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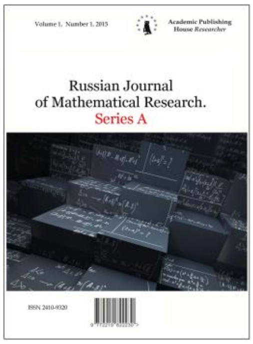

An Infinite Number of Ways of Algebraic Factorization of a Number and Radical Solution of Higher Degree Polynomial Equations

Samuel Bonaya Buya [a, *]

[a] Teacher Ngao Girls' secondary school, Kenya

Abstract

In this paper a method is proposed by which any number not equal to zero can have an infinite number of algebraic factorizations. A factorization method of radical solution of polynomial equations is then presented. The method provides radical solution of polynomial equations of degree than 4. As a demonstration the Bring-Jerrard quintic equation formula is derived by factorization.

Keywords: factorization of numbers; radical solution of higher degree polynomial equations; algebraic number theory.

1. Introduction

In number theory integer factorization is the decomposition of a composite number into a product of smaller integers. In this research factorization will be broadened to decomposition to algebraic numbers. The numbers that will be involved in factorization will include all categories of numbers with the exception of zero.

The factorization method has been used with some measure of success to solve some quadratic equations. This research examines the possibility of using the factorization method as a complete method of solving algebraic equations. This requires some re-examination at the process of factorizing numbers in general, symmetric equations in general and Galois Theory.

In this research a method of factorization will be presented in which a number can be factorized into n algebraic factors. The major aim of doing this is to first to make some contribution to algebraic number theory. As is quoted in Gauss sum Gedachtiniss (1856) by Sartorius von Waltershausen Variants: Mathematics is the queen of the sciences and number theory is the queen of mathematics.

In this research it will be shown that a number can be have infinite number of algebraic factors.

An identity will be proposed by which this can be achieved. This knowledge will be used to solve algebraic equations. It will also be used to demonstrate how we can easily obtain radical solution of the Bring-Jerrard quintic equation.

2. Methodology

Consider the number a, not equal to zero. The following identity can be used to factorize it:

* Corresponding author
E-mail addresses: sbonayab@gmail.com (S.B. Buya)

$$a=b^2-(b^2-a)=(b+\sqrt{(b^2-a)})(b-\sqrt{(b^2-a)})\vdots a,b\in \square$$

1

$$b+\sqrt{(b^2-a)}=c^2-(c^2-(b+\sqrt{(b^2-a)}))=(c+\sqrt{(c^2-(b+\sqrt{(b^2-a)}))}(c-\sqrt{(c^2-(b+\sqrt{(b^2-a)}))})$$

(2)

Take the case a = 1.

$$1=(2^2-(2^2-1)=(2+\sqrt{(2^2-1)})(2-\sqrt{(2^2-1)})$$

$$1=(3^2-(3^2-1)=(3+\sqrt{(3^2-1)})(3-\sqrt{(3^2-1)})$$

$$1=(101^2-(101^2-1)=(101+\sqrt{(101^2-1)})(101-\sqrt{(101^2-1)})\vdots$$

$$(101+\sqrt{(101^2-1)})=5^2-(5^2-(101+\sqrt{(101^2-1)})$$

$$=(5+\sqrt{5^2-(101+\sqrt{(101^2-1)}})(5-\sqrt{5^2-(101+\sqrt{(101^2-1)}})$$

$$=(7+\sqrt{7^2-(101+\sqrt{(101^2-1)}})(7-\sqrt{7^2-(101+\sqrt{(101^2-1)}}).....$$

In the above example 1 is divisible to $(2+\sqrt{(2^2-1)})$; $(2-\sqrt{(2^2-1)})$; $(3+\sqrt{(3^2-1)})$; $(101+\sqrt{(101^2-1)})$ and so on. The number 1 is divisible to infinite number of algebraic numbers.

From equation 1 the a number a is divisible to some infinite set of algebraic numbers, $b+\sqrt{(b^2-a)}$

The above identity 1 has application in solving polynomial equations

Example 1

Solve the quadratic equation $x^2+2x+6=0$ by factorization

Solution

$$6=(\tfrac{2}{2})^2-(-5)=(\tfrac{2}{2}+\sqrt{(-5)})(\tfrac{2}{2}-\sqrt{(-5)})$$

$$2=\tfrac{2}{2}+\tfrac{2}{2}$$

$$x_{1,2}=-(\tfrac{2}{2}\pm\sqrt{(-5)})$$

Example 2

Solve the cubic equation $x^3+2x+6=0$ by factorization

Solution

$$6=(\tfrac{2}{2})^2-(-5)=(\tfrac{2}{2}+\sqrt{(-5)})(\tfrac{2}{2}-\sqrt{(-5)})$$

$$(\tfrac{2}{2}+\sqrt{(-5)})=(\tfrac{2}{4})^2-((\tfrac{2}{4})^2-(\tfrac{2}{2}+\sqrt{(-5)})$$

$$=(\tfrac{2}{4}+\sqrt{((\tfrac{2}{4})^2-(\tfrac{2}{2}+\sqrt{(-5)})}(\tfrac{2}{4}-\sqrt{(\tfrac{2}{4})^2-(\tfrac{2}{2}+\sqrt{(-5)}})$$

