BULLETIN OF THE INTERNATIONAL MATHEMATICAL VIRTUAL INSTITUTE
ISSN (p) 2303-4874, ISSN (o) 2303-4955
www.imvibl.org /JOURNALS / BULLETIN
Vol. 8(2018), 89-97
DOI: 10.7251/BIMVI1801089P

Former
BULLETIN OF THE SOCIETY OF MATHEMATICIANS BANJA LUKA
ISSN 0354-5792 (o), ISSN 1986-521X (p)

4-PRIME CORDIAL LABELING OF SOME SPECIAL GRAPHS

R.Ponraj, Rajpal Singh, and R.Kala

Abstract. Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \ldots, k\}$ be a function. For each edge uv, assign the label $\gcd(f(u), f(v))$. f is called k-prime cordial labeling of G if $|v_f(i) - v_f(j)| \leqslant 1$, $i, j \in \{1, 2, \ldots, k\}$ and $|e_f(0) - e_f(1)| \leqslant 1$ where $v_f(x)$ denotes the number of vertices labeled with x, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labeled with 1 and not labeled with 1. A graph with admits a k-prime cordial labeling is called a k-prime cordial graph. In this paper we investigate 4-prime cordial labeling behavior of union of two bipartite graphs, union of trees, durer graph, tietze graph, planar grid $P_m \times P_n$, subdivision of wheels and subdivision of helms.

1. Introduction

Graphs consider here are finite, simple and undirected only. Let G be a (p, q) graph where p refers the number of vertices of G and q refers the number of edge of G. The number of vertices of a graph G is called order of G, and the number of edges is called size of G. The subdivision graph $S(G)$ of a graph G is obtained by replacing each edge uv by a path uwv. The *Join* of two graphs $G_1 + G_2$ is obtained from G_1 and G_2 and whose vertex set is $V(G_1 + G_2) = V(G_1) \cup V(G_2)$ and edge set $E(G_1 + G_2) = E(G_1) \cup E(G_2) \cup \{uv : u \in V(G_1), v \in V(G_2)\}$. The graph $C_n + K_1$ is called a *wheel*. In a wheel, the vertex of degree n is called the central vertex and the vertices on the cycle C_n are called rim vertices. The *helm* H_n is the graph obtained from a wheel by attaching a pendent edge at each vertex of the n-cycle. Let G_1, G_2 respectively be (p_1, q_1), (p_2, q_2) graphs. The corona of G_1 with G_2, $G_1 \odot G_2$ is the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2. The product graph $G_1 \Box G_2$ is defined as follows: Consider any two

1991 *Mathematics Subject Classification.* 05C78.
Key words and phrases. Cycle, wheel, join, sunflower graph, lotus inside a circle.

points $u = (u_1, u_2)$ and $v = (v_1, v_2)$ in $V = V_1 \times V_2$. Then u and v are adjacent in $G_1 \times G_2$ whenever $[u_1 = v_1 \text{ and } u_2 \text{ adj } v_2]$ or $[u_2 = v_2 \text{ and } u_1 \text{ adj } v_1]$. Cahit introduced the concept of cordial labeling of graphs [**1**]. Sundaram, Ponraj, Somasundaram have introduced the notion of prime cordial labeling [**13**] and product cordial labeling [**14**]. A 2-prime cordial labeling is a product cordial labeling. Also Prajapathi et al have studied edge product cordial labeling of some cycle related graphs [**10, 11**]. Recently Ponraj et al. [**4**], introduced k-prime cordial labeling of graphs. In [**5, 6, 7, 8, 9**] Ponraj et al. have studied the 4-prime cordial labeling behavior of complete graph, book, flower, mC_n, wheel, gear, double cone, helm, closed helm, butterfly graph, and friendship graph and some more graphs. In this paper we have studied 4-prime cordiality of union of two bipartite graphs, union of trees, durer graph, tietze graph, planar grid $P_m \square P_n$, subdivision of wheels and subdivision of helms and some more graphs. Let x be any real number. Then $\lfloor x \rfloor$ stands for the largest integer less than or equal to x and $\lceil x \rceil$ stands for smallest integer greater than or equal to x. Terms not defined here follow from Harary [**3**] and Gallian [**2**].

2. Main results

First investigation is about $G \cup G$, where G is bipartite.

