The Load Capacity for the System of Rotating Discs under the Second Order Rotatory Theory of Hydrodynamic Lubrication

Dr. Mohammad Miyan
Associate Professor, Department of Mathematics,
Shia P.G.College, University of Lucknow, Lucknow,
Uttar Pradesh, India -226020,
Email: miyanmohd@rediffmail.com

Abstract: The system of the lubrication of discs can be made kinematically equivalent to gears if they have the same radius at their contact line and rotate at same angular velocities as the gears. For the system of discs, we will take the origin at the surface of disc of radius R on the line of centers of the two discs. In the present paper, there are some new solutions with the help of geometrical figures, derivation for the expression for the load capacity, calculated tables and graphs for the load capacity in the view of second order rotatory theory of hydrodynamic lubrication. The analysis of equation for load capacity, tables and graphs reveal that the load capacity are not independent of viscosity and increases slightly with viscosity. Also the load capacity increases with increasing values of rotation number. The relevant tables and graphs confirm these results in the present paper.

Keywords: Load capacity, Lubricating discs, Reynolds equation, Rotation number, Taylor's number, Viscosity.

1. Introduction:

The concept of two dimensional classical theories of lubrication [3], [7] was given by Osborne Reynolds [8]. In the wake of an experiment by Beauchamp Tower [9], he had given a differential equation that was said as Reynolds Equation [8]. The basic mechanism and formation of the fluid film was observed by that experiment by considering some assumptions that the film thickness of fluid is much smaller than its axial and longitudinal dimensions and if lubricant layer is to produce pressure between the bearing and the shaft then the layer will vary the thickness of the fluid film.

After some period Osborne Reynolds again revised his own differential equation that was the much improved version and was said as: Generalized Reynolds Equation [4], [7]. The differential equation depends on viscosity, density, film thickness, transverse and surface velocities. The concept of the rotation [1] of the fluid film about an axis, which lies across the fluid film, gives some excellent solutions in the lubrication problems of the fluid mechanics. The origin of rotation was observed by some theorems of vorticity in the rotating fluid dynamics. The rotation induces component of vorticity in the direction of rotation of fluid film and effects arising from it are predominant, for large Taylor's Number, it results in streamlines becoming confined to the plane transverse to direction of rotation of the fluid film.

The latest extended version of the Generalized Reynolds Equation [4], [7] is known as the Extended Generalized Reynolds Equation that takes into account of effects of uniform rotation about an axis, which lies across the fluid film and depends on rotation number M [1], which is the square root of the classical Taylor's Number. The generalization of the theory of hydrodynamic lubrication is said as the Rotatory Theory of Hydrodynamic Lubrication [1], [2]. The concept of the Second Order Rotatory Theory of Hydrodynamic Lubrication [2], [5] was given by retaining terms containing up to second powers of M and by neglecting higher powers of M.

The lubrication of discs can be made kinematically equivalent to gears if they have the same radius at their contact line and rotate at the same angular velocities as the gears. For the system of discs, we will take the origin at the surface of disc of radius R on the line of centers of the two discs. The geometry of the system of discs is given by the figure (1.1) and figure (1.2).

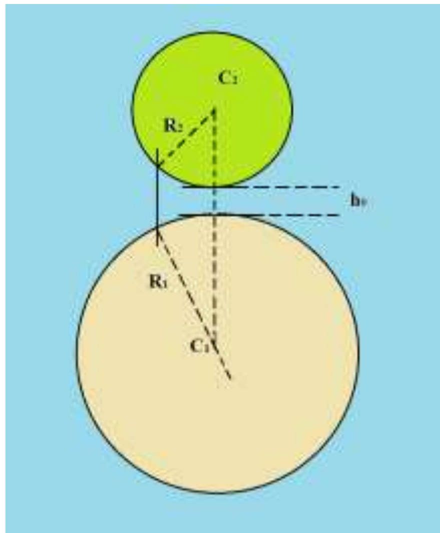

Figure- (1.1) (Geometry of system of discs)

