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Heat Transfer to Magnetohydrodynamic Flow in a Horizontal Channel with Constant Heat Flux

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ABSTRACT

Unsteady flow of an electrically conducting viscous incompressible fluid with small electrical conductivity and electromagnetic force in a horizontal channel under the influence of constant head flux is investigated. Approximate solutions for the velocity and temperature fields as well as heat transfer co-efficient have been derived. Results obtained are discussed with the help of graphs and tables.

Key words: MHD flow, heat transfer, horizontal channel, magnetic field, constant heat flux

INTRODUCTION

Radiative heat transfer flow is very important in manufacturing industries, nuclear power plants, gas turbines and various propulsion devices for aircrafts, missiles, satellites and space vehicles. In the light of these various applications Soundalgekar and Takhar [1] Studied radiative free convective flow past a semi-infinite vertical plate. Hossain and Takhar [2] analyzed the effect of radiation on mixed convection along a vertical plate with uniform plate temperature. Raptis and Perdikis [3] analyzed the effect of thermal radiation and free convection flow past a moving plate.

In many industrial applications, it is found that an infinite vertical plate oscillating in its own plane receives heat at constant rate and hence this study was presented by Soundalgekar and Patil [4] for ordinary fluid. However, in many applications, such as in nuclear engineering, the presence of applied transverse magnetic field plays an important role. Hence Soundalgekar [5] studied the effect of transversely applied uniform magnetic field on the flow past an infinite vertical oscillating isothermal plate. Soundalgekar et al [6] studied the effects of constant heat flux on the flow of an electrically conducting fluid under transversely applied uniform magnetic field past an infinite vertical plate oscillating in its own plane. Free convection effects on flow past an impulsively started semi-infinite inclined plate with constant heat flux was studied by Ganeshan et al [7]. Natural convection flow past an impulsively started vertical plate with variable heat flux was discussed by Muthukumarswami [8]. Chaudhary and Gupta [9] analyzed the unsteady mixed convective MHD flow past a vertical porous oscillating plate with variable suction and constant heat flux.

During the past few decades the study of MHD flows has stimulated considerable interest due to its applications in cosmic fluid dynamics, meteorology, Solar physics and in the motion of Earth's core as studied by Cramer and pai [10]. In a broad sense, MHD flows have applications in three different subject areas such as Astrophysics, Geophysics and Engineering problems. In the light of these applications, MHD flows in a channel has been studied by many authors. Some of them are Nigam and Singh [11], Soundalgekar and Bhat [12], Vajravelu [13] and Attia and Kotb [14]. Bodosa and Borkakati [15] studied the MHD flow and heat transfer of a visco-elastic fluid past between two horizontal plates with heat source or sinks. Chaudhary et al [16] have studied the Ohmic dissipation effect on unsteady flow of a viscous, incompressible and electrically conducting fluid through a channel filled with porous medium under the influence of transverse magnetic field and heat source. Deka and Deka [17] have investigated the heat transfer characteristics of a viscous incompressible and electrically conducting fluid through a horizontal channel bounded by two long vertical parallel porous plates at constant temperature in presence of heat source, suction and a uniform transverse magnetic field.

The purpose of this investigation is to study the heat transfer effects on flow of an electrically conducting viscous incompressible fluid with small electrical conductivity and electromagnetic force in a horizontal channel under the influence of constant heat flux.

MATHEMATICAL ANALYSIS

We consider the unsteady flow of an electrically conducting viscous incompressible fluid in a channel bounded by two long vertical parallel plates. The x-axis is taken along the vertical plate (placed at y=0) in vertically upward direction and the y-axis is taken normal to it. A transverse magnetic field of uniform strength B_0 is applied along the y-axis. Also the whole system is kept under the influence of constant heat flux.

