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Characteristic Polynomial for Detecting Isomorphism among 12-Link, 1-Freedom Simple Jointed Kinematic Chains (SJKCs)

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ABSTRACT

In this paper the complete collection of 12 link, 1-degree of freedom category of simple jointed kinematic chains (SJKCs) consisting of 6856 distinct chains are studied with an attempt to assess the efficacy of the characteristic polynomial based approaches using the link-adjacency, degree and structural matrix representations of kinematic chains for the detection of structural isomorphism amongst chains. All the cospectral pairs and triplets of kinematic chains where the isomorphism test failed to distinguish are reported in the form of a concise notation developed by the author for the purpose of documentation.

Key words: Isomorphism test, kinematic chain, cospectral chain, characteristic polynomial

INTRODUCTION

The problem of isomorphism in kinematics arises in the number synthesis of linkages. This problem usually involves generation of some kind of a unique identifier for every kinematic chain so that the task of comparing the structures of two chains for structural similarity is akin to comparing their respective unique identifiers.

In the area of number synthesis, a number of researchers have developed methodologies for enumerating the complete collection of SJKCs, mechanisms, multiple jointed chains, cam-modulated linkages, linkages possessing various types of kinematic links and pairs, and epicyclical gear transmission systems [1]. There are quite a few papers that have attempted synthesis of chains with more than 11 links. The author of this paper also has developed a method of synthesis in his doctoral thesis [2] and has derived chains with greater than 11 links. One of the results of the work is the complete collection of chain having 12-links and several degrees of freedom. In his work [2], the author has also developed a new isomorphism test based on representation-set of chain and has used it to derive all of the above categories of chains. In this paper, the author applies the characteristic polynomial based isomorphism tests on the entire collection of 12-link, 1-freedom chains to check the robustness of the tests.

The need for a reliable and computationally efficient isomorphism test arises in the structural synthesis of kinematic chains - the reason being, most synthesis methodologies available in the literature generate a large number of duplicate chains which have to be subsequently eliminated in order to tabulate the total number distinct chains possible in the given category. Many attempts [2-20] have been made in literature to develop reliable and computationally efficient tests for detecting structural isomorphism. These tests fall under four groups namely: 1) Characteristic polynomial based approaches, 2) Code-based approaches, 3) Hamming-number based approaches, and 4) Distance or Path based approaches.

Uicker and Raicu [3] were the first to investigate the properties of the characteristic polynomial of the adjacency matrix of a kinematic chain. Murthyunjaya and Raghavan [7] applied Bocher's formula for the determination of the characteristic coefficients and presented a counter example for the uniqueness of the characteristic polynomial and showed that polynomial is unique for closed and connected kinematic graphs. Yan and Hall [8] presented rules and theorems by which characteristic polynomial of kinematic chain and its coefficients are determined. Mruthyunjaya and Balasubramanian [4] worked on characteristic polynomial of a vertex-vertex degree matrix, and detected counter examples in the 10-link category of chains. Dubey and Rao [9] considered characteristic polynomial of distance matrix. Ambekar and Agrawal [10] proposed max-code and min-code methods for the detection of isomorphism. Kim and Kawk [12] proposed heuristic algorithm that uniquely labels the links of a chain which leads to a unique code. Shin and Krishnamurthy [13] presented the standard code theory for the detection of isomorphism. Rao [14] introduced the concept of Hamming distances from information and communication theory to the study of kinematic structure. Rao and Varadaraju [15] defined link hamming string as an index for testing isomorphism. Rao [16] illustrated a method by using chain Hamming matrix by which reliability of isomorphism test based on primary Hamming string is increased. Yan and Hwang [17] defined the linkage path code of a kinematic chain. Yadav et al [18] presented a sequential three-step test for isomorphism. Shende and Rao [19] proposed a method based on summation polynomials. Patil, et al [20] have proposed the Eigen values and Eigen vectors to identify isomorphic chains. They applied the technique to several isomorphic cases of chains to show that the effectiveness of the method in identifying kinematic chains.