THEOREM 2.1. *If G is bipartite then $G \cup G$ is 4-prime cordial.*

PROOF. Let $V(G) = V_1 \cup V_2$, $|V_1| = m$, $|V_2| = n$, $m \leqslant n$. First consider the first copy G. Assign the label 2 to the $\left\lceil \frac{m+n}{2} \right\rceil$ vertices of the first copy and 4 to the $\left\lfloor \frac{m+n}{2} \right\rfloor$ remaining vertices of the first copy. We now move to the second copy G. In this copy, assign the label 1 to all the m vertices of the set V_1. Then assign the label 3 to the m vertices of the set V_2. Next assign the label 1 to the $\left\lceil \frac{n-m}{2} \right\rceil$ vertices of the set V_2. Finally assign the label 3 to the remaining $\left\lfloor \frac{n-m}{2} \right\rfloor$ vertices of the set V_2. Obviously this vertex labeling is a 4-prime cordial labeling of $G \cup G$. □

COROLLARY 2.1. *If T is a tree, then $T \cup T$ is 4-prime cordial.*

PROOF. As T is bipartite, the proof follows from theorem 2.1. □

Next is the Dürer graph. Let C_n be the cycle $u_1 u_2 \dots u_1$ where $n \equiv 0 \pmod 6$. Then DG_n is the graph with vertex set $V(DG_n) = V(C_n) \cup \{v_i : 1 \leqslant i \leqslant n\}$, $E(DG_n) = E(C_n) \cup \{u_i v_i : 1 \leqslant i \leqslant n\} \cup \{u_i u_{i+2} : 1 \leqslant i \leqslant n-2\} \cup \{u_n u_2, u_{n-1} u_1\}$. DG_6 is called the Dürer graph.

THEOREM 2.2. *The graph DG_n is 4-prime cordial.*

PROOF. Clearly DG_n has $2n$ vertices and $3n$ edges. Assign the label 1 to the vertices $v_1, v_3, v_5 \dots v_{n-1}$. Then assign the label 3 to the vertices $v_2, v_4, v_6 \dots v_n$. We now move to the cycle vertices. Assign the label 2 to the vertices $u_1, u_2, u_3 \dots u_{\frac{n}{2}}$ and assign the label 4 to the vertices $u_{\frac{n}{2}+1}, u_{\frac{n}{2}+2}, \dots u_n$. This vertex labeling f is a 4-prime cordial labeling follow from $v_f(1) = v_f(2) = v_f(3) = v_f(4) = \frac{n}{2}$ and $e_f(0) = e_f(1) = \frac{3n}{2}$. □

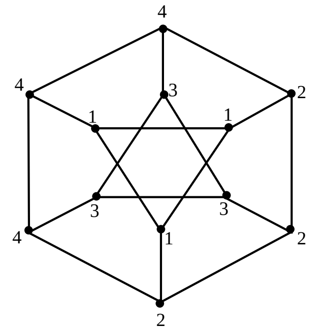

FIGURE 1

A 4-prime cordial of Dürer graph DG_6 is given in figure 1.

We now investigate the Tietze graph. Let C_n be the cycle $u_1u_2\ldots u_1$ where $n \equiv 0 \pmod 9$. Then TG_n is the graph with vertex set $V(TG_n) = V(C_n) \cup \{v_{3i-2} : 1 \leqslant i \leqslant \frac{n}{3}\}$, $E(TG_n) = E(C_n) \cup \{u_{3i-2}v_{3i-2} : 1 \leqslant i \leqslant \frac{n}{3}\} \cup \{u_{3i-1}u_{3i+3} : 1 \leqslant i \leqslant \frac{n-3}{3}\} \cup \{u_{n-1}u_3\}$. The graph TG_9 is called the Tietze graph.

THEOREM 2.3. *TG_n is 4-prime cordial.*

PROOF. The order and size of TG_n are $\frac{4n}{3}$ and $2n$ respectively. Assign the label 3 to the vertices $v_1, v_4, v_7, \ldots, v_{n-2}$. Next we move to the cycle vertices. Assign the label 1 to the vertices $u_1, u_4, u_7, \ldots, u_{n-2}$. For the remaining $\frac{2n}{3}$ cycle vertices, assign the label 2 to any of $\frac{n}{3}$ vertices and 4 to another $\frac{n}{3}$ vertices. This labeling pattern f is a 4-prime cordial labeling since $v_f(1) = v_f(2) = v_f(3) = v_f(4) = \frac{n}{3}$ and $e_f(0) = e_f(1) = n$. □

A 4-prime cordial labeling of the Tietze graph TG_9 is given in figure 2.