It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. Then under Boussinesq's incompressible fluid model, the flow can be shown to be governed by the equations:

$$\frac{\partial u}{\partial t}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2 u}{\rho} \tag{1}$$

$$\frac{\partial T}{\partial t}=\frac{k}{\rho Cp}\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{Cp}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B_0^2 u^2}{\rho Cp}+Q\left((T-T_0\right) \tag{2}$$

With the boundary conditions

$$\left.\begin{aligned} u=0,\ \frac{\partial\theta}{\partial T}=-\frac{q}{K} \quad &\text{at} \quad y=0 \\ u=0,\ T=T_w \quad &\text{at} \quad y=a \end{aligned}\right\} \tag{3}$$

Here u is the fluid velocity in the x-direction, t the time, T the fluid temperature, P the pressure, ρ the fluid density, σ_e the electrical conductivity of the fluid, B_0 $(=\mu_e H_0)$ the electromagnetic induction, μ_e the magnetic permeability, H_0 the intensity of the magnetic field, K the thermal conductivity, q the constant surface heat flux, Cp the specific heat at constant pressure, T_0 and T_w are the wall temperatures, a the width of the channel and Q the volumetric rate of heat generation.

Now let us introduce the following non-dimensional quantities:

$$\bar{x}=\frac{x}{a},\ \bar{y}=\frac{y}{a},\ \bar{\theta}=\frac{T-T_0}{qa/K},\ E=\frac{K\nu^2}{q\,Cp\,a^3},\ \bar{t}=\frac{t\nu}{a^2}$$

$$\bar{u}=\frac{ua}{\nu},\ \bar{P}=\frac{Pa^2}{\rho\nu^2},\ M^2=\frac{\sigma B_0^2 a^2}{\rho\nu},\ Pr=\frac{\rho\nu Cp}{K},\ \alpha=\frac{Qa^2}{\nu}$$

Putting these in equations (1), (2) and (3) we get the following dimensionless governing equations:

$$\frac{\partial u}{\partial t}=-\frac{\partial P}{\partial x}+\frac{\partial^2 u}{\partial y^2}-M^2 u \tag{4}$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2}+E\left(\frac{\partial u}{\partial y}\right)^2+EM^2u^2+\alpha\theta \qquad \text{(neglecting the bars for clarity)} \tag{5}$$

The corresponding boundary conditions in non-dimensional form are

$$\left.\begin{aligned} u=0,\quad \frac{d\theta}{dy}=-1 \qquad &\text{on} \quad y=0 \\ u=0,\quad \theta=1 \qquad &\text{on} \quad y=1 \end{aligned}\right\} \tag{6}$$

In order to solve equations (4) and (5) following [Chaudhary et al, 2006] let

$$-\frac{\partial P}{\partial x}=\lambda e^{i\omega t},\ u(y,t)=u_0(y)e^{i\omega t},\ \theta(y,t)=\theta_0(y)e^{i\omega t}$$

where λ is constant and ω is the frequency of oscillation.
Then

$$u(y,t)=\left[\frac{\lambda}{m_2^2}\frac{\sinh(m_2 y)}{\sinh(m_2)}\left((\cosh(m_2)-1)\right)+\frac{\lambda}{m_2^2}\left(1-\cosh(m_2 y)\right)\right]e^{i\omega t} \tag{7}$$

$$\theta(y,t)=\left[\frac{\cos(m_1 y)}{\cos(m_1)}\left(e^{-i\omega t}+\frac{\sin(m_1)}{m_1}A_0+e^{i\omega t}EPrA_1\right)-\frac{\sin(m_1 y)}{m_1}A_0-e^{i\omega t}EPrA(y)\right]e^{i\omega t} \tag{8}$$

where $m_1=(\alpha-i\omega)Pr,\ m_2=\sqrt{M^2+i\omega},\ k_1=\frac{\lambda^2}{m_2^2},\ k_2=\frac{\cosh(m_2)-1}{\sinh(m_2)}$

$$A_0 = e^{-i\omega t} + e^{i\omega t} E Pr\left(\frac{2k_1k_2m_2}{m_1^2+4m_2^2} + \frac{M^2}{m_2^2}\frac{2k_1k_2m_2}{m_1^2+m_2^2} - \frac{M^2}{m_2^2}\frac{2k_1k_2m_2}{m_1^2+4m_2^2} \right)$$

