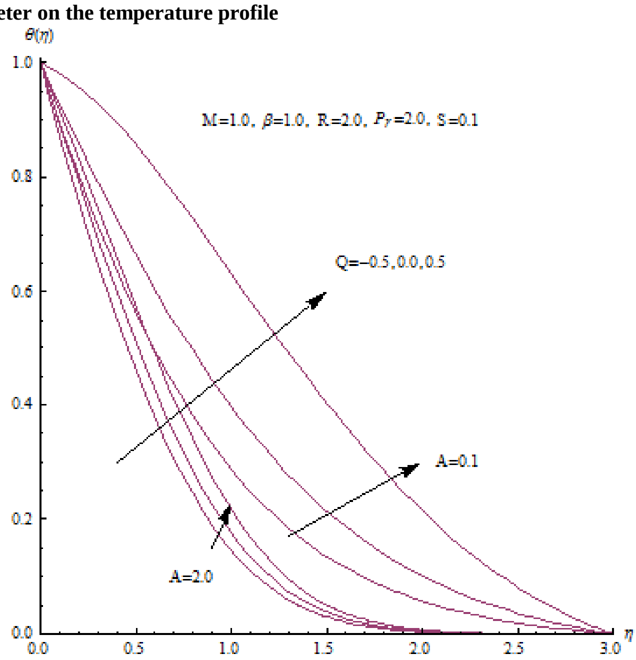


Fig. 7 Effect of heat generation parameter on the temperature profile

Table 1 shows the data of heat and mass transfer rate. From the table, we could determine that the value Skin-friction coefficient increases and Nusselt number decreases at $A = 0.1$, but the value Skin-friction coefficient decreases and Nusselt number increases at $A = 2.0$ with an increasing in Casson parameter. The value Skin-friction coefficient and Nusselt number decrease at $A = 0.1$, while the value Skin-friction coefficient and Nusselt number increase at $A = 2.0$ with the increase of each of magnetic field and porous medium parameters. With increase of Prandtl number parameter, we found that Nusselt number decreases at $A = 0.1$, while Nusselt number increases at $A = 2.0$. And Nusselt number decreases with an increasing in heat generation parameter at $A = 0.1$ *and* $A = 2.0$

Table -1 Numerical of the values the local Skin-friction coefficient ($C_f R_e^{-1/2}$) and the local Nusselt number ($N_u R_e^{-1/2}$) with

β, M, S, R, P_r and Q

β	M	S	R	P_r	Q	$C_f R_e^{1/2}$		$N_u R_e^{-1/2}$	
						$A=0.1$	$A=2.0$	$A=0.1$	$A=2.0$
1.0	1.0	0.1	2.0	2.0	0.5	-1.92168	3.21179	0.155282	0.778019
2.0	1.0	0.1	2.0	2.0	0.5	-1.65903	2.78142	0.108913	0.794826
5.0	1.0	0.1	2.0	2.0	0.5	-1.48204	2.78077	0.0700599	0.807753
1.0	**1.0**	0.1	2.0	2.0	0.5	-1.92168	3.21179	0.155282	0.778019
1.0	**2.0**	0.1	2.0	2.0	0.5	-2.29916	3.50697	0.0974868	0.786815
1.0	**3.0**	0.1	2.0	2.0	0.5	-2.62596	3.77972	0.0508974	0.794377
1.0	1.0	**0.1**	2.0	2.0	0.5	-1.92168	3.21179	0.155282	0.778019
1.0	1.0	**0.5**	2.0	2.0	0.5	-2.08014	3.33291	0.130568	0.781709
1.0	1.0	**1.0**	2.0	2.0	0.5	-2.26403	3.47855	0.1027	0.785997
1.0	1.0	0.1	**1.0**	2.0	0.5	-	-	0.182213	0.684612
1.0	1.0	0.1	**2.0**	2.0	0.5	-	-	0.155282	0.778019
1.0	1.0	0.1	**5.0**	2.0	0.5	-	-	0.154387	0.873981
1.0	1.0	0.1	2.0	**1.0**	0.5	-	-	0.217459	0.581646
1.0	1.0	0.1	2.0	**2.0**	0.5	-	-	0.155282	0.778019
1.0	1.0	0.1	2.0	**4.0**	0.5	-	-	0.132542	1.04326
1.0	1.0	0.1	2.0	2.0	**-0.5**	-	-	1.04143	1.26752
1.0	1.0	0.1	2.0	2.0	**0.0**	-	-	0.702494	1.04151
1.0	1.0	0.1	2.0	2.0	**0.5**	-	-	0.155282	0.778019

CONCLUSION

The MHD boundary layer flow of Casson fluid over a stretching sheet in the presence of transverse magnetic field has been investigated. The main findings of the present study can be summarized as follows:

- Momentum boundary layer thickness increases at $A=2.0$, and decreases at $A=0.1$ with the increase in values of Casson parameter, magnetic field parameter and the porous medium parameter.
- The thermal boundary layer thickness at $A=0.1$ increases with the increase in values of Casson parameter, magnetic field parameter, the porous medium parameter and the heat generation parameter, and decreases with the increase in values of the velocity ratio parameter.
- The Skin-friction at surface increases at $A=0.1$ and decreases at $A=2.0$ with an increasing in Casson parameter, and vice versa with the increase of each of magnetic field and porous medium parameters.
- The Nusselt number decreases at $A=0.1$, and increases at $A=2.0$ with the increase of Casson parameter, magnetic field parameter and the porous medium parameter.

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