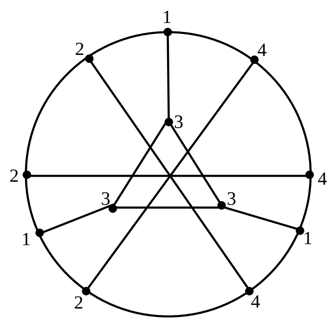

FIGURE 2

THEOREM 2.4. *A planar grid $P_m \square P_n$ is 4-prime cordial.*

PROOF. Let (i, j) denotes the vertex in the i^{th} row and j^{th} column. Clearly $P_m \Box P_n$ has mn vertices $2mn - (m + n)$ edges.

Case 1. $m \equiv 0 \pmod 4$.

Let $m = 4t$, $t \geqslant 1$.

Subcase 1a. n is even. Consider the first row vertices. Assign the labels 3 to the first $\frac{n}{2} + 1$ vertices of the first row from left to right. Next assign the labels 1 and 3 alternatively to the vertices $(1, \frac{n}{2}+2), (1, \frac{n}{2}+3), \ldots, (1, n)$. We now assign the label to the second row depends on the labels of the vertex $(1, n)$. If the vertex $(1, n)$ received the label 1 then assign the labels 3 and 1 alternatively to the vertices of the second row from right to left until we reach the vertex $(2, \frac{n}{2} + 2)$; otherwise assign 1 and 3 alternatively until we reach the vertex $(2, \frac{n}{2} + 2)$. Then assign the label 1 to the vertices $(2, 1), (2, 2), \ldots, (2, \frac{n}{2} + 1)$. Next we move to the third row. If the $(2, \frac{n}{2} + 2)$ vertex received the label 1, then assign the labels 3 and 1 alternatively to the vertices of the third row completely from left to right; otherwise assign 1 and 3 alternatively to the vertices of the third row. Next we move to the fourth row. If the $(3, n)$ received the label 1 then assign 3 and 1 alternatively to the vertices of the forth row completely from right to left otherwise assign 1 and 3 alternatively from right to left. That is if the i^{th} row is labeled from right to left, then $(i + 1)^{th}$ is labeled from left to right. Also if the i^{th} $(i \geqslant 3)$ row is labeled from right to left, then the $(i + 1)^{th}$ row is labeled 1 and 3 alternatively or 3 and 1 alternatively depends on the label of (i, n) is 3 or 1. The same procedure is continued until we labeled $(2r)^{th}$ row vertices. Now our attention is turn to the $(2r + 1)^{th}$ row. Assign the labels 2 to the vertices of the $(2r + 1)^{th}$, $(2r + 2)^{th}$, $\ldots$, $(3r)^{th}$ row. Finally assign the labels 4 to the vertices of the $(3r + 1)^{th}$, $(3r + 2)^{th}$, $\ldots$, $(4r)^{th}$ rows.

Subcase 1b. n is odd.

In this case assign the label 3 to the first $\frac{n+1}{2}$ vertices of the first row from left to right. Next assign the labels alternatively 1 and 3 to the vertices $(1, \frac{n+1}{2} + 1), (1, \frac{n+1}{2} + 2), \ldots, (1, n)$. The second and subsequent rows are labeled as in case 1 pattern.

Case 2. $m \equiv 1 \pmod 4$.

As in case 1, assign the label to the vertices of the first $(m - 1)^{th}$ rows. In the last row we assign the labels is as follows:

Subcase 2a. $n \equiv 0 \pmod 4$.

Let $n = 4r$. Assign the label 2 to the first r column vertices. That is in the m^{th} row, first r column vertices received the label 2. Now assign the label 4 to the vertices of the $(r + 1)^{th}$, $(r + 2)^{th}$, $\ldots$, $(r)^{th}$ column. Next assign the labels 1 and 3 alternatively to the vertices of $(2r + 1)^{th}$, $(2r + 2)^{th}$, $\ldots$, $(4r)^{th}$ column vertices.