$$A_1 = \frac{k_1k_2^2}{2m_1^2} + \frac{k_1k_2^2\cosh(2m_2)}{2(m_1^2+4m_2^2)} - \frac{k_1}{2m_1^2} + \frac{k_1\cosh(2m_2)}{2(m_1^2+4m_2^2)} - \frac{k_1k_2\sinh(2m_2)}{m_1^2+4m_2^2} + \frac{M^2k_1}{m_2^2}\left(\frac{k_2^2}{2m_1^2} - \frac{k_2^2\cosh(2m_2)}{2(m_1^2+4m_2^2)} - \frac{1}{m_1^2} - \frac{1}{2m_1^2} - \frac{\cosh(2m_2)}{2(m_1^2+4m_2^2)} + \frac{2\cosh(m_2)}{m_1^2+m_2^2} - \frac{2k_2\sinh(m_2)}{m_1^2+m_2^2} + \frac{k_2\sinh(2m_2)}{m_1^2+4m_2^2} \right)$$

$$A(y) = \frac{k_1k_2^2}{2m_1^2} + \frac{k_1k_2^2\cosh(2m_2y)}{2(m_1^2+4m_2^2)} - \frac{k_1}{2m_1^2} + \frac{k_1\cosh(2m_2y)}{2(m_1^2+4m_2^2)} - \frac{k_1k_2\sinh(2m_2y)}{m_1^2+4m_2^2} + \frac{M^2k_1}{m_2^2}\left(\frac{k_2^2}{2m_1^2} - \frac{k_2^2\cosh(2m_2y)}{2(m_1^2+4m_2^2)} - \frac{1}{m_1^2} - \frac{1}{2m_1^2} - \frac{\cosh(2m_2y)}{2(m_1^2+4m_2^2)} + \frac{2\cosh(m_2y)}{m_1^2+m_2^2} - \frac{2k_2\sinh(m_2y)}{m_1^2+m_2^2} + \frac{k_2\sinh(2m_2y)}{m_1^2+4m_2^2} \right)$$

RESULTS AND DISCUSSION

In order to have physical insight into the problem, numerical values of u and θ have been computed for different values of Pr, α, t, ω, M, λ and E, and these are plotted on graphs from Fig. 1 to Fig. 9.

Fig. 1 is the velocity profiles for different values of M (M= .1, .2,.4,.5,.8,1.0,2.0; ω=1, λ=1, t=.2). From this figure we observe that transient fluid velocity increases with an increase in the values of Hartmann number M. It is also observed that fluid velocity profile is parabolic with maximum magnitude along the channel centre line and minimum at walls.

Fig. 2 represents the velocity profiles for different values of ω, λ and t (ω=1, λ=1, t=.2; ω=1, λ=1, t=.4; ω=1, λ=1, t=1; ω=2, λ=1, t=.2; ω=10, λ=1, t=.2; ω=1, λ=2, t=.2; M=1). From this figure we see that fluid velocity increases with increasing values of ω and λ; but decreases with increasing values of t.

Fig. 3 to Fig. 9 are graphical representations of the temperature profiles for different values of the parameters involved. Fig. 3 is the temperature profiles for different values of Pr (Pr=.71, 1, 2, 7, 9, 10; α = 1, ω = 1, M = 1, λ = 1, t = .2, E = .01). It is noted from Fig.3 that fluid temperature decreases for increasing values of the Prandtl number Pr. Fig.3 also shows that temperature profile plummets for large values of Pr.

Fig. 4 represents the temperature profiles for different values of non-dimensional heat generation parameter α (α =0, .1, 1, 2, 5 ; Pr = .71, ω = 1, M = 1, λ = 1, t = .2, E = .01). From this figure it is seen that temperature decreases due to an increase in the values of α.

Fig. 5 is the temperature profiles for different values of M (M=0, .1, 1, 1.5, 2 ; Pr = .71, α = 1, ω = 1, t = .2, E = .01). From this figure we observe that temperature decreases when the values of the Hartmann number M increases. But this change in temperature is very low.