$$6=(\tfrac{2}{4}+\sqrt{((\tfrac{2}{4})^2-(\tfrac{2}{2}+\sqrt{(-5)})}(\tfrac{2}{4}-\sqrt{(\tfrac{2}{4})^2-(\tfrac{2}{2}+\sqrt{(-5)}})(\tfrac{2}{4}-\sqrt{(\tfrac{2}{4})^2-(\tfrac{2}{2}+\sqrt{(-5)}})$$

$$x_{1,2}=(\tfrac{2}{4}\pm\sqrt{((\tfrac{2}{4})^2-(\tfrac{2}{2}+\sqrt{(-5)})}$$

$$x_3=\frac{6}{x_1x_2}$$

Example 4

Solve the Bring-Jerrard quintic equation $x^5+bx+c=0$

Solution

$$c=(\frac{b}{2})^2-((\frac{b}{2})^2-c)=(\frac{b}{2}+\sqrt{(\frac{b}{2})^2-c})(\frac{b}{2}-\sqrt{(\frac{b}{2})^2-c}):$$

$$(\frac{b}{2}+\sqrt{(\frac{b}{2})^2-c})=((\frac{b}{4})^2-((\frac{b}{4})^2-(\frac{b}{2}+\sqrt{(\frac{b}{2})^2-c}))$$

$$=(\frac{b}{4}+\sqrt{(\frac{b}{4})^2-(\frac{b}{2}+\sqrt{(\frac{b}{2})^2-c})}(\frac{b}{4}-\sqrt{(\frac{b}{4})^2-(\frac{b}{2}+\sqrt{(\frac{b}{2})^2-c})}):$$

$$\frac{b}{4}+\sqrt{(\frac{b}{4})^2-(\frac{b}{2}+\sqrt{(\frac{b}{2})^2-c}}=((\frac{b}{8})^2-((\frac{b}{8})^2-(\frac{b}{4}+\sqrt{(\frac{b}{4})^2-(\frac{b}{2}+\sqrt{(\frac{b}{2})^2-c})}))$$

$$=(\frac{b}{8}+((\frac{b}{8})^2-(\frac{b}{4}+\sqrt{(\frac{b}{4})^2-(\frac{b}{2}+\sqrt{(\frac{b}{2})^2-c})})))(\frac{b}{8}-((\frac{b}{8})^2-(\frac{b}{4}+\sqrt{(\frac{b}{4})^2-(\frac{b}{2}+\sqrt{(\frac{b}{2})^2-c})}))):$$

$$\frac{b}{8}+((\frac{b}{8})^2-(\frac{b}{4}+\sqrt{(\frac{b}{4})^2-(\frac{b}{2}+\sqrt{(\frac{b}{2})^2-c})}))$$

$$=((\frac{b}{16})^2-((\frac{b}{16})^2-(\frac{b}{8}+((\frac{b}{8})^2-(\frac{b}{4}+\sqrt{(\frac{b}{4})^2-(\frac{b}{2}+\sqrt{(\frac{b}{2})^2-c})}))))$$

$$=(\frac{b}{16}+\sqrt{(((\frac{b}{16})^2-(\frac{b}{8}+((\frac{b}{8})^2-(\frac{b}{4}+\sqrt{(\frac{b}{4})^2-(\frac{b}{2}+\sqrt{(\frac{b}{2})^2-c})}))))})\times$$

$$(\frac{b}{16}-\sqrt{(((\frac{b}{16})^2-(\frac{b}{8}+((\frac{b}{8})^2-(\frac{b}{4}+\sqrt{(\frac{b}{4})^2-(\frac{b}{2}+\sqrt{(\frac{b}{2})^2-c})}))))}):$$

$$x_1=-(\frac{b}{16}+\sqrt{(((\frac{b}{16})^2-(\frac{b}{8}+((\frac{b}{8})^2-(\frac{b}{4}+\sqrt{(\frac{b}{4})^2-(\frac{b}{2}+\sqrt{(\frac{b}{2})^2-c})}))))}))$$

$$x_2=-(\frac{b}{16}-\sqrt{(((\frac{b}{16})^2-(\frac{b}{8}+((\frac{b}{8})^2-(\frac{b}{4}+\sqrt{(\frac{b}{4})^2-(\frac{b}{2}+\sqrt{(\frac{b}{2})^2-c})}))))}))$$

$$x_3=-(\frac{b}{16}+\sqrt{(((\frac{b}{16})^2-(\frac{b}{8}-((\frac{b}{8})^2-(\frac{b}{4}+\sqrt{(\frac{b}{4})^2-(\frac{b}{2}+\sqrt{(\frac{b}{2})^2-c})}))))}))$$

$$x_4=-(\frac{b}{16}-\sqrt{(((\frac{b}{16})^2-(\frac{b}{8}-((\frac{b}{8})^2-(\frac{b}{4}+\sqrt{(\frac{b}{4})^2-(\frac{b}{2}+\sqrt{(\frac{b}{2})^2-c})}))))}))$$

$$x_5=-\frac{c}{x_1x_2x_3x_4}$$

The above solution method can be extended to the general quintic equation and higher degree polynomial equations.

3. Conclusion

A number can be decomposed into algebraic factors in an infinite number of ways. The factorization method is a candidate method of solving algebraic equations.

Higher degree polynomial equations have radical solution.

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