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Model Reference Adaptive Controller of Glucose Insulin System

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ABSTRACT

In this paper one present an asymptotic output tracking algorithm based on direct model reference adaptive control procedure (DMRAC). The new algorithm is applied to control the level of the blood glucose which represents a nonlinear system and its regulation is a big challenge for decades. The regulation is able using the virtual linearization concept. Simulations examples are given to demonstrate the usefulness of the algorithm.

Key words: Glucose-insulin system, insulin, regulation, Lyapunov stability, positive real systems, model reference adaptive control, unmodelled dynamics

INTRODUCTION

The simple MRAC of MIMO plants was first proposed by Sobel et al [1]. This class of algorithms requires neither full state access nor satisfaction of the perfect model following conditions. Asymptotic stability is ensured provided that the plant is almost strictly positive real (ASPR). Barkana [2] extended the original algorithm to a class of plants which violates this condition. This approach involved designing a supplementary feed forward filter to be included in parallel with the original plant resulting in a new augmented plant which had to satisfy the same strictly positive real condition, unfortunately, the tracking error was not the true difference between the plant and the model outputs since it included the contribution of the supplementary feed forward filter which leads to an asymptotically stable error [3-6].

One of the major diseases in the Western world today is diabetes. Several million people suffer from the disease and the number is increasing. Culture is mainly due to the lifestyle in the western world, with lots of unhealthy food. Because it's a big problem, many researchers are trying to find ways to diagnose and treat disease. One approach is to design a mathematical model describing the glucose-insulin system. Diabetes is a malfunction of this system. These mathematical models can be used to diagnose, but also to create simulators to test various types of treatment. One of the mathematical models describing the glucose-insulin system with a small number of parameters is called minimal model of Bergman, it was introduced in the eighties [7]. This is the model that will be described and analyzed in this paper.

The Adaptive control is a robust approach used for uncertain linear and nonlinear systems. It takes a place increasingly important among the methods of controller synthesis. In this paper one synthesize a controller that is able to regulate the blood glucose by injecting an adequate quantity of insulin which represents the input to our system. One must also test it's robustness with regards to disturbance rejection.

DIRECT MODEL REFERENCE ADAPTIVE CONTROL

The model reference adaptive control is considered for the non-linear plant

$$\begin{aligned} \dot{x}_p(t) &= A_p(t)x_p(t) + B_p(t)u_p(t) + f(x_p) \\ y_p(t) &= C_p x_p(t) \end{aligned} \tag{1}$$

where $x_p(t)$ is the $(n\times1)$ state vector, $u_p(t)$ is the $(m\times1)$ control vector, $y_p(t)$ is the $(q\times1)$ plant output vector, $f(x)$ is an $(n\times1)$ vector of nonlinearities and A_p , B_p are matrices with appropriate dimensions. We assume that the parameters of the linear part of the plant model are uncertain, i.e., only known within certain finite bounds. The range of the plant parameters is assumed to be known and bounded with

$$\underset{-ij}{a} \le a_p(i,j) \le \bar{a}_{ij}, i,j = 1,...,n \quad \& \quad \underset{-ij}{b} \le b_p(i,j) \le \bar{b}_{ij}, i,j = 1,...,n \tag{2 -3}$$

Assumption 1

The non-linear function $f(x)$ is Lipschitz that means $\left| f(X_1) - f(X_2) \right| < L \left| X_1 - X_2 \right|$ where $L > 0$ is the constant of Lipschitz, $|(.)|$ is the Euclidean norm and X_1, X_2 belongs to a compact set $\Omega \in R^n$.

The objective of this paper is to find, without explicit knowledge of A_p, B_p and the non-linearity $f(x_p)$, the control $u_p(t)$ such that the plant output vector $y_p(t)$ follows the reference model given by:

$$\dot{x}_m(t) = A_m x_m(t) + B_m(t) u_m \quad \text{and} \quad y_m(t) = C_m x_m(t) \tag{4}$$

The output y_m is the desired response to the set point command u_m. The model incorporates the desired behaviour of the plant, but its choice is not restricted. In particular, the order of the plant may be much larger than the order of the reference model. The ideal control law that generates perfect output tracking and ideal state trajectories is assumed to be a linear combination of the model states and model input, i.e [8].

$$\begin{bmatrix} x_p^* \\ u_p^* \end{bmatrix} = \begin{bmatrix} S_{11}(t) & S_{12}(t) \\ S_{21}(t) & S_{22}(t) \end{bmatrix} \begin{bmatrix} x_m(t) \\ u_m(t) \end{bmatrix} \tag{5}$$