Fig. 6 is the graphic representation of temperature profile for different values of ω (ω=1, 2, 3, 5, 10 ; Pr = .71, α = 1, M = 1, t = .2, E = .01) It is seen from this figure that temperature decreases for increasing values of ω.

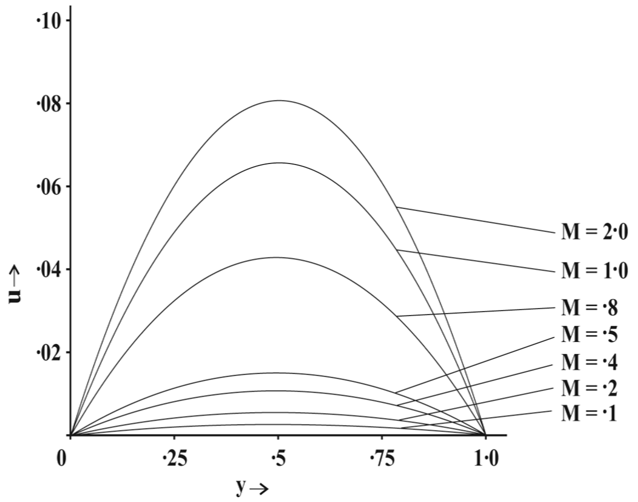

Fig. 1 Velocity Profiles for different M (ω=1, λ=1, t=.2)

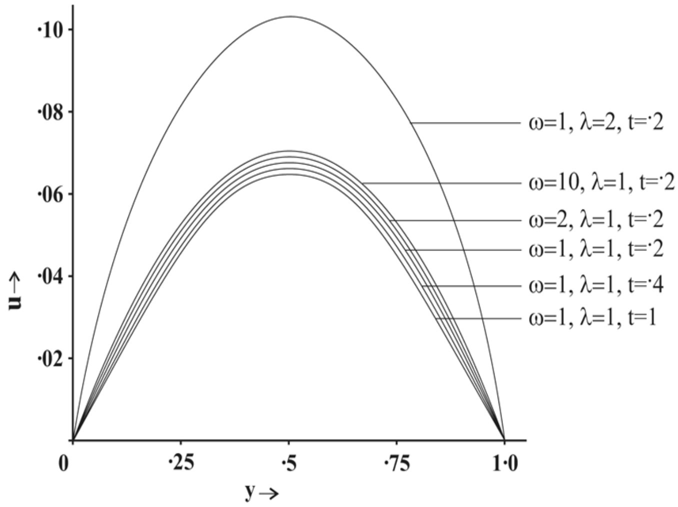

Fig. 2 Velocity Profiles for different ω, λ, t (M= 1)

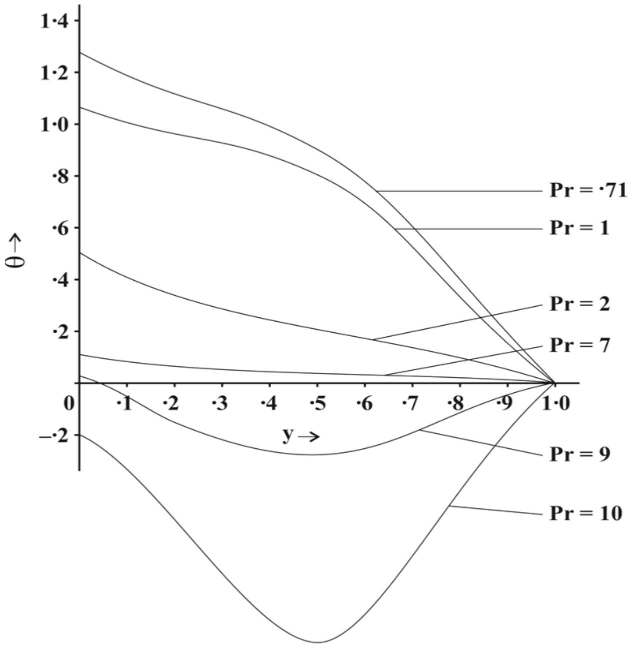

Fig. 3 Temperature profiles for different values of Pr
(α = 1, ω = 1, M = 1, λ = 1, t = 0.2, E = .01)

