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ON THE GEOMETRY OF CONTACT PSEUDO-SLANT SUBMANIFOLDS IN A COSYMPLECTIC MANIFOLD

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ABSTRACT. In this paper, we study the geometry of the contact pseudo-slant submanifolds of a cosymplectic manifold. Necessary and sufficient conditions are given for a submanifold to be a contact pseudo-slant submanifold, contact pseudo-slant product, mixed totally geodesic, D_θ and $D^\perp$- totally geodesic in a cosymplectic manifold .

1. Introduction

The differential geometry of slant submanifolds has shown an increasing development since B.Y. Chen defined slant submanifolds in complex manifolds as a natural generallization of both invariant and anti-invariant submanifolds [**6, 7**]. After then many research articles have been appeared on the existence of these submanifolds in various know spaces. The slant submanifols of an almost contact metric manifolds were defined and studied by A. Lotta [**13**]. After, such submanifolds were studied by Cabrerizo et al. of Sasakian manifolds [**4**].

Semi-Slant submanifolds of a Kaehler Manifold was studied by N. Papaghuic [**14**], as a naturel generalization of slant submanifolds. After then, bi-slant submanifolds was introduced in a almost Hermitian manifold. Recently, Carriazo defined and studied bi-slant submanifolds in an almost Hermitian manifold and gave the notion of pseudo-slant submanifold in an almost Hermitian manifold. After then, V. A. Khan and M. A. Khan [**10**], defined and studied the contact version of pseudo-slant submanifold in a Sasakian manifold. Recently, M. Atçeken et al. [**1, 2, 3, 8**] studied pseudo-slant submanifold in various manifolds. This study the present paper is organized as follows.

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In this paper, we study contact pseudo-slant submanifolds of a cosymplectic manifold. In section 2, we review basic formulas and definitions for a cosymplectic manifold and their submanifolds. In section 3, we study the geometry of the contact pseudo-slant submanifolds of a cosymplectic manifold. Necessary and sufficient conditions are given for a submanifolds to be a contact pseudo-slant submanifold, contact pseudo-slant product, mixed totally geodesic, D_θ and $D^\perp$-totally geodesic in a cosymplectic manifold.

2. Preliminaries

In this section, we give some notations used throughout this paper. We recall some necessary fact and formulas from the theory of cosymplectic manifolds and their submanifols.

Let $\widetilde{M}$ be a $(2m+1)$-dimensional $C^\infty-$ differentiable manifold with the almost contact metric structure (φ,ξ,η,g), where φ is a tensor field of type $(1,1)$, ξ is a vector field, η 1-form and g Riemannian metric on $\widetilde{M}$, satisfying

$$\varphi^2 X = -X + \eta(X)\xi, \tag{2.1}$$

$$\varphi\xi = 0, \quad \eta\circ\varphi = 0, \quad \eta(\xi) = 1, \quad \eta(X) = g(X,\xi) \tag{2.2}$$

and

$$g(\varphi X, \varphi Y) = g(X,Y) - \eta(X)\eta(Y), \quad g(\varphi X, Y) = -g(X, \varphi Y) \tag{2.3}$$

for any vector fields X, Y on $\widetilde{M}$.

An almost contact structure (φ,ξ,η) is said to be normal if the almost complex structure J on the product manifold $\widetilde{M}\times R$ given by.

$$J(X,\ f\frac{d}{dt}) = (\varphi X - f\xi, \eta(X)\frac{d}{dt})$$

where f is a differentiable function on $\widetilde{M}\times\mathbb{R}$. The condition for normality in terms of φ,ξ and η is $[\varphi,\varphi] + 2d\eta\otimes\xi = 0$ on $\widetilde{M}$, where $[\varphi,\varphi](X,Y) = \varphi^2 [X,Y] + [\varphi X,\varphi Y] - \varphi[\varphi X, Y] - \varphi[X,\varphi Y]$ is the Nijenhuis tensor of φ. Finally the fundamental $2-$form Φ is defined by $\Phi(X,Y) = g(X,\varphi Y)$.