CONTRIBUTIONS OF THIS WORK

In this paper, with an intention to check the effectiveness of characteristic polynomial based approaches to the isomorphism test, the complete collection of 6856 chains in the 12-link, 1-freedom category was employed to run the three popular tests based on the link adjacency matrix (Uicker, et al [3]), the degree matrix (Mruthyunjaya et al [4]) and finally the structural matrix (Yan et al [5]). A number of cospectral cases were detected during the exercise.

When characteristic polynomial of the link adjacency matrix was employed as the isomorphism test it is found that there were 15 triplets of cospectral SJKCs from among the 6856, 12 link, 1-freedom category. The 15 triplets are tabulated in Table-1. For the purpose of reporting the cospectral chains, a concise notation specifically developed for the purpose is used in this paper. The notation and the way to decode them are also explained in the paper. It is also found that there are 183 pairs of cospectral SJKCs among the 12 link, 1-freedom category. These are tabulated in Table-2. These findings show that the test is not effective in capturing all the possible chain variants for categories having more than 9 links.

Table -1 Cospectral Triplets among 12-Link, 1-Freedom Chains

1 :	GKWe4252b81 ,	J+oI+G9+5+Z ,	J+o2+G8GA4o
2 :	aG8ae58+4+U ,	5+K1GK1Gj81 ,	6GOWuD++++p
3 :	OHW1+e8o4GX ,	SHe+Y+A+5+f ,	SHe+W+g844X
4 :	OHW1+e8o41X ,	OHW1+e8o4+f ,	OHW1+e8Ia1X
5 :	OHW12e82a+Z ,	O1W1Ge8n2K1 ,	O1W1Ge8n251
6 :	O1W1+g824af ,	O1W1+ef25+Z ,	SLe+W+A+44Z
7 :	O1W1+e9I51X ,	O1W1+e92KKX ,	SHe+W+AG4KX
8 :	O1W1+e9I44Z ,	O1W1+e92K4Z ,	O1W1+e8352f
9 :	3+C2Wg22WWZ ,	5+K1WO2I24f ,	5+K1GK2KA4X
10:	MGu+G4W4D+X ,	MGu+G4+4L+f ,	M4u+O4+4a+Z
11:	MGu+G4W45GX ,	MGu+G4+45Wf ,	MGu+G4W44KX
12:	XG2YgLGX++1 ,	+K1JW4I+9OX ,	248I1WQ424i
13:	H8YWG83L+GX ,	mWa+C253GG5 ,	CXC+I8X8e+K
14:	44GH2GaABC+ ,	4GH18I42+pa ,	GKXI4H+8Cu+
15:	64O8oCG+++U ,	2KGn9XM6+++ ,	+K+m91HXWp+

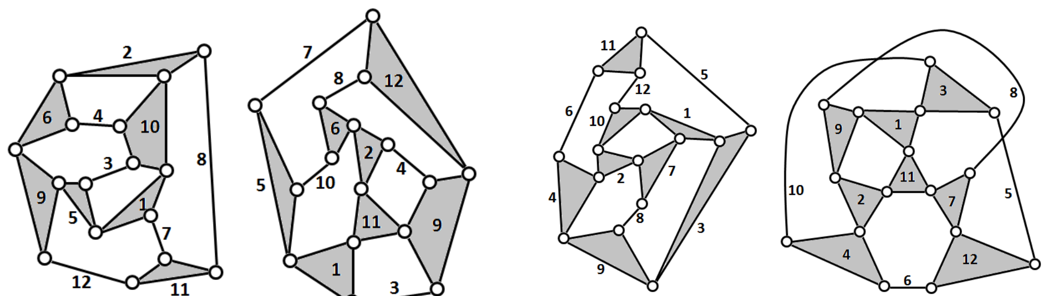
Table -2 Cospectral Pairs of 12-Link, 1-Freedom Chains