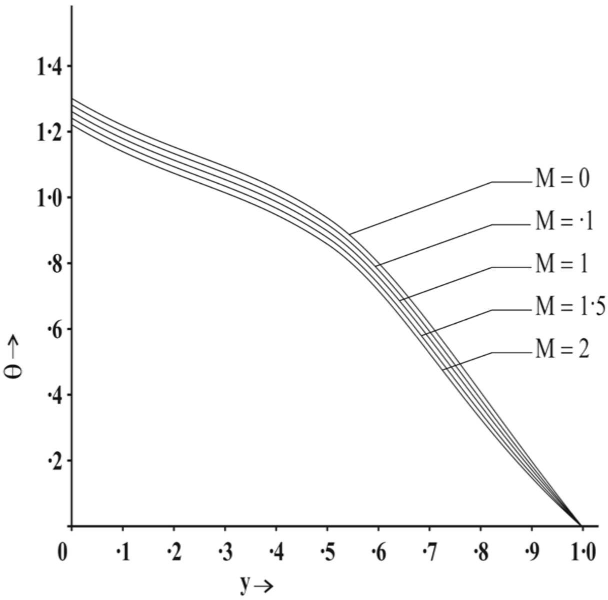

Fig. 5 Temperature profiles for different values of M
(Pr = .71, α = 1, ω = 1, t = .2, E = .01)

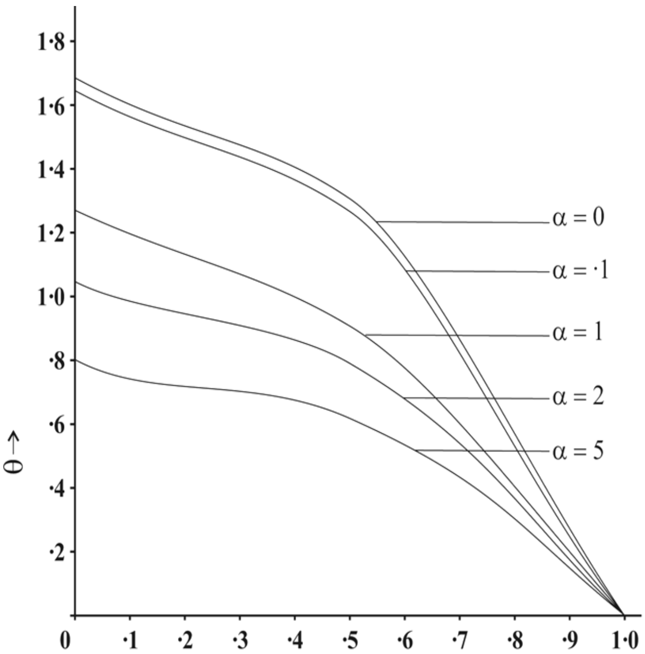

Fig. 4 Temperature profiles for different α
(Pr = .71, ω = 1, M = 1, λ = 1, t = .2, E = .01)

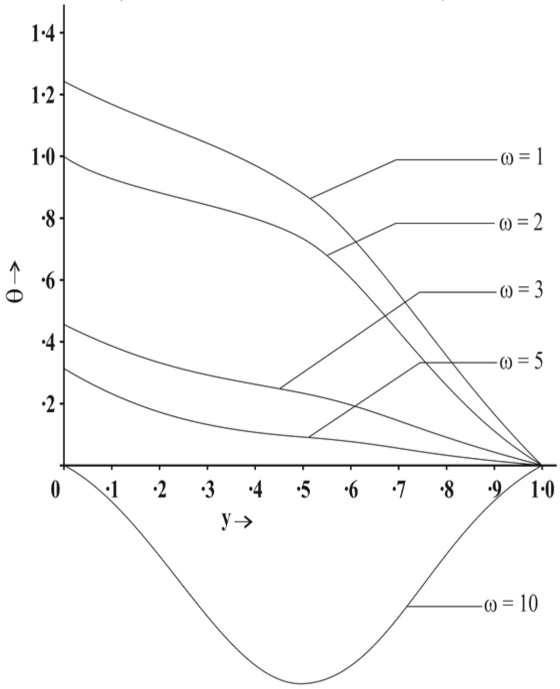

Fig. 6 Temperature profile for different values of ω
(Pr = .71, α = 1, M = 1, t = .2, E = .01)