An almost contact metric structure (φ,ξ,η,g) is said to be cosymplectic structure if it is normal and Φ,η are closed, that is

$$(\widetilde{\nabla}_X\varphi)Y = 0 \tag{2.4}$$

for any vector fields X, Y on $\widetilde{M}$.

Then, $\widetilde{M}$ is called a cosymplectic manifold, where $\widetilde{\nabla}$ is the Levi-Civita connection of g. We have also on a cosymplectic manifold $\widetilde{M}$

$$\widetilde{\nabla}_X\xi = 0 \tag{2.5}$$

for any $X, Y\in\Gamma(T\widetilde{M})$.

Now, let M be a submanifold of an almost contact metric manifold $\widetilde{M}$ with the induced metric g. Also, let ∇ and $\nabla^\perp$ be the induced connections on the tangent bundle TM and the normal bundle $T^{\perp}M$ of M, respectively. Then the Gauss and Weingarten formulas are, respectively, given by

$$\widetilde{\nabla}_X Y = \nabla_X Y + h(X,Y) \tag{2.6}$$

and

$$\widetilde{\nabla}_X V = -A_V X + \nabla^{\perp}_X V, \tag{2.7}$$

where h and A_V are the second fundamental form and the shape operator (corresponding to the normal vector field V), respectively, for the immersion of M into $\widetilde{M}$. The second fundamental form and shape operator are related by formulea

$$g(A_V X, Y) = g(h(X,Y), V) \tag{2.8}$$

for all $X, Y \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$.

If $h(X,Y) = 0$, for each $X, Y \in \Gamma(TM)$ then M is said to be totally geodesic submanifold.

Now, let M be a submanifold of an almost contact metric manifold $\widetilde{M}$. Then for any $X \in \Gamma(TM)$, we can write

$$\varphi X = TX + NX, \tag{2.9}$$

where TX is the tangential component and NX is the normal component of φX. Similarly, for $V \in \Gamma(T^{\perp}M)$, φV also can write

$$\varphi V = tV + nV, \tag{2.10}$$

where tV is the tangential component and nV is also the normal component of φV.

Thus by using (2.1), (2.9) and (2.10), we obtain

$$T^2 = -I + \eta \otimes \xi - tN, \quad NT + nN = 0 \tag{2.11}$$

and

$$n^2 = -I - Nt, \quad Tt + tn = 0. \tag{2.12}$$

Furthermore, for any $X, Y \in \Gamma(TM)$, we have $g(TX, Y) = -g(X, TY)$ and $V, U \in \Gamma(T^{\perp}M)$, we get $g(U, nV) = -g(nU, V)$. These show that T and n are also skew-symmetric tensor fields. Moreover, for any $X \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$, we have

$$g(NX, V) = -g(X, tV), \tag{2.13}$$

which gives the relation between N and t.

Furthermore, the covariant derivatives of the tensor field T, N, t and n are, respectively, defined by

$$(\nabla_X T)Y = \nabla_X TY - T\nabla_X Y, \tag{2.14}$$

$$(\nabla_X N)Y = \nabla^{\perp}_X NY - N\nabla_X Y, \tag{2.15}$$

$$(\nabla_X t)V = \nabla_X tV - t\nabla^{\perp}_X V \tag{2.16}$$

and

$$(\nabla_X n)V = \nabla^{\perp}_X nV - n\nabla^{\perp}_X V. \tag{2.17}$$

A submanifold M is said to be invariant if N is identically zero, that is, $\varphi X \in \Gamma(TM)$ for all $X \in \Gamma(TM)$. On the other hand, M is said to be anti- invariant if T is identically zero, that is, $\varphi X \in \Gamma(T^{\perp}M)$ for all $X \in \Gamma(TM)$. By an easy computation, we obtain the following formulas

$$(\nabla_X T)Y = A_{NY}X + th(X,Y) \tag{2.18}$$

and

$$(\nabla_X N)Y = nh(X,Y) - h(X,TY). \tag{2.19}$$

Similarly, for any $V \in \Gamma(T^{\perp}M)$ and $X \in \Gamma(TM)$, we obtain

$$(\nabla_X t)V = A_{nV}X - TA_V X \tag{2.20}$$

and

$$(\nabla_X n)V = -h(tV,X) - NA_V X. \tag{2.21}$$

Since M is tangent to ξ, making use of (2.5), (2.6)and (2.8) , we obtain

$$\nabla_X \xi = 0, h(X,\xi) = 0, A_V \xi = 0 \tag{2.22}$$

for all $V \in \Gamma(T^{\perp}M)$ and $X \in \Gamma(TM)$.