1:	QHmWI8K2++X ,	+I+m52f+c4X	62:	SHe+W+g+44f ,	M+u+G414aWZ	123:	C+m3W4+Ia8Z ,	1W622Wg1Y49
2:	G2WG429+aXp ,	+K1G51P+c4X	63:	S1f+Y+A+5+Z ,	SHe+Y+A+4Gc	124:	I1822S611+f ,	4GH1CIaA5+1
3:	JGo2+H8+42f ,	L+i+G4f+44f	64:	S1f+W+A+5Wf ,	i5849+44a8X	125:	I182+S6112f ,	4GH1CIa25+3
4:	JGo2+GeW4GX ,	J+o2AG8+a1X	65:	S1f+W+A+4af ,	S1eGW+A15+Z	126:	C+m3W4W35+f ,	248JW3H+CO4
5:	JGo2+Ge+aGX ,	J+o2AG8W41X	66:	S1e+e+AG5+f ,	T1e+A+8+aC9	127:	L4q1GK1+W+Z ,	M8u+Y8+1496
6:	JGoI+G8+5+Z ,	+m31Gma44Gc	67:	S1e+W+A15Wf ,	T1e+8+8G4CD	128:	3+C2a612G4f ,	1W6+A2gXA14
7:	JGo2+G8G5+f ,	C+m3Y4+2a8f	68:	S1eGW+A144f ,	T1e+8+8WKC3	129:	T1e+8+8W4i9 ,	i18K8+K4a8X
8:	JGo2+G8+52f ,	C4m3W4+2a8f	69:	S1eGW+A144Z ,	S1e+Y+A144Z	130:	O1W12e9+aD1 ,	O1W12e984C6
9:	JGo22GO+44X ,	KCf+e58+4GX	70:	S1e+Y+A144f ,	S1e+e+A+KGc	131:	KWo+8284a+p ,	K+m+3+nXca+
10:	JGo2+Ge+4KX ,	J+o2AG8+51X	71:	S1e+W+g+64f ,	S1e+8+C252Z	132:	H8ac+G114e9 ,	G2WE+G221iJ
11:	4I8f3Wm+2c+ ,	H8aWG455+f1	72:	S1e+W+AG64Z ,	T1e+8+884i5	133:	+e2cW61LW11 ,	84WJW31+j81
12:	J+oI8G8+5+Z ,	Z+I62G8+A8c	73:	O1W1+Wj244f ,	2WA2WeX+b83	134:	+e2cW61LW+3 ,	84WJW318b81
13:	J+o28G8+5WZ ,	i184A+K4aC1	74:	4eAWoC882+X ,	MGu+I4+a4+Z	135:	i5848+44aC9 ,	+i2aeX1B4+1
14:	J4o28G8+44Z ,	L+i+I4114Gc	75:	1W61Kb8G2+f ,	MGu+G4Wa4+Z	136:	i5848+44aC3 ,	+e2aeX1J4G1
15:	J+oI8G8+44Z ,	J+o2AG8+44Z	76:	58AY6H11411 ,	J+o2+GeW28Z	137:	i1848+a4aD1 ,	n8Y28I4AG+5