Where the $S_{ij}(t)$ matrices satisfy

$$\begin{aligned} S_{11} A_m + \dot{S}_{11} &= A_p S_{11} + B_p S_{21} \\ S_{11} B_m + \dot{S}_{12} &= A_p S_{12} + B_p S_{22} \\ C_p S_{11} &= C_m \\ C_p S_{12} &= 0 \end{aligned} \tag{6}$$

Then the adaptive control law based on the command generator tracker (CGT) approach is given as [9-10].

$$u_p(t) = K_e(t) e_y(t) + K_x(t) x_m(t) + K_u(t) u_m(t) \tag{7}$$

Note that adaptive law (7) has been applied for linear system and one try to extend it to non-linear system described by (1). The tracking error is given by $e_y(t) = y_m(t) - y_p(t)$ and $K_e(t)$, $K_x(t)$ and $K_u(t)$ are adaptive gains and concatenated into the matrix $K(t)$ as

$$K(t) = \left[K_e(t) \ K_x(t) \ K_u(t) \right] \tag{8}$$

Defining the vector $r(t)(n_r \times 1)$ as: $r(t) = \left[(y_m(t) - y_p(t))^T \ x_m^T(t) \ u_m^T(t) \right]^T$ (9)

The control $u_p(t)$ is written in a compact form as

$$u_p(t) = K(t) r(t) \tag{10}$$

where

$$K(t) = K_p(t) + K_i(t) \tag{11}$$

$$K_p(t) = \left[y_m(t) - y_p(t) \right] r^T(t) T_p, T_p \ge 0 \tag{12}$$

$$\dot{K}_i(t) = \left[y_m(t) - y_p(t) \right] r^T(t) T_i, \ T_i > 0 \tag{13}$$

STUDY OF THE STABILITY

The first step of the demonstration is to design a positive definite quadratic form in the state variables $e_x(t)$ and $K_I(t)$ of the adaptive system. Before doing this, it is assumed that T_i^{-1} is a symmetric positive definite matrix. Then an appropriate choice of the Lyapunov function is:

$$V = e_x^T P(t) e_x + Tr\left[S(K_I - \tilde{K}) T_i^{-1} (K_I - \tilde{K})^T S^T \right] \tag{14}$$

where Tr : represents the trace of a matrix

It's time derivative is: $\dot{V} = \dot{e}_x^T P e_x + e_x^T P \dot{e}_x + e_x^T \dot{P}(t) e_x + 2Tr\left[S(K_I - \tilde{K}) T_i^{-1} \dot{K}_I S^T \right]$ (15)

Where $P(t)$ is a symmetric positive definite matrix of size $n \times n$, $\tilde{K}$ is a matrix of dimension $m \times n_r$ and S is a non-singular matrix of dimension $m \times m$.

Since the matrix $\tilde{K}$ appears only in the function V and not in the control algorithm, it is called fictitious gain matrix, it has the same dimension as K where

$$\tilde{K} r = \tilde{K}_e C_p e_x + \tilde{K}_u u_m + \tilde{K}_x x_m \tag{16}$$

And the three gains $\tilde{K}_x$, $\tilde{K}_u$ and $\tilde{K}_e$ are as $\tilde{K}$ fictitious. Then we take the equation of the error using the fact that for $e_x = x_p^* - x_p$ to find

$$\begin{aligned} \dot{e}_x &= A_p x_p^* + B_p u_p^* + f(x_p^*) - A_p x_p - B_p u_p - f(x_p) \\ &= A_p \left[x_p^* - x_p\right] + B_p \left[u_p^* - u_p\right] + f(x_p^*) - f(x_p) \\ &= A_p e_x + B_p \left[u_p^* - u_p\right] + f(x_p^*) - f(x_p) \end{aligned} \tag{17}$$

If we set $df = f(x_p^*) - f(x_p)$

Substituting u_p^* from (5) and u_p from (7), one gets:

$$e_x = A_p e_x + B_p \left[S_{21} x_m + S_{22} u_m - K_X x_m - K_u u_m - K_e C_p e_X\right] + df \tag{18.a}$$

$$= A_p e_x + B_p \left[S_{21} x_m + S_{22} u_m - K_I r - C_p e_x r^T T_p r\right] + df \tag{18.b}$$