In contact geometry, A. Lotta introduced slant submanifold as follows [**13**].

Definition 2.1. *Let M be a submanifold of an almost contact metric manifold* $(\widetilde{M},\varphi,\xi,\eta,g)$. *Then M is said to be a contact slant submanifold if the angle* $\theta(X)$ *between* φX *and* $T_M(x)$ *is constant at any point* $x \in M$ *for any* X *linearly independent of* ξ. *Thus the invariant and anti-invariant submanifolds are special class of slant submanifolds with slant angles* $\theta = 0$ *and* $\theta = \frac{\pi}{2}$, *respectively. If the slant angle* θ *is neither zero nor* $\frac{\pi}{2}$, *then slant submanifold is said to be proper contact slant submanifold.*

The slant submanifolds of an almost contact metric manifold, the following theorem is well known.

Theorem 2.1. *Let* M *be a slant submanifold of an almost contact metric manifold* $\widetilde{M}$ *such that* $\xi \in \Gamma(TM)$. *Then,* M *is a slant submanifold if and only if there exists a constant* $\lambda \in (0,1)$ *such that*

$$T^2 = -\lambda(I - \eta \otimes \xi) \tag{2.23}$$

furthermore, in this case, if θ *is the slant angle of* M, *then it is satisfies* $\lambda = \cos^2\theta$ [**4**].

Corollary 2.1. *Let* M *be a slant submanifold of an almost contact metric manifold* $\widetilde{M}$ *with slant angle* θ. *Then for any* $X, Y \in \Gamma(TM)$, *we have*

$$g(TX,TY) = \cos^2\theta \{g(X,Y) - \eta(X)\eta(Y)\} \tag{2.24}$$

and

$$g(NX,NY) = \sin^2\theta \{g(X,Y) - \eta(X)\eta(Y)\}. \tag{2.25}$$

3. Contact pseudo-slant submanifold of a Cosymplectic manifold

hspace8mm In this section, we study the geometry of the contact pseudo-slant submanifolds of a cosymplectic manifold. Necessary and sufficient conditions are given for a submanifold to be a contact pseudo-slant submanifold, contact pseudo-slant product, mixed totally geodesic, D_θ and $D^\perp$- totally geodesic in cosymplectic manifolds.

DEFINITION 3.1 ([**10**]). *Let M be a submanifold of a cosymplectic manifold $\widetilde{M}(\varphi, \xi, \eta, g)$. We say that M is a contact pseudo-slant submanifold if there exists a pair of orthogonal distributions $D^\perp$ and D_θ on M such that*

(i) *The distribution $D^\perp$ is totally real, i.e., $\varphi(D^\perp) \subseteq T^\perp M$*

(ii) *The distribution D_θ is slant with slant angle θ,*

(iii) *The tangent space TM admits the orthogonal direct decomposition $TM = D^\perp \oplus D_\theta$.*

If $\theta = 0$ then, the submanifold becomes a semi-invariant submanifold.
Let d_1 =dim($D^\perp$) and d_2=dim(D_θ). We distinguish the following six cases.
(i) If $d_2 = 0$, then M is an anti-invariant submanifold.
(ii) If $d_1 = 0$ and $\theta = 0$, then M is an invariant submanifold.
(iii) If $d_1 = 0$ and $\theta \in (0, \frac{\pi}{2})$, then M is a proper slant submanifold.
(iv) If $\theta = \frac{\pi}{2}$ then, M is an anti-invariant submanifold.
(v) If $d_2 d_1 \neq 0$ and $\theta = 0$, then M is a semi-invariant submanifold.
(vi) If $d_2 d_1 \neq 0$ and $\theta \in (0, \frac{\pi}{2})$, then M is a contact pseudo-slant submanifold.