16:	J+o28G8G44Z ,	J+o2+G9+52f	77:	58AY4H114W9 ,	J+o2+GeW2OX	138:	i1848+aaC8X ,	i1848+c4WS1
17:	J+o28G8+46Z ,	L+i+G41H4Gc	78:	34C2Wg2Y++f ,	3+C2Wg2Y+2f	139:	i1848+aa49X ,	i1848+64Wi9
18:	J+o2+G9+5WZ ,	XG2YgL+X++9	79:	3+C2Wg2YG+Z ,	+C+p8H441C9	140:	i5848+64WC9 ,	O5W1+Y44a8Z
19:	J4o2+G8+64Z ,	Z+G62G9+4CE	80:	34C2Wg22W+f ,	L4i+G41144Z	141:	i5848+64WC3 ,	O1W1+Y4Ka8Z
20:	J+o2+G8G64Z ,	+A+h8Ga4Wg8	81:	3+C2Wg2IW+Z ,	+6+RW4Y+Hee	142:	i184A+64WC3 ,	i9841+53425
21:	KGf+a58G4+f ,	OHWH+eA+a1X	82:	3+C2Wg22W2f ,	L4i+G411C4X	143:	XG2YeL+n+11 ,	XG2YeL+X+X1
22:	L+i+O48+a2Z ,	L+i+G49+K4Z	83:	34C2Wg22+4Z ,	Z+I64G8+I8Z	144:	S1e+G81114p ,	4C8mf6H114+
23:	L+i+G4f+5+f ,	OHWH+eA+44Z	84:	+I19AYa2146 ,	+O1We64YWY9	145:	H8aWG455+A3 ,	4I98b5GGGY+
24:	ZGG6+GA+4af ,	Z4H6+GA+5+Z	85:	3+C2Yg22+4Z ,	Z+I64G8+I9X	146:	+e2YeK2DH+1 ,	HKb+G42gX+1
25:	Z+H6+GA+L+f ,	J+o22G8WA8X	86:	44GH2GaAA4K ,	+I1851KQQ4+	147:	Iaq+W48X249 ,	Iaq+W48X243
26:	Z+H6+GAG44Z ,	Z+G6+GB+52f	87:	3+C2WeZ2+4Z ,	+q31Gm4aC+X	148:	IWq+W48X2b1 ,	aYCKe51+W+I
27:	Z+G68Gg+5+Z ,	Z+G6+IA+4af	88:	5+K1YO32+4Z ,	+m31Gm4aK+Z	149:	IWq+W48X2a3 ,	J+o2+G1X42D
28:	Z+G6+IA+L+f ,	Z+G6+IA+K1c	89:	5+K1WO32G4f ,	+m3+I4aag41	150:	GGX2WO2I1S1 ,	GGX2WO2I1C9
29:	Z+G62IA+44f ,	Z+I64Ge828X	90:	5+K1WO32+af ,	+a2GmC+eg1K	151:	GGX2WO2I1C3 ,	3GKYOL+G++f