Subcase 2b. $n \equiv 1 \pmod 4$.

Let $n = 4r + 1$. In this case assign the label 2 to the first $r + 1$ column vertices of the m^{th} row. Then assign 4 to the next r column vertices. Finally assign the label 1 and 3 alternatively to the $(2r + 1)^{th}$, $\ldots$, $(4r)^{th}$ column vertices.

Subcase 2c. $n \equiv 2 \pmod 4$.

Let $n = 4r + 2$. Assign the label 2 to the first $r + 1$ column vertices of the m^{th} row and 4 to the $(r + 2)^{th}, \ldots, (2r + 2)^{th}$ column vertices. Finally assign the label 1 and 3 alternatively to the remaining column vertices.

Subcase 2d. $n \equiv 3 \pmod 4$.

Let $n = 4r + 3$. As in subcase 2c, assign the label to the vertices. the label 2 to the first $r + 1$ column vertices of the m^{th} row and 4 to the $(r + 2)^{th}, \ldots, (2r + 2)^{th}$ column vertices. Finally assign the label 1 and 3 alternatively to the remaining column vertices.

Case 3. $m \equiv 2 \pmod 4$.

As in case 2, assign the label to the first $(m - 1)^{th}$ row vertices. We now consider the m^{th} row. Assign the label to this m^{th} row vertices as in subcase 2a to 2d of case 2.

Case 4. $m \equiv 3 \pmod 4$.

Similar to case 3. □

THEOREM 2.5. *Let G be any 4-prime cordial graph. Then $G \odot K_1$ is also a 4-prime cordial.*

PROOF. Let $V(G \odot K_1) = V(G) \cup \{v_i : 1 \leqslant i \leqslant p\}$ and $E(G \odot K_1) = E(G) \cup \{u_iv_i : 1 \leqslant i \leqslant p\}$. Let f be a 4-prime cordial labeling of G.

Case 1. $p \equiv 0 \pmod 4$.

Let $p = 4t$. Assign the label 2 to the vertices v_i such a way that their support u_i rceived the label 4. Similarly assign the label 4 to the vertices u_i whose support also received the label 2. Next assign the label 3 to the vertices such that the vertices u_i received the label 1. Finally assign the label 1 to the non labeled vertices. Clearly this vertex labeling g is a 4-prime cordial labeling since $e_g(0) = e_f(0) + \frac{p}{2}$ and $e_g(1) = e_f(1) + \frac{p}{2}$.

Case 2. $p \equiv 1 \pmod 4$.

Let $p = 4t + 1$. The following types arises.

TYPE A: $v_f(1) = t + 1$, $v_f(2) = v_f(3) = v_f(4) = t$
TYPE B: $v_f(2) = t + 1$, $v_f(1) = v_f(3) = v_f(4) = t$
TYPE C: $v_f(3) = t + 1$, $v_f(1) = v_f(2) = v_f(4) = t$
TYPE D: $v_f(4) = t + 1$, $v_f(1) = v_f(2) = v_f(3) = t$

TYPE A: $v_f(1) = t + 1$, $v_f(2) = v_f(3) = v_f(4) = t$.

Subcase A(i): q is even.

In this case the vertex labeling g in case 1 is automatically 4-prime cordial labeling of $G \odot K_1$. Since $e_g(1) = e_g(0) + 1$ and $v_g(1) = v_g(3) = 2t + 1$, $v_g(2) = v_g(4) = 2t$.

Subcase A(ii): q is odd.

When $e_f(0) = e_f(1) + 1$, then clearly the vertex labeling g is 4-prime cordial labeling of $G \odot K_1$. In the case of $e_f(1) = e_f(0) + 1$, interchange the labels of any two vertices v_r and v_t where label of $v_r = 1$ and label of $v_t = 2$. That is after interchange the vertex v_r received the label 2 whereas the vertex v_t received the label 1.

TYPE B: $v_f(2) = t + 1$, $v_f(1) = v_f(3) = v_f(4) = t$

Clearly g is a 4-prime cordial labeling of $G \odot K_1$ when $e_f(1) = e_f(0) + 1$. Otherwise interchange the labels of any two vertices v_r and v_t such that label of $v_r = 3$ and label of $v_t = 4$.