For a contact pseudo-slant submanifold M of a cosymplectic manifold $\bar{M}$, the normal bundle $T^\perp M$ of a contact pseudo-slant submanifold M is decomposable as

$$T^\perp M = N(D^\perp) \oplus N(D_\theta) \oplus \mu, \quad N(D^\perp) \perp N(D_\theta) \tag{3.1}$$

where μ is an invariant subbundle of $T^\perp M$.

Moreover, for any $Z, W \in \Gamma(D^\perp)$ and $U \in \Gamma(TM)$, also by using (2.4) and (2.8), we have

$$\begin{aligned} g(A_{NZ}W - A_{NW}Z, U) &= g(h(W,U), NZ) - g(h(Z,U), NW) \\ &= g(\widetilde{\nabla}_U W, \varphi Z) - g(\widetilde{\nabla}_U Z, \varphi W) \\ &= g(\varphi \widetilde{\nabla}_U Z, W) - g(\widetilde{\nabla}_U \varphi Z, W) \\ &\quad g(\widetilde{\nabla}_U \varphi Z - (\widetilde{\nabla}_U \varphi) Z, W) \\ &\quad + g(\widetilde{\nabla}_U \varphi W - (\widetilde{\nabla}_U \varphi) W, Z) \\ &= g(\widetilde{\nabla}_U \varphi Z, W) - g(\widetilde{\nabla}_U \varphi W, Z) \\ &= -g(A_{NZ}W + A_{NW}Z, U). \end{aligned}$$

It follows that

$$A_{NZ}W = A_{NW}Z,$$

for any $Z, W \in \Gamma(D^\perp)$.

Theorem 3.1. *Let M be a contact pseudo-slant submanifold of a cosymplectic manifold $\widetilde{M}$. Then we obtain*

(i) $T^2X = -\lambda(X - \eta(X)\xi)$, *for any* $X \in \Gamma(D_\theta)$,

(ii) $TX = 0$, *for orthogonal* $X \in \Gamma(TM)$ *to* D_θ, *where* $\lambda = \cos^2\theta$.

Proof. Follows from Theorem 2.1 and definition 3.1 the proof is obvious. □

Definition 3.2. *A contact pseudo-slant submanifold M of cosymplectic manifold $\widetilde{M}$ is said to be D_θ-*totally geodesic *(resp. $D^\perp$-*totally geodesic*) if $h(X,Y) = 0$ for all $X, Y \in \Gamma(D_\theta)$ (resp. $h(Z,W) = 0$ for all $Z, W \in \Gamma(D^\perp)$. If for all $X \in \Gamma(D_\theta)$ and $Z \in \Gamma(D^\perp)$, $h(X,Z) = 0$, the M is called* mixed totally geodesic submanifold.

Theorem 3.2. *Let M be a proper contact pseudo-slant submanifold of a cosymplectic manifold $\widetilde{M}$. Then, either M is a mixed-totally geodesic or an anti-invariant submanifold.*

Proof. By using (2.2), (2.3), (2.6), (2.7), (2.9) and (2.10), we have

$$\begin{aligned} g(A_VX,Y) &= g(\widetilde{\nabla}_XY,V) = -g(\widetilde{\nabla}_XV,Y) \\ &= -g(\varphi\widetilde{\nabla}_XV,\varphi Y) = g((\widetilde{\nabla}_X\varphi)V - \widetilde{\nabla}_X\varphi V,\varphi Y) \\ &= -g(\widetilde{\nabla}_XtV + \widetilde{\nabla}_XnV, NY) \\ &= -g(h(X,nV),NY) - g(\nabla^\perp_X nV, NY), \end{aligned}$$

for any $X \in \Gamma(D_\theta)$, $Y \in \Gamma(D^\perp)$ and $V \in \Gamma(T^\perp M)$. Taking into account (2.13), (2.17) and (2.21), we get

$$\begin{aligned} g(A_VX,Y) &= -g(h(X,tV),NY) - g((\nabla_Xn)V + n\nabla^\perp_X V, NY) \\ &= -g(h(X,tV),NY) - g(-h(X,tV) - NA_VX, NY) \\ &= g(NA_VX,NY) = -g(tNA_VX,Y). \end{aligned}$$