30:	+O1YKI14aW9	1W62KK45+GL	91:	3+C2WeY224Z	L+i+G41H45X	152:	9H326H424G1	9H326H42411
31:	KGeWa68+4+U	3GKYOL++++p	92:	L8iW828G489	T5e+8+8+CC6	153:	HCYWG8J5++X	i18+X4HGmYW
32:	XG24W46c+Wf	1W61Ie25+4Z	93:	MGu+K4W44+Z	1G52mn+6+2Z	154:	HCYWG8358+X	i18+e6+mX51
33:	K8ma2+A5981	O5W1+eB+5+f	94:	MGu+K4+K4+Z	1K52mn+6+1X	155:	H8YWI835+GX	L8i+82884A5
34:	OHW1+egW4+Z	O1W1+gAGa+f	95:	MGu+I5+44+Z	Kao+8284a+Z	156:	H8YWG835++p	L+g1+W1XWY9
35:	G6WQ+G225Ie	G6WQ+G115Ie	96:	MGu+G5+K4+Z	Kao+82O4a+X	157:	+K1J8GK4+P6	+I192mK4WgW
36:	OHW1+eA+q+Z	S1e+e+AGD+X	97:	MGu+G5+442f	M4u+G41444Z	158:	+A+h8Ga4+gC	Z+G62G9+4S6
37:	OHW12eA+44Z	O1W18eA+52f	98:	MGu+G5+442Z	Kao+8284a+f	159:	5+K1GK1+WYU	5+K1GK+GILK
38:	O1W18eA+4af	O1W12eA15+Z	99:	MGu+I4G45+X	MGu+I4+45+f	160:	58AY4H11+Y9	K4eG824YWHO
39:	OHWH+e8Y41X	OHWH+e8Y4+f	100:	MGu+I4+45GX	MGu+I4+44KX	161:	54AI2GW8BC+	K8eW8242+pm
40:	OHW12e8Y41X	OHW12e8Y4+f	101:	MGu+G4WC5+X	MGu+G4+K5+f	162:	1W6+51QXI28	L+i+G41H41b
41:	OHW1+eeY41X	OHW1+eeY4+f	102:	H8aYAI8+491	i18K8+44a8Z	163:	G6WQ4H66++e	GOXY+G+OTe+
42:	OHW1+e8Ia+Z	K+m+Y81Xc51	103:	M4u+O4+a4+Z	CnA2WO+W+2Z	164:	+A+hW3X+Se+	M+u41+242eb
43:	OHW1+ee25+f	T5e+8+8WCC1	104:	M+u+O4+a4XX	M+u+I4+464Z	165:	N+w+C3+++Y3	+O1Wg6aYWH+
44:	OHW1+e8I5GX	T1e+A+8W4S1	105:	M+u+O4W4a+f	T1e+8+e+aC3	166:	144G51N3H1+	+6+U+8eIa8Y
45:	OHW1+e82L+Z	S1f+Y+A+41c	106:	M+u+O4+Ka+f	T1e+8+e+aC9	167:	4O9Wa542+Y9	GOXWY4aYWH+
46:	OHW12e8244Z	O1W18e8252f	107:	M+u+O4+4aXX	M+u+O4+45XX	168:	i18a8+K44A5	i9848+64+OL
47:	OHW1+e8I4KX	SHe+W+g+4KX	108:	G6WQ4H4228e	GOXY+G4IGHO	169:	+K1KXX+QC8W	+C+qXX+BC2W
48:	O1W1Ge8n249	O1W1+ef25+f	109:	M+u+O4W45+f	H8ac+G1H483	170:	42GCYX+IK8e	i184C+64+OL
49:	O1W1Ge8X24J	S5e8W41+Y4X	110:	M+u+O4+K5GX	M+u+O4+K51X	171:	i184814C4A5	i1848+64WOL
50:	O5W18e8244f	L8iWA288481	111:	M+u+O4+K5+f	H8ac+G114A3	172:	i18481444Q5	+Y2CYX+JK+Y
51:	O1W18ee244f	O1W18ee24Gc	112:	M+u+O4+45Wf	M+u+O4+452f	173:	i18481444AD	i1848+6419L
52:	O1W18e8I4KX	SHeGW+A+4KX	113:	M+u+O4W444f	2K96W5+iW81	174:	4K9Gc5aA8++	O1W14Y+GIcA
53:	O1W18e8I45X	O1W1+ef+aC9	114:	M+u+O4+K44f	2G96W5+qW91	175:	4G924H+89x+	1G4WO51GmYA
54:	O1W18e82K4f	O1W12e98aC1	115:	M+u+G414aXX	M+u+G4+46aZ	176:	+O1Y8I4IGH6	+O1YeM+GGH5
55:	O1W1+g8I4KX	SLe+W+A+51X	116:	M+u+I4145+f	KWo+A284aGX	177:	5+C2GK1XWY9	+O1WQ9b2WH+
56:	O5W1+e925+Z	SHe+W+g+44Z	117:	M+u+G4145XX	M+u+G4+4M4Z	178:	M8u+X8+14A5	+O1YeM+GSG4
57:	O1W12e925+Z	SHeGW+A+44Z	118:	M+u+I41444f	KWo+A284i+X	179:	3KIoeL++++A	+m3+a92j8Y+
58:	O1W1+e9I5+Z	SHe+W+AG44Z	119:	M+u+G414KKX	M+u+G4+552f	180:	M8u+W8+14AD	+C+pW4o+9OW
59:	O1W1+e9252Z	SHe+W+A+4aZ	120:	2WA2WeY424Z	1G52aK2Cm81	181:	O1W14Y4IIL+	5+K1GK+WYPK
60:	SHe+Y+A+5+f	SHe+W+g844X	121:	5+K1GK2K24f	1G52aK24m89	182:	2KGn9XM6+++	+K+m91HXWp+
61:	SHe+W+g+5+Z	M+u+G414aWf	122:	C+m3W4W2a8Z	44GGmCQ1Y41	183:	54CIHKH114+	5GKXOL+GGH+