TYPE C: $v_f(3) = t + 1$, $v_f(1) = v_f(2) = v_f(4) = t$

In the case of $e_f(0) = e_f(1)+1$, clearly g is a 4-prime cordial labeling of $G \odot K_1$. For the case $e_f(1) = e_f(0) + 1$, interchange the labels of v_r and v_t such that label of $v_r = 1$ and label of $v_t = 3$.

TYPE D: $v_f(4) = t + 1$, $v_f(1) = v_f(2) = v_f(3) = t$

As in TYPE B we get a 4-prime cordial labeling of $G \odot K_1$.

Case 3. $p \equiv 2 \pmod 4$.

Let $p = 4t + 2$. In this case, the following six types are arises:

TYPE A: $v_f(1) = v_f(2) = t + 1$, $v_f(3) = v_f(4) = t$
TYPE B: $v_f(1) = v_f(3) = t + 1$, $v_f(2) = v_f(4) = t$
TYPE C: $v_f(1) = v_f(4) = t + 1$, $v_f(2) = v_f(3) = t$
TYPE D: $v_f(2) = v_f(3) = t + 1$, $v_f(1) = v_f(4) = t$
TYPE E: $v_f(2) = v_f(4) = t + 1$, $v_f(1) = v_f(3) = t$
TYPE F: $v_f(3) = v_f(4) = t + 1$, $v_f(1) = v_f(2) = t$

TYPE A: $v_f(1) = v_f(2) = t + 1$, $v_f(3) = v_f(4) = t$

In this case $v_g(1) = v_g(2) = v_g(3) = v_g(4) = 2t + 1$ and $e_f(0) = e_f(1) = q + 2t + 1$. Hence g is a 4-prime cordial labeling of $G \odot K_1$.

TYPE B: $v_f(1) = v_f(3) = t + 1$, $v_f(2) = v_f(4) = t$

In this case interchange the labels of any vertices v_i and v_j such that label of v_i is 1 and label of v_j is 3.

TYPE C: $v_f(1) = v_f(4) = t + 1$, $v_f(2) = v_f(3) = t$

Clearly the vertex labeling in case 1 is also 4-prime cordial labeling of this type.

TYPE D: $v_f(2) = v_f(3) = t + 1$, $v_f(1) = v_f(4) = t$.

In this type also the vertex labeling g in case 1, a 4-prime cordial labeling of $G \odot K_1$.

TYPE E: $v_f(2) = v_f(4) = t + 1$, $v_f(1) = v_f(3) = t$

Interchanging the labels of two vertices u_i and v_j such that label of $v_i = 1$ and label of $v_j = 2$.

TYPE F: $v_f(3) = v_f(4) = t + 1$, $v_f(1) = v_f(2) = t$

Obviously the labeling f as in case 1 is a 4-prime cordial labeling.

Case 4. $p \equiv 3 \pmod 4$.

Let $p = 4t + 3$. The following types are occurs.

TYPE A: $v_f(1) = v_f(2) = v_f(3) = t + 1$, $v_f(4) = t$
TYPE B: $v_f(1) = v_f(2) = v_f(4) = t + 1$, $v_f(3) = t$
TYPE C: $v_f(1) = v_f(3) = v_f(4) = t + 1$, $v_f(2) = t$
TYPE D: $v_f(2) = v_f(3) = v_f(4) = t + 1$, $v_f(1) = t$

TYPE A and TYPE C:

Interchange the labels of two vertices v_r and v_t such that label of $v_r = 1$ and label of $v_t = 3$.

TYPE B and TYPE D:

In this type interchange the label of v_r and v_t in such a way that label of $v_r = 2$ and label $v_t = 3$. □

Now we investigate the subdivision of wheel and helm.

THEOREM 2.6. *$S(W_n)$ is 4-prime cordial.*

PROOF. Let C_n be the cycle $u_1u_2\ldots u_nu_1$ and $W_n = C_n + K_1$ where $V(K_1) = \{u\}$. Let $v_i, w_i (1 \leqslant i \leqslant n)$ be the vertices which subdivide the rim edges and spokes edges respectively. Note that $S(W_n)$ has $3n+1$ vertices and $4n$ edges.