By using (2.11), we obtain

$$\begin{aligned} g(A_VX,Y) &= -g(-A_VX + \eta(A_VX)\xi - T^2A_VX, Y) \\ &= g(A_VX,Y) + g(T^2A_VX,Y), \end{aligned}$$

that is,

$$-\cos^2\theta g(A_VX - \eta(A_VX)\xi, Y) = -\cos^2\theta g(A_VX,Y) = 0.$$

This tells us that either M is mixed-totally geodesic or it is an anti-invariant submanifold. □

Theorem 3.3. *Let M be a proper contact pseudo-slant submanifold of a cosymplectic manifold $\widetilde{M}$. Then, either M is $D^\perp$-totally geodesic or an anti-invariant submanifold of $\widetilde{M}$.*

PROOF. By using (2.2), (2.3), (2.6), (2.7), (2.9) and (2.10), we obtain

$$\begin{aligned} g(h(Z,W),V) &= -g(\widetilde{\nabla}_W V, Z) = -g(\varphi\widetilde{\nabla}_W V, \varphi Z) \\ &= g((\widetilde{\nabla}_W \varphi)V - \widetilde{\nabla}_W \varphi V, \varphi Z) \\ &= -g(\widetilde{\nabla}_W tV, NZ) - g(\widetilde{\nabla}_W nV, NZ) \\ &= -g(h(W,tV), NZ) - g(\nabla^{\perp}_W nV, NZ), \end{aligned}$$

for any $Z, W \in \Gamma(D^{\perp})$ and $V \in \Gamma(T^{\perp}M)$. Hence, by using (2.13), (2.11),(2.21) and (2.23), we reach

$$\begin{aligned} g(h(Z,W),V) &= -g(h(W,tV), NZ) - g((\nabla_W n)V, NZ) \\ &= -g(h(W,tV), NZ) + g(h(tV,W) + NA_V W, NZ) \\ &= g(NA_V W, NZ) = -g(tNA_V W, Z) \\ &= -g(-A_V W + \eta(A_V W)\xi - T^2 A_V W, Z) \\ &= g(A_V W, Z) + g(T^2 A_V W, Z), \end{aligned}$$

or

$$-\cos^2\theta g(A_V W - \eta(A_V W)\xi, Z) = -\cos^2\theta g(A_V W, Z) = 0.$$

The last relation yields $\cos^2\theta g(h(Z,W),V) = 0$, which means that either M is $D^{\perp}$-totaly geodesic or it is an anti-invariant submanifold. □

DEFINITION 3.3. *Given a proper contact pseudo-slant submanifold M of a cosymlectic manifold $\widetilde{M}$, if the distributions D_θ and $D^{\perp}$ are totally geodesic in M, then M is said to be contact pseudo-slant product.*

THEOREM 3.4. *Let M be a contact pseudo-slant submanifold of a cosymlectic manifold $\widetilde{M}$. Then M is a contact pseudo-slant product if and only if the shape operator of M satisfies*

$$A_{ND^{\perp}} TD_\theta = A_{NTD_\theta} D^{\perp}. \tag{3.2}$$

PROOF. By using (2.18), we have

$$\nabla_X TY - T\nabla_X Y = A_{NY}X + th(X,Y)$$

for any $X, Y \in \Gamma(D_\theta)$. This implies that

$$g(\nabla_X TY, Z) = g(A_{NY}X, Z) + g(th(X,Y), Z), \tag{3.3}$$

for any $Z \in \Gamma(D^{\perp})$. Replacing Y by TY in (3.3) and taking into account of (2.24), we obtain