Fig. 7 represents the temperature profiles for different values of λ (λ= 1, 2, 3, 4; Pr = .71, α = 1, ω = 1, M = 1, t = .2, E = .01). From this figure we observe that fluid temperature slowly increases for increasing values of λ .

Fig. 8 represents the temperature profiles for different values of t (t=0, .2, .4, .8, 1.0 ; Pr=.71, α= 1, ω=1, M=1, λ=1, E =.01). Fig. 8 shows that fluid temperature gradually decreases for increasing values of t. However this change in temperature is very low.

Fig. 9 is the temperature profiles for different values of E (E = 0, .01, .1, 1.0, 10.0; Pr=.71, α=1, ω=1, M=1, λ=1, t=.2). This figure shows that fluid temperature slightly increases for increasing values of E. However in this case, too, this change in temperature is very low.

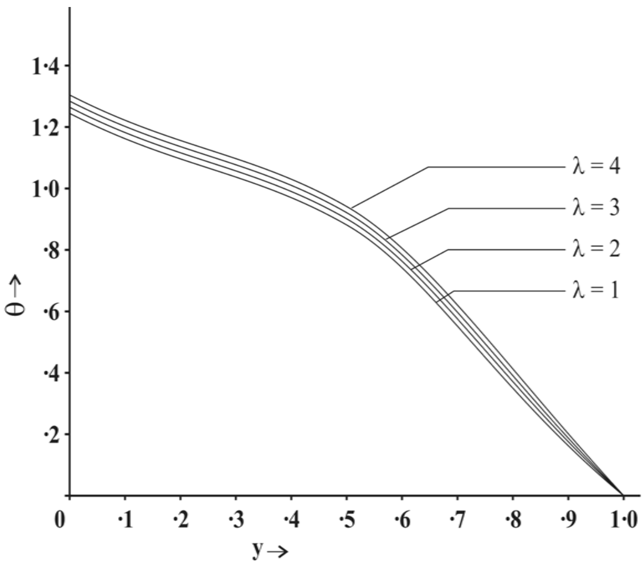

Fig. 7 Temperature profiles for different values of λ (Pr = .71, α = 1, ω = 1, M = 1, t = .2, E = .01)

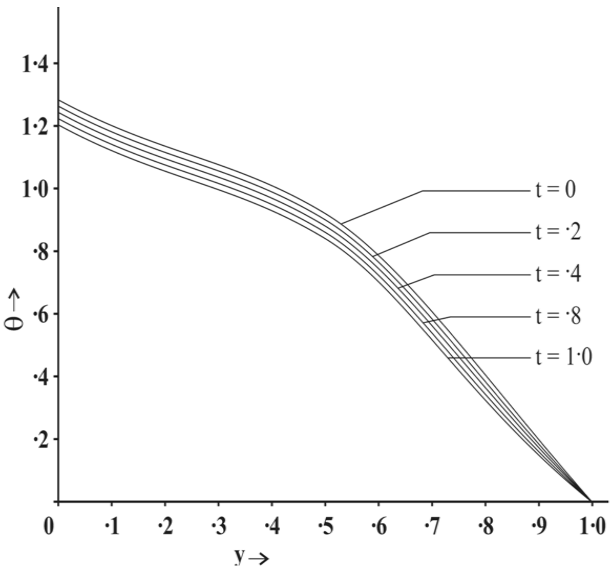

Fig. 8 Temperature profiles for different values of t (Pr=.71, α= 1, ω=1, M=1, λ=1, E =.01)

