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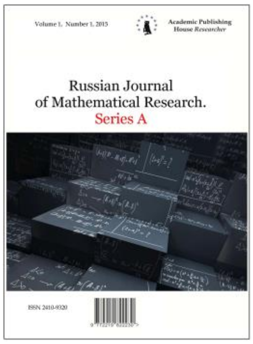


# Disproof of the Riemann Hypothesis

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**Abstract**

In this research Riemann hypothesis is investigated for a proof. A functional extension of the Riemann zeta function is proposed for which non trivial zeroes can be generated. It found that non trivial zeroes can also be generated outside the critical strip. Thus it is found that the Riemann hypothesis is found to be incomplete.

**Keywords:** disproof of Riemann hypothesis, Gamma function, number theory.

## 1. Introduction

In mathematics a Dirichlet series is any series of the form $\sum_{1}^{\infty}\frac{a_n}{n^s}$ where s is a complex number and $a_n$ is a complex sequence. These series play an important role in analytic number theory. The Riemann zeta function $\zeta(s)$, analytically continues the sum of the Dirichlet series. The Riemann zeta function is given by:

$$\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^s}$$

The Riemann zeta function also can be defined by the integral

$$\zeta(s)=\frac{1}{\Gamma(s)}\int_0^{\infty}\frac{1}{e^x-1}x^{s-1}dx \wedge \Gamma s=\int_0^{\infty}x^{s-1}e^{-x}dx$$

The Riemann's functional equation is given by:

$$\zeta(s)=2^s\pi^{s-1}\sin(\frac{\pi s}{2})\Gamma(1-s)\zeta(1-s)$$

The functional equation shows that the Riemann zeta function has trivial zeroes at negative even integer values of s. It is postulated that any non-trivial zeroes lie in the open strip $\{s\in Z:0<\mathrm{Re}(s)<1\}$ Riemann hypothesis asserts that for any non trivial zero s has $\mathrm{Re}(s)=\frac{1}{2}$ .

In 1914, Godfrey Harold Hardy proved that $\zeta(\frac{1}{2},it)$ has infinitely many zeroes.

Could it be possible that the Riemann zeta function could have infinitely many zeroes outside the critical strip? Could there be functional extensions of the Riemann zeta function that can generate non trivial zeroes?

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Many constructions of the gamma function have been proposed. The gamma function has been defined in the Euler integral of the second kind:

$$\Gamma s = \int_0^\infty x^{s-1} e^{-x} dx$$

The same gamma function has been defined in infinite products due to Euler and has been found to be valid for all complex numbers except for non-positive integers. The product definition of the gamma function is given by:

$$\Gamma(z) = \frac{1}{z} \prod_{n=1}^{\infty} \frac{(1+\frac{1}{n})z}{1+\frac{1}{z}}$$

The gamma function has been given a Weierstrass's definition. It has been expressed in terms of the generalized Laguerre polynomials. In terms of Euler's reflection formula the gamma function has been defined as:

$$\Gamma(1-z)\Gamma(z) = \frac{\pi}{\sin(\pi z)}$$

The Riemann zeta function should have functional extensions that can generate the same results and more as the original function. In this paper a product extension of the function is proposed for generating the non trivial zeroes of the Riemann zeta function.

## 2. Results

**Functional extensions of the Riemann zeta function that generates the non-trivial zeroes of the Riemann hypothesis**

The family of functions 1 in variable x and primes p below will be will be examined

$$f(x,p) = (x^2 + (p-x)^2)^{(1/2)} = x(1+\frac{(p-x)^2}{x^2})^{1/2}$$

1

The algebraic solution $f(x,p) = (x^2 + (p-x)^2)^{(1/2)} = 0$ will be examined.

$$f(x,p) = (x^2 + (p-x)^2)^{(1/2)} = 0 \rightarrow$$

$$2x^2 - 2px + p^2 = 0 \rightarrow$$

$$x = \frac{4p \pm \sqrt{4p^2 - 8p^2}}{4} = p \pm ip$$

2

An extension of the function in **Z** will be considered. There are many different ways representing such a function. One way is given in 3 below:

$$g(x,p) = (x^2 + (p-x)^2)^{(1/2)}(x^2 + (p-x)^2)^{mx}$$

3

There is a justification of integrating the function 3 in an extension of the Riemann –zeta function for non-trivial zeroes. This because:

$$g(0,p) = (0^2 + (p-0)^2)^{(1/2)}(0^2 + (p-0)^2)^{0.m} = p$$

4

The function 3 can therefore be adopted in the Riemann zeta function to obtain a possible extension that can be used to account for the non trivial zeroes of the Riemann zeta function.

$$\zeta(s)=\sum_{n=1}^{n=m}\frac{1}{n^s}=\prod_{j=1}^{j=\infty}\frac{1}{1-p_j^{-s}}$$

$$if: p_j^{\,s}(x,p_j)=(x^2+(p_j-x)^2)^{(1/2)}(x^2+(p_j-x)^2)^{imx}$$

$$\zeta(s,x)=\prod_{j=1}^{j=\infty}\frac{1}{1-(x^2+(p_j-x)^2)^{-(\frac{1}{2}+imx)}}$$

$$\zeta(s,x)=\prod_{j=1}^{j=\infty}\frac{(x^2+(p_j-x)^2)^{\frac{1}{2}}(x^2+(p_j-x)^2)^{imx}}{(x^2+(p_j-x)^2)^{\frac{1}{2}}(x^2+(p_j-x)^2)^{imx}-1}\rightarrow$$

$$\zeta(s,x)=\prod_{j=1}^{j=\infty}\frac{(x^2+(p_j-x)^2)^{\frac{1}{2}+imx}}{(x^2+(p_j-x)^2)^{\frac{1}{2}+imx}-1}\rightarrow\zeta(s,0)=\prod_{i=1}^{i=\infty}\frac{p_j}{p_j-1}$$

$$\zeta(s,x)=0\rightarrow(x^2+(p_j-x)^2)^{\frac{1}{2}}(x^2+(p_j-x)^2)^{imx}=0$$

$$\rightarrow x=\frac{4p_j\pm\sqrt{4p_j^{\,2}-8pj^2}}{4}=p_j\pm ip_j$$

$$\rightarrow s=\tfrac{1}{2}\mp m(p_j+ip_j)$$

In the above function extension of the Riemann zeta function a trivial zero is obtained whenever $s=\tfrac{1}{2}\mp m(p_j+ip_j)$. The real part of s in this case is given by $\mathrm{Re}(s)=\tfrac{1}{2}\mp mp_j$

The function 3 above is a case of disproof of Riemann hypothesis.

In an paper entitled “An elegant and short proof of Riemann hypothesis” the author used the above function to come up with some special general case in which Riemann hypothesis could be verified. He showed that $\zeta(\tfrac{1}{2},it)$ has infinitely many zeroes. There are cases however (as in the above) in which Riemann hypothesis can be disproved. The Riemann zeta function can also have infinitely many non trivial zeroes outside the critical strip. Thus the hypothesis is disproved.

## 3. Conclusion

Non trivial zeroes can be generated outside the critical strip. Riemann hypothesis is incomplete.

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