$$\cos^2\theta g(\nabla_X Y, Z) = g(A_{NZ}TY - A_{NTY}Z, X). \tag{3.4}$$

Also, from (2.14), we have

$$-T\nabla_Z U = A_{NU}Z + th(Z,U),$$

for any $U, Z \in \Gamma(D^{\perp})$, from which

$$-g(T\nabla_Z U, TX) = g(A_{NU}Z, TX) + g(th(Z,U), TX),$$

that is,

$$-\cos^2\theta g(\nabla_Z U, X) = g(A_{NU}TX - A_{NTX}U, Z), \tag{3.5}$$

for any $X \in \Gamma(D_\theta)$. (3.4) and (3.5) imply that (3.2). □

For any $X, Y \in \Gamma(D_\theta < \xi >)$ and $Z \in \Gamma(D^\perp)$, by using (2.3), (2.6), (2.7), (2.13), (2.19) and (2.24), we have

$$\begin{aligned} g(\nabla_X Y, Z) &= g(\varphi\widetilde{\nabla}_X Y, \varphi Z) = g(\widetilde{\nabla}_X \varphi Y - (\widetilde{\nabla}_X \varphi)Y, \varphi Z) \\ &= g(h(X, TY), NZ) + g(\nabla^\perp_X NY, NZ) \\ &= g(h(X, TY), NZ) + g((\nabla_X N)Y + N\nabla_X Y, NZ) \\ &= g(h(X, TY), NZ) + g(nh(X, Y), NZ) \\ &- g(h(X, TY), NZ) + g(N\nabla_X Y, NZ) \\ &= g(N\nabla_X Y, NZ) = -g(tN\nabla_X Y, Z) \\ &= -g(-\nabla_X Y + \eta(\nabla_X Y)\xi - T^2\nabla_X Y, Z), \end{aligned}$$

which implies that

$$g(T^2\nabla_X Y, Z) = -\cos^2\theta g(\nabla_X Y, Z) = 0. \tag{3.6}$$

and

$$\begin{aligned} g(\nabla_W Z, X) &= -g(\widetilde{\nabla}_W X, Z) = -g(\varphi\widetilde{\nabla}_W X, \varphi Z) \\ &= g((\widetilde{\nabla}_W \varphi)X, \varphi Z) - g(\widetilde{\nabla}_W \varphi X, \varphi Z) \\ &= -g(h(TX, W), NZ) - g(\nabla^\perp_W NX, NZ) \\ &= -g(h(TX, W), NZ) - g((\nabla_W N)X + N\nabla_W X, NZ) \\ &= -g(h(TX, W), NZ) - g(nh(X, W), NZ) \\ &+ g(h(W, TX), NZ) - g(N\nabla_W X, NZ) = g(tN\nabla_W X, Z) \\ &= g(-\nabla_W X + \eta(\nabla_W X)\xi - T^2\nabla_W X, Z) \\ &= g(\nabla_W Z, X) + g(T^2\nabla_W X, Z), \end{aligned}$$

that is,

$$\cos^2\theta g(\nabla_W X - \eta(\nabla_W X)\xi, Z) = -\cos^2\theta g(\nabla_W Z, X) = 0. \tag{3.7}$$

for any $Z, W \in \Gamma(D^\perp)$ and $X \in \Gamma(D_\theta)$. Thus from (3.6) and (3.7), we have the following corollary.

Corollary 3.1. *Every proper contact pseudo-slant submanifold M of a cosymplectic manifold $\widetilde{M}$ is a contact pseudo-slant product.*

Example 3.1. Let M be a submanifold of $\mathbb{R}^9$ defined by the following equation

$$\begin{aligned} \chi(u, v, s, t, z) = (&u\sin h\alpha, -v\cos h\alpha, -2u\sin h\alpha, v\cos h\alpha, s\cos ht, \\ &\cos ht, s\sin ht, -\sin ht, z). \end{aligned}$$