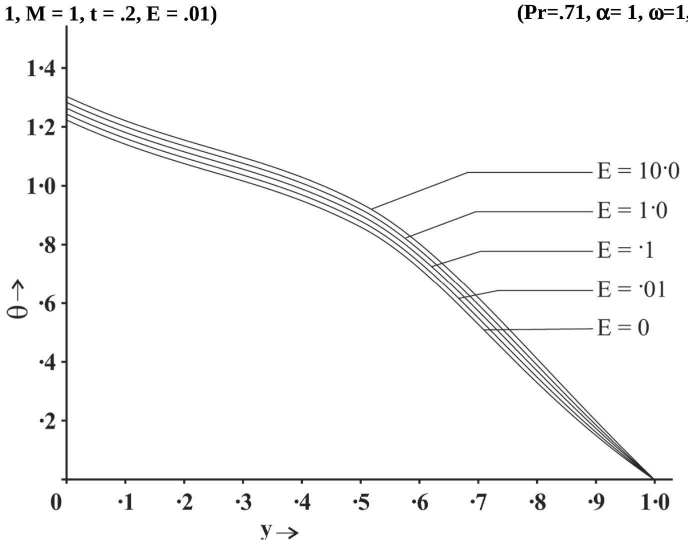

Fig. 9 Temperature profiles for different values of E (Pr=.71, α=1, ω=1, M=1, λ=1, t=.2)

HEAT TRANSFER COEFFICIENT

Having known the velocity and temperature field, we now study the heat transfer co-efficient (Nusselt Number) which is given by

$$Nu = -\left.\frac{\partial \theta}{\partial y}\right|_{y=0} = -\left.\frac{\partial \theta_0}{\partial y}\right|_{y=0} e^{i\omega t} = 2A_0 e^{i\omega t} - 1$$

$$\text{where} \quad A_0 = e^{-i\omega t} + e^{i\omega t} E Pr\left(\frac{2k_1k_2m_2}{m_1^2+4m_2^2} + \frac{M^2}{m_2^2}\frac{2k_1k_2m_2}{m_1^2+m_2^2} - \frac{M^2}{m_2^2}\frac{2k_1k_2m_2}{m_1^2+4m_2^2}\right)$$

We have computed the numerical values of Nu for different values of Pr, α, ω, M, t and E (taking λ=1) and these are listed in the following table.

From this table it is observed that heat transfer coefficient increases with an increase in the heat generation parameter α or time t or the Eckert number E; but decreases when Pr or M or ω increases.

CONCLUSIONS

The results of this paper may be summarized as follows:

- Fluid velocity increases when M or ω or λ increases, but decreases when t increases.
- Fluid temperature increases when E or λ increases, but decreases when Pr or α or M or ω or t increases.
- Heat transfer co-efficient increases when α or t or E increases, but decreases when Pr or M increases.

Table -1 Values of Nu (λ=1)

Pr	α	M	ω	T	E	Nu
·71	1	·1	1	·2	·01	1·0058121
·71	1	·5	1	·2	·01	1·0052410
·71	1	1	1	·2	·01	1·0049132
·71	1	1	1	·2	·1	1·0493116
·71	1	1	1	·2	1·0	1·4933122
·71	1	2	1	·2	·01	1·0026356
·71	0	1	1	·2	·01	·9965681
·71	·1	1	1	·2	·01	1·0032943
·71	2	1	1	·2	·01	1·0056274
7	1	1	1	·2	·01	1·0000453
10	1	1	1	·2	·01	1·0000431
20	1	1	1	·2	·01	1·0000276
·71	·1	2	1	·2	·01	1·0025173
·71	·1	1	2	·2	·01	·9981622
·71	1	1	2	·2	·01	1·0002825
·71	1	1	5	·2	·01	·9990203
·71	·1	2	1	·2	·1	1·0251393
·71	·1	2	1	·2	1·0	1·2713524
·71	·1	2	1	1·0	1·0	1·4913117
·71	1	1	1	1·0	·01	1·0052251
·71	1	1	1	1·0	1·0	1·5240391

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