As the next step, characteristic polynomial of the degree matrix proposed in [4] and that of the structural matrix proposed in [5] was examined by applying it to the 12 link, 1-freedom chains. The intent was to check for existence of counter examples. It was found that there existed two pairs of chains wherein both the tests failed to distinguish the structural distinctiveness. The two chains are illustrated in Fig. 1. A very surprising discovery from this exercise is that both of the above two tests produced exactly identical results, in every sense. The two cases of counter-examples shown in Fig. 1 were common to both isomorphism tests. It may be noted that the size of structural matrix (which is equal to the sum of number of links and joints in the chain) is larger than that of the degree matrix for the same chain (which is the size of the link adjacency matrix). That is, for a 12-link, 1-freedom category the structural matrix is a square matrix of size 28 while it is 12 in the case of degree matrix. Although the degree matrix of a chain is easily derivable from its link adjacency matrix, the former contain some additional information in it. The latter, loaded with extra information, serves as the better starting point for the computation of the characteristic coefficients. It is due to this that the characteristic polynomial of degree matrix has been found to have fewer counter-examples.

In the same manner the structural matrix of the chain, although derivable from link-adjacency matrix, contains a lot more information about the chain in its initial form itself. The method, in fact, generates a matrix which is larger in size than what the representation requires. The objective was to pack as much information about the chain as possible into the matrix representation from which characteristic polynomial of the matrix is computed. The test based on this matrix is believed to have no counter-examples. The findings of the present work in the 12-link, 1-freedom category reveals that the degree matrix, despite its smaller size, is computationally as efficient as the method of characteristic polynomial suggested in [3], and as robust as that of structural matrix based method [5].



58AY4H11+Y9 K4eG824YWHO H8aY8I+GK83 GKXI4H+8Cu+

Fig. 1 Counter-examples for the degree matrix [3] and structural matrix [2] based approaches

A PROPOSAL – CONCISE NOTATION FOR REPORTING SJKCs

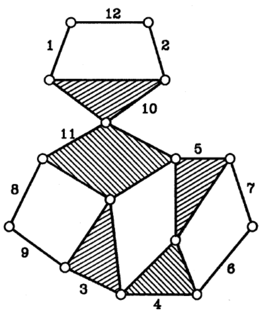
A compact notation is proposed in this paper to represent SJKCs. The notation helps in greatly reducing the space requirements for reporting the results. To illustrate the notation we consider the example of the 12-link, 3-d.o.f chain shown in Fig. 2 along with its link adjacency matrix. The compact notation for this chain is **+A+iAm4K+A4**. The letters included in the notation are evaluated by the following procedure.

Commencing from row-1 of the upper triangular link adjacency matrix (diagonal elements excluded), the matrix elements are first written row by row as a single binary string as follows:

000000001010000000101100001010110000000100010100000000001010000100

The above binary string is then divided into units containing six binary digits. A 12-link KC represented by 12×12 matrix will have 66 binary digits (=11×12÷2) leading to eleven six-bit-units as shown below.

[000000][001010][000000][101100][001010][110000][000100][010100][000000][001010][000100]



$$\begin{pmatrix} 0&0&0&0&0&0&0&0&0&1&0&1\\ 0&0&0&0&0&0&0&0&0&1&0&1\\ 0&0&0&1&0&0&0&0&1&0&1&0\\ 0&0&1&0&1&1&0&0&0&0&0&0\\ 0&0&0&1&0&0&1&0&0&0&1&0\\ 0&0&0&1&0&0&1&0&0&0&0&0\\ 0&0&0&0&1&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&1&0\\ 0&0&1&0&0&0&0&1&0&0&0&0\\ 1&1&0&0&0&0&0&0&0&0&1&0\\ 0&0&1&0&1&0&0&1&0&1&0&0\\ 1&1&0&0&0&0&0&0&0&0&0&0 \end{pmatrix}$$

Fig. 2 10-Link, 3-DOF SJKC and its Link-Adjacency Matrix

Let $x_1x_2x_3x_4x_5x_6$ represent a six-bit-unit where x_i (i = 1,2,…,6) are the six binary digits. From the x_i's we can generate an integer d_j as follows.

$$d_j = \sum_{i=1}^{6} x_i \times 2^{6-i} \quad (1)$$

Depending upon the six-bit-units x_i's, d_j can take any integer value from 0 (zero) through 63 ($2^6 - 1$). Knowing d_j the following substitution is carried out.