Case 1. $n \equiv 0 \pmod 4$

Let $n = 4t$, $t \geqslant 1$. Assign the label 2 to the central vertex. Next assign the label 2 to the vertices $w_1, w_2, w_3, \ldots, w_{3t}$ and 4 to t vertices $w_{3t+1}, w_{3t+2}, \ldots, w_{4t}$. Next we move to the rim vertices. Assign the label 4 to the vertices u_1,u_2,..., u_t, v_1,v_2, ...,v_t. In the case of even values of t, assign the label 3 to the vertices u_{t+1}, v_{t+1}, u_{t+2}, v_{t+2}, ..., $u_{\frac{3t+2}{2}}$, $v_{\frac{3t+2}{2}}$. For the odd t, assign the label 3 to the vertices $u_{t+1}, u_{t+2}, \ldots, u_{\frac{3t+3}{2}}$ and $v_{t+1}, v_{t+2}, \ldots, v_{\frac{3t+1}{2}}$. We now assign the label 3 to the vertices $v_{\frac{3t+4}{2}}, v_{\frac{3t+6}{2}}, \ldots, v_{\frac{7t-2}{2}}$ or $u_{\frac{3t+5}{2}}, u_{\frac{3t+7}{2}}, \ldots, u_{\frac{7t-1}{2}}$ according as t is even or odd. Finally assign the label 1 to the non labeled vertices.

Case 2. $n \equiv 1 \pmod 4$

Let $n = 4t+1$. Assign the label to the vertices $u, u_i, v_i, w_i (1 \leqslant i \leqslant n-1)$ as in case 1. Next assign the labels $4, 1, 3$ respectively to the vertices w_n, u_n, v_n. Finally interchange the labels of the vertices $u_{\frac{7t-2}{2}}$ and u_n or $u_{\frac{7t-1}{2}}$ and u_n according as t is even or odd.

Case 3. $n \equiv 2 \pmod 4$

As in case 2 assign the label to the vertices $u, u_i, v_i, w_i (1 \leqslant i \leqslant n-1)$. Next assign the labels $2, 3, 1$ respectively to the vertices w_n, u_n, v_n.

Case 4. $n \equiv 3 \pmod 4$

Assign the label to the vertices $u, u_i, v_i, w_i (1 \leqslant i \leqslant n-1)$ as in case 3. Finally assign the labels $2, 1, 4$ respectively to the vertices w_n, u_n, v_n. The table 1 is establish that this vertex labeling f is a 4-prime cordial labeling.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$v_f(4)$	$e_f(0)$	$e_f(1)$
$n = 4t$	$3t$	$3t+1$	$3t$	$3t$	$8t$	$8t$
$n = 4t+1$	$3t+1$	$3t+1$	$3t+1$	$3t+1$	$8t+2$	$8t+2$
$n = 4t+2$	$3t+2$	$3t+2$	$3t+2$	$3t+1$	$8t+4$	$8t+4$
$n = 4t+3$	$3t+3$	$3t+3$	$3t+2$	$3t+2$	$4t+6$	$4t+6$

TABLE 1

□

THEOREM 2.7. *The subdivision of helm, $S(H_n)$, is 4-prime cordial.*

PROOF. Let $V(S(H_n)) = V(S(W_n)) \cup \{x_i, y_i : 1 \leqslant i \leqslant n\}$ and $E(S(H_n)) = E(S(W_n)) \cup \{x_i y_i, u_i x_i : 1 \leqslant i \leqslant n\}$. Clearly $S(H_n)$ has $5n+1$ vertices and $6n$ edges. We now give the 4-prime cordial labeling to the vertices of $S(H_n)$ as follows: Assign the labels to the vertices of $S(W_n)$ as in Theorem 2.6.

Case 1. $n \equiv 0 \pmod 4$.

Assign the labels 2 to the pendent vertices $y_1, y_2, \ldots, y_t$. Next assign the label 3 to any of the t non labeled x_i vertices such that the label of whose neighbor's is 3. Now assign the label 3 to the the corresponding vertices y_i. That is the t pairs of vertices (x_i, y_i) received the label 3. Next assign the label 4 to the any of $2t$ non labeled vertices and 1 to the remaining $2t$ vertices.