Then the adaptive system is described by:

$$\dot{e}_x = A_p e_x + B_p \left[S_{21} x_m + S_{22} u_m - K_I r - C_p e_x r^T T_p r\right] + df \tag{19}$$

$$\dot{K}_I = C_p e_x r^T T_i \tag{20}$$

Introducing (19) and (20) in (15), one gets:

$$\begin{aligned} \dot{V} &= \left[A_p e_x + B_p (S_{21} x_m + S_{22} u_m - K_I r - C_p e_x r^T T_p r\right]^T P e_x \\ &+ e_x^T P \left[A_p e_x + B_p (S_{21} x_m + S_{22} u_m - K_I r - C_p e_x r^T T_p r\right] \\ &+ e_x^T \dot{P}(t) e_x + 2Tr\left[S(K_I - \tilde{K}) T_i^{-1} (C_p e_x r^T T_i)^T S^T \right] + df \end{aligned} \tag{21}$$

We can write it as:

$$\begin{aligned} \dot{V} &= e_x^T A_p^T P e_x + (x_m^T S_{21}^T B_p^T + u_m^T S_{22}^T B_p^T - r^T K_I^T B_p^T - r^T T_p^T r e_x^T C_p^T B_p^T) P e_x + e_x^T P A_p e_x + e_x^T P B_p (S_{21} x_m + \\ & S_{22} u_m - K_I r - C_p e_x r^T T_p r) + e_x^T \dot{P}(t) e_x + 2Tr\left[S(K_I - \tilde{K}) T_i^{-1} T_i^T r e_x^T C_p^T S^T \right] + df \end{aligned} \tag{22}$$

Knowing that for two vectors $U(l,1)$ and $V(1,l)$ then $Tr[U.V] = V.U$ therefore

$$\begin{aligned} \dot{V} &= e_x^T (PA_p + A_p^T P) e_x + e_x^T P B_p S_{21} x_m + e_x^T P B_p S_{22} u_m - e_x^T P B_p K_I r - e_x^T P B_p C_p e_x r^T T_p r + x_m^T S_{21}^T B_p^T P e_x \\ &+ u_m^T S_{22}^T B_p^T P e_x - r^T K_I^T B_p^T P e_x - r^T T_p^T r e_x^T C_p^T B_p^T P e_x + 2 e_x^T C_p^T S^T S(K_I - \tilde{K}) r + e_x^T \dot{P}(t) e_x + df \end{aligned} \tag{23.a}$$

That means

$$\begin{aligned} \dot{V} &= e_x^T (PA_p + A_p^T P) e_x + 2 e_x^T P B_p (S_{21} x_m + S_{22} u_m) \\ &- 2 e_x^T P B_p C_p e_x r^T T_p r + 2 e_x^T \left[C_p^T S^T S - P B_p\right] K_I r - 2 e_x^T C_p^T S^T S \tilde{K} r + e_x^T \dot{P}(t) e_x + df \end{aligned} \tag{23.b}$$

By setting: $C_p = G B_p^T P \quad \forall A_p, B_p$ où $G = (S^T S)^{-1}$

The derivative of the Lyapunov function becomes:

$$\begin{aligned} \dot{V} &= e_x^T (PA_p + A_p^T) e_x + 2 e_x^T P B_p (S_{21} x_m + S_{22} u_m) \\ &- 2 e_x^T P B_p (S^T S)^{-1} B_p^T P e_x r^T T_p r - 2 e_x^T C_p^T S^T S \tilde{K} r + df \end{aligned} \tag{24}$$

Substituting $\tilde{K}_r = \tilde{K}_e C_p e_x + \tilde{K}_u u_m + \tilde{K}_x x_m$ in the previous equation, one get:

$$\dot{V} = e_x^T \left[P(A_p - B_p \tilde{K}_e C_p) + (A_p - B_p \tilde{K}_e C_p)^T P \right] e_x$$
$$- 2e_x^T PB_p (S^T S)^{-1} B_p^T P e_x r^T T_p r +$$
$$2e_x^T PB_p \left[(S_{21} - \tilde{K}_x) x_m + (S_{22} - \tilde{K}_u) u_m \right] + e_x^T \dot{P}(t) e_x + df \tag{25}$$

Thus, if we set $\left[(S_{21} - \tilde{K}_x) x_m + (S_{22} - \tilde{K}_u) u_m \right] = 0$ or $\tilde{K}_x = S_{21}$ and $\tilde{K}_u = S_{22}$ (none of which is required for implementation), the derivative of V becomes:

$$\dot{V} = e_x^T \left[\dot{P} + P(A_p - B_p \tilde{K} e\, C_p) + (A_p - B_p \tilde{K} e\, C_p)^T P \right] e_x - 2e_x^T PB_p (S^T S)^{-1} B_p^T P e_x r^T T_p r + df \tag{26}$$