- If d_j = 0, substitute with the '+' character. This is done to avoid confusion between '0' (zero) with the alphabet 'O'.
- If d_j = 1, 2, 3, ..., 9, no substitution need be made.
- If d_j = 10,11,12,…, 35, substitute it with the upper case alphabets A, B, C,…, Z respectively.
- If d_j = 36, 37, 38 ..., 61, substitute it with the lower case alphabets a, b, c,…, y, z respectively.
- If d_j = 62, 63, substitute them with the characters '*', '=' respectively.

After the above substitutions, concatenate d_j (j = 1, 2,..., 11) obtained from the eleven six-bit-units. The sequence of occurrence of d_j's is to be retained. The string consisting of eleven characters given above (**+A+iAm4K+A4**) provides information about the structure of kinematic chain in the condensed notation.

FORTRAN PROGRAM FOR DECODING THE NOTATION

```
!------ THIS PROGRAM READS THE STRING (For example: +A+iAm4K+A4) AND
!------ OUTPUTS THE LINK LABELS THAT ARE ADJACENT TO EACH OTHER
        INTEGER*4 M
        CHARACTER KCN*11, MATRIX*66, BINUM*6
        OPEN(UNIT=5,FILE='12LINK.INP',STATUS='OLD',ACCESS='SEQUENTIAL')
        READ (5,*) KCN                          ! THE STRING IS READ HERE
        DO I = 1,11
          M = ICHAR(KCN(I:I))
          IF(M == 43) THEN                      !if '+', set M to 0
            M = 0
          ELSEIF(M > 48 .AND. M < 58) THEN      !if '1-9' do not change
            M = M - 48
          ELSEIF(M > 64 .AND. M < 91) THEN      !if 'A-Z' set to '10-35' resp.
            M = M - 55
          ELSEIF(M > 96 .AND. M < 123) THEN     !if 'a-z' set to '36-61' resp.
            M = M - 61
          ELSEIF(M == 42) THEN                  !if '*' set to 62
            M = 62
          ELSEIF(M == 61) THEN                  !if '=' set to 63
            M = 63
          ENDIF
          BINUM = '000000'
          CALL TOMATRIX(M,BINUM)
          MATRIX(6*(I-1)+1:6*I) = BINUM(1:6)
        ENDDO
```

```
            IENT = 0
          DO I = 1,11
            DO J = I+1,12
              IENT = IENT + 1
              IF(MATRIX(IENT:IENT) == '1') THEN
                WRITE(*,*) I,J                ! OUTPUTS ADJACENT LINKS
              ENDIF
            ENDDO
          ENDDO
          STOP
          END
!$$$$$$ =======================================================
          SUBROUTINE TOMATRIX(M,BINUM)
          INTEGER*4 M, IDIV, IREM
          CHARACTER BINUM*6
          DO J = 1,6
            IDIV = M/2
            IREM = MOD(M,2)
            IF(IREM == 1) THEN
              BINUM(7-J:7-J)= '1'
            ELSE
              BINUM(7-J:7-J)= '0'
            ENDIF
            IF(IDIV /= 0) THEN
              M = IDIV
            ELSE
            RETURN
            ENDIF
          ENDDO
          END
```

CONCLUSION

It is found that there are altogether 228 (= 45+183) instances of counter-examples in the 12-link, 1-freedom category of chains where characteristic polynomial of link adjacency matrix based test fail to distinguish. This makes the test unsuitable for cases wherein the chain contains more than 10 links – since in these cases a large number of chain variants exist. However, the degree matrix and structural matrix based tests perform better in distinguishing chain structures probably due to the fact that they contain extra structural information about the chain. Only two pairs of isomorphic chains were detected in this category of chains, and the two counter-examples happen to be the same for both the tests!! This is a surprise finding from this exercise.

If the two chain structures (counter-examples) are compared through visual inspection, it is clearly evident that they are non-isomorphic. The disposition of polygonal links within the chain is different in both the cases. Isomorphism methods that rely on such techniques to differentiate between chains will quickly rule out any similarities in the chain pairs. The results of this work, reported in the form of a concise notation that can be decoded and sketched easily, can be employed as the cases to check against for any new isomorphism methods proposed in future.

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