Case 2. $n \equiv 1 \pmod 4$.

Assign the label $u, u_i, v_i, w_i (1 \leqslant i \leqslant n)$ as in case 2 of Theorem 2.6. Next assign the label to the vertices $x_i, y_i (1 \leqslant i \leqslant n-1)$ as in case 1. Finally assign the labels $3, 1$ respectively to the vertices x_n and y_n.

Case 3. $n \equiv 2 \pmod 4$.

As in case 2 assign the labels to the vertices $u, u_i, v_i, w_i, x_i, y_i (1 \leqslant i \leqslant n-1)$ and u_n, v_n, w_n as in case 2 of Theorem 2.6. Now assign the labels $2, 4$ to the vertices x_n and y_n respectively.

Case 4. $n \equiv 3 \pmod 4$.

Assign the labels to $u, u_i, v_i, w_i, x_i, y_i (1 \leqslant i \leqslant n-1)$ as in case 3 and u_n, v_n, w_n as in case 3 of Theorem 2.6. Next assign the label $4, 3$ to the vertices x_n and y_n respectively. Finally interchange the labels of u_n and v_n. □

References

[1] I.Cahit.. Cordial Graphs: A weaker version of Graceful and Harmonious graphs, *Ars combin.,* **23** (1987) 201-207.

[2] J.A.Gallian. A Dynamic survey of graph labeling, *The Electronic Journal of Combinatorics*, **17** (2015) #Ds6.

[3] F.Harary. *Graph theory.* Addision wesley, New Delhi 1969.

[4] R.Ponraj, Rajpal Singh, R.Kala and S. Sathish Narayanan. k-prime cordial graphs, *J. Appl. Math. & Informatics*, **34 (3-4)** (2016) 227 - 237.

[5] R.Ponraj, Rajpal Singh and S. Sathish Narayanan. 4-prime cordiality of some classes of graphs, *Journal of algorithms and computation*, **48(1)**(2016), 69- 79.

[6] R.Ponraj, Rajpal Singh and S. Sathish Narayanan. 4-prime cordiality of some cycle related graphs, *Applications and Applied Mathematics*, **12(1)**(2016), 230-240.

[7] R.Ponraj and Rajpal Singh. 4-Prime cordial graphs obtained from 4-prime cordial graphs, *Bull. Int. Math. Virtual Inst.*, **8**(1)(2018), 1-9.

[8] R.Ponraj, Rajpal Singh and R.Kala. Some more 4-Prime cordial graphs, *International Journal of Mathematical Combinatorics*, **2** (2017) 105-115.

[9] R.Ponraj, Rajpal Singh and R.Kala. Some new families of 4-Prime cordial graphs, *Submitted to the Journal.*

[10] U.M.Prajapati and N.B.Patel. Some Edge Product Cordial Graphs in the Context of Duplication of Some Graph Elements, *Open Journal of Discrete Mathematics*, **6**(2016), 248-258.

[11] U.M.Prajapati and N.B.Patel. Edge Product Cordial Labeling of Some Cycle Related Graphs, *Open Journal of Discrete Mathematics*, **6**(2016), 268-278.

[12] M.A.Seoud and M.A.Salim. Two upper bounds of prime cordial graphs, *JCMCC*, **75**(2010) 95-103.
[13] M.Sundaram, R.Ponraj and S.Somasundaram. Prime cordial labeling of graphs, *J. Indian Acad. Math.,* **27**(2005) 373-390.
[14] M. Sundaram, R. Ponraj and S. Somasundaram. Product cordial labeling of graphs, *Bull. Pure and Appl. Sci. (Math. & Stat.)*, **23E** (2004) 155-163.

Received by editors 27.12.2016; Accepted 12.07.2017; Available online 17.07.2017.

R.Ponraj. Department of Mathematics, Sri Paramakalyani College, Alwarkurichi-627412, India.
E-mail address: ponrajmaths@gmail.com

Rajpal Singh. Research Scholar, Department of Mathematics, Manonmaniam Sundaranar University, Tirunelveli-627012, India.
E-mail address: rajpalsingh@gmail.com

R.Kala. Department of Mathematics, Manonmaniam Sundaranar University, Tirunelveli - 627012, India.
E-mail address: karthipyi91@yahoo.co.in