This derivative consists of three terms. If T_p is a positive semi-definite matrix, then the second term is negative semi-definite in e_x^T. In the same manner, if the first quadratic term is negative definite in e_x^T that means there exist a matrix $Q = Q^T \geq 0$ so that $\left[\dot{P} + P(A_p - B_p \tilde{K}_e C_p) + (A_p - B_p \tilde{K}_e C_p)^T P \right] = -Q$

And taking into account that the third term df verifies the assumption (1), so, the derivative of the Lyapunov function verifies

$$\dot{V} \leq -e_x^T Q e_x + |df| \leq -e_x^T Q e_x + L\left|x - x^*\right| = -e_x^T Q e_x + L|e_x|$$
$$\leq -\lambda_{\min}(Q)|e_x|^2 + L|e_x| \leq 0 \Rightarrow |e_x| \geq \frac{L}{\lambda_{\min}(Q)}$$

Where $\lambda_{\min}$ stands for the lowest eigenvalue of Q which is a positive number since $|e_x| \geq L/\lambda_{\min}(Q)$, the vector $e_x(t)$ and the matrix $K_I(t)$ are bounded. We summarize the stability concept in the following theorem -

Theorem

The adaptive control given by (10) applied to the non-linear uncertain system (1) that verifies the assumption 1 leads to a ultimately stable error between the system and the model if and only if there exist two matrix $P(t) = P(t)^T > 0$ and $Q = Q^T \geq 0$ so that

$$\left[\dot{P} + P(A_p - B_p \tilde{K} e\, C_p) + (A_p - B_p \tilde{K} e\, C_p)^T P \right] = -Q$$
$$PB_p = (GC)^T \qquad T_p \geq 0, \quad T_i > 0\ , G = G^T > 0$$
$$\left| f(x_1) - f(x_2) \right| < L\left| x_1 - x_2 \right|, \;\; L > 0 \;\; x_1, x_2 \in R^2$$

In the following session we introduce the mathematical model of the Bergman Minimal Model.

DYNAMICS OF THE GLUCOSE INSULUN SYTEM

Bergman Minimal Model

There are many model of the glucose insulin system, the simple one is called minimal model of Bergman [7] described by the following equations

$$\dot{G}(t) = -p_1(G(t) - G_b) - X(t)G(t) + D(t) \tag{29}$$

$$\dot{X}(t) = -p_2 X(t) + p_3(I(t) - I_b) \tag{30}$$

$$\dot{I}(t) = -n(I(t) - I_b) + \gamma[G(t) - h]^+ t + u(t) \tag{31}$$

$D(t)$ is a disturbance that can be modeled by a decreasing exponential function of the following form: $D(t) = A\exp(-Bt), B > 0$, which represents

(1) The meals Fisher standards [11]. $B = 0.05$ (2) The effects of exercise [12]. $B = 0.11$

The description of the parameters and terms in equations (29-31) are given in the table -1.

Table -1Parameter Description and Terms of the Bergman Minimal Model

Parameter	Unit	Description
t	*min*	The time
$G(t)$	*mg/dl*	concentration of glucose in the blood
G_b	*mg/dl*	steady state concentration of glucose in the blood.
$X(t)$	*1/min*	the effect of active insulin.
$I(t)$	*µU/ml*	The concentration of insulin in the blood.
I_b	*µU/ml*	steady state concentration of insulin in the blood.
$I_2(t)$	*µU/ml*	active concentration of insulin
p_1	*1/min*	independent glucose disposal Speed insulin.
P_2	*1/min*	release rate of active insulin.
P_3	$(min^{-2})\ (\mu U/ml)^{-1}$	The increase in the ability to absorb caused by insulin
n	*1/min*	rate of prime insulin decrease in plasma
γ	$(\mu U/ml)\ min^{-2}\ (mg/dl)^{-1}$	release rate of insulin from pancreatic β-cells after glucose injection to the glucose concentration above the threshold
h	*mg/dl*	glucose threshold value which the pancreatic β – cells release insulin
$u(t)$	*µU/ml*	defines the injection of insulin and replaces the normal regulation of insulin of the body

This model can be used to simulate the glucose-insulin system for a type 1 diabetic on treatment. It can be used to test the predictive controller's models [13]. And as a tool in the search for an artificial pancreas. This model also adopts the problem with the minimal model of glucose.