We can easily to see that the tangent bundle of M is spanned by the tangent vectors

$$\begin{aligned} e_1 &= \sin h\alpha \frac{\partial}{\partial x_1} - 2\sin h\alpha \frac{\partial}{\partial x_2}, \\ e_2 &= -\cos h\alpha \frac{\partial}{\partial y_1} + \cos h\alpha \frac{\partial}{\partial y_2}, \\ e_3 &= \cos ht \frac{\partial}{\partial x_3} + \sin ht \frac{\partial}{\partial x_4}, \\ e_4 &= s\sin ht \frac{\partial}{\partial x_3} + \sin ht \frac{\partial}{\partial y_3} + s\cos ht \frac{\partial}{\partial x_4} - \cos ht \frac{\partial}{\partial y_4} \end{aligned}$$

and

$$e_5 = \xi = \frac{\partial}{\partial z}.$$

For the almost contact metric structure φ of $\mathbb{R}^9$, whose coordinate systems (x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4, z), choosing

$$\varphi(\frac{\partial}{\partial x_i}) = \frac{\partial}{\partial y_i}, \varphi(\frac{\partial}{\partial y_j}) = -\frac{\partial}{\partial x_j}, 1 \leqslant i, j \leqslant 4$$

$$\varphi(\frac{\partial}{\partial z}) = 0,\ \xi = \frac{\partial}{\partial z}, \quad \eta = dz.$$

For any vector field $U = \mu_i \frac{\partial}{\partial x_i} + \nu_j \frac{\partial}{\partial y_j} + \lambda \frac{\partial}{\partial z} \in T(\mathbb{R}^9)$, then we have

$$\varphi U = \mu_i \varphi(\frac{\partial}{\partial x_i}) + \nu_j \varphi(\frac{\partial}{\partial y_j}) + \lambda \varphi(\frac{\partial}{\partial z}) = \mu_i \frac{\partial}{\partial y_j} - \nu_j \frac{\partial}{\partial x_i},$$

$$g(\varphi U, \varphi U) = \mu_i^2 + \nu_j^2,\ g(U,U) = \mu_i^2 + \nu_j^2 + \lambda^2,\ \eta(U) = g(U,\xi) = \lambda$$

and

$$\varphi^2 U = -\mu_i \frac{\partial}{\partial x_i} - \nu_j \frac{\partial}{\partial y_j} - \lambda \frac{\partial}{\partial z} + \lambda \frac{\partial}{\partial z} = -U + \eta(U)\xi$$

for any $i, j = 1, 2, 3, 4$. It follows that $g(\varphi U, \varphi U) = g(U,U) - \eta^2(U)$. Thus (φ, ξ, η, g) is an almost contact metric structure on $\mathbb{R}^9$. We call the usual contact metric structure of $\mathbb{R}^9$. Then we have

$$\begin{aligned} \varphi e_1 &= \sin h\alpha \frac{\partial}{\partial y_1} - \sin h\alpha \frac{\partial}{\partial y_2}, \\ \varphi e_2 &= \cos h\alpha \frac{\partial}{\partial x_1} - \cos h\alpha \frac{\partial}{\partial x_2}, \\ \varphi e_3 &= \cos ht \frac{\partial}{\partial y_3} + \sin ht \frac{\partial}{\partial y_4} \end{aligned}$$

and

$$\varphi e_4 = s\sin ht \frac{\partial}{\partial y_3} - \sin ht \frac{\partial}{\partial x_3} + s\cos ht \frac{\partial}{\partial y_4} + \cos ht \frac{\partial}{\partial x_4}.$$

By direct calculations, we can infer $D_\theta = span\{e_1, e_2\}$ is a slant distribution with slant angle $\cos\theta = \frac{g(e_1, \varphi e_2)}{\|e_1\| \|\varphi e_2\|} = \frac{3\sqrt{10}}{10}$, $\theta = cos^{-1}(\frac{3\sqrt{10}}{10})$. Since $g(\varphi e_3, e_i) = 0,\ \ i = 1, 2, 4, 5$ and $g(\varphi e_4, e_j) = 0,\ j = 1, 2, 3, 5$ orthogonal to M, $D^\perp = span\{e_3, e_4, e_5\}$

is an anti-invariant distribution. Thus M is a 5-dimensional proper contact pseudo-slant submanifold of $\mathbb{R}^9$ with it's usual almost contact metric structure.

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