Virtual Linearization of the Glucose Insulin System

In order to write the state space description of glucose insulin system in the form given by (1), one use the virtual linearization procedure [14] which is described below. The equation of the glucose insulin system can be written as

$$\begin{aligned}\dot{x} &= A(x) + B(x)u + f(x) + p = \frac{A_i(x)}{x_i}x_i + B_i(x)u + f_i(x) + p_i \\ &= A_i(t)x_i + B_i(t)u + f_i(x) + p_i, \quad i = 1..3\end{aligned} \tag{32}$$

Where $A_i(t) = \dfrac{A_i(x)}{x_i}$ with $x_i \xrightarrow{\lim} 0 \dfrac{A_i(x)}{x_i} < \infty$

Which is in the same form as in (1) without perturbation. Note that this way of rewriting the system does nothing but rearrange the terms in each equation so that when x and u are specified, the systems appears to be linear.

Simulation

In the simulation, it is required that the glucose concentration tracks its basal reference $G_b=70$ *mg/dl* by injecting the sufficient amount of insulin. Fig.ure 1 shows the evolution of the glucose concentration of a patient person with and without correction and one see that the controller is able to lead the glucose to its normal value. Fig.ure 2 shows the evolution of the insulin with and without correction which decreases and reach its equilibrium point. Fig.ure 3 shows controlled input which decreases and reach zero in steady state.

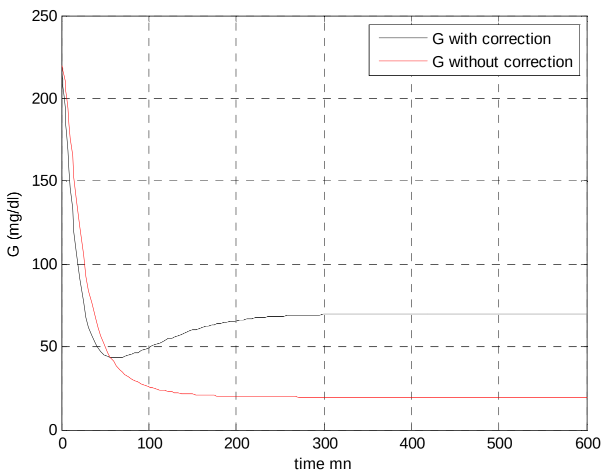

Fig. 1 Glucose concentration for a patient person with and without correction

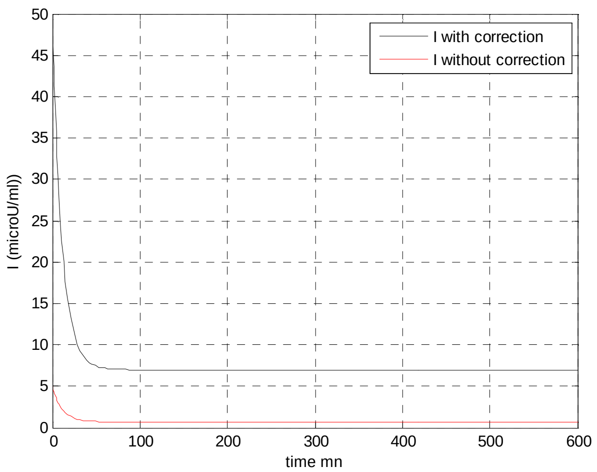

Fig. 2 Concentration of insulin in the blood

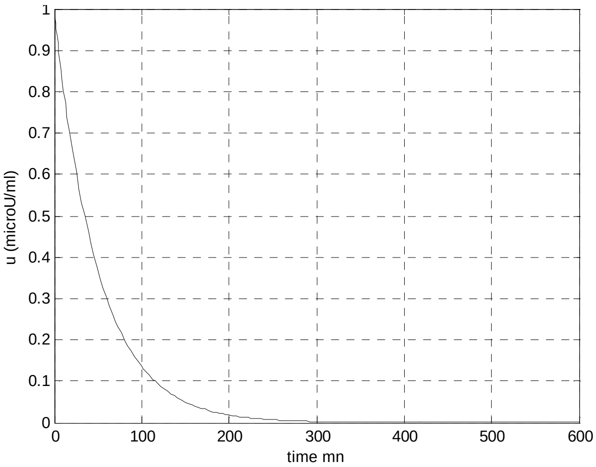

Fig. 3 Command (insulin injection) to the blood glucose insulin system

CONCLUSION

This paper presents the adaptive command which will be applied for a perturbed system. The Lyapunov theory has been addressed in order to achieve a robust command against the uncertainty which is inherent in all real system. The adaptive command has been applied to control the concentration of the glucose of a patient person. The simulation results confirm the robustness of the developed controller.

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