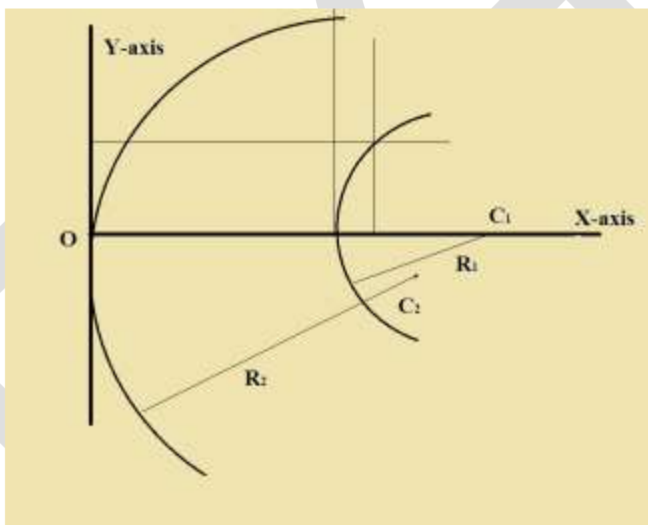

Figure- (1.2) (Film thickness of system of discs)

The film thickness 'h' is given by

$$h = h_0\left[1 + \frac{y^2}{2h_0}\left(\frac{1}{R_1} \mp \frac{1}{R_2}\right)\right] \qquad (1.1)$$

$$\left(\frac{1}{R_1} \mp \frac{1}{R_2}\right) = \frac{1}{R} \qquad (1.2)$$

$$tan\theta = \frac{y}{\sqrt{2Rh_0}} \qquad (1.3)$$

$$h = h_0 sec^2\,\theta \qquad (1.4)$$

Let us suppose that the disc is stationary at the lower surface transverse to the fluid film where sliding is absent and U=+U (constant). Suppose the variation of pressure in x-direction is very small as compared to the variation of pressure in y-direction. So the terms containing pressure gradient $\partial p/\partial x$ can be neglected in comparison to the terms containing $\partial p/\partial y$ in the differential equation of pressure, hence P may be taken as function of y alone.

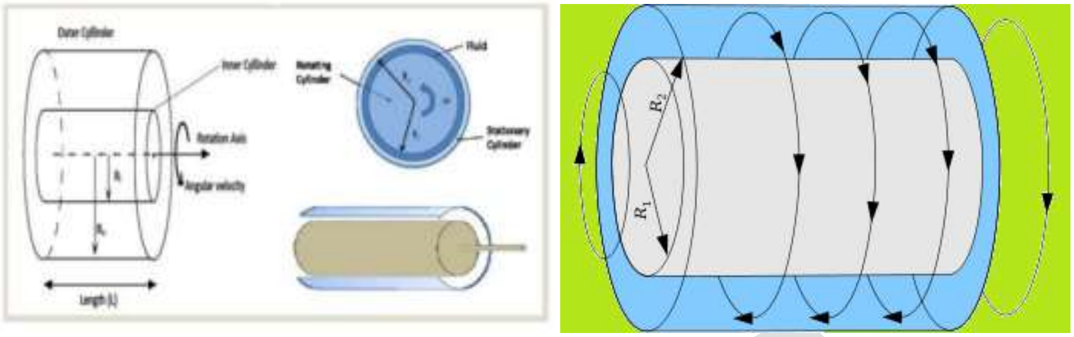

Figure- (1.3) (System of rotation of discs [11], [12])

2. Governing Equations and Boundary Conditions:

The Extended Generalized Reynolds Equation [4] for the second order rotatory theory of hydrodynamic lubrication is given by equation (2.1).

$$\frac{\partial}{\partial x}\left[-\sqrt{\frac{2\mu}{M\rho}}\,\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}\;-\;\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}\;+\;\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial x}\right]+\frac{\partial}{\partial y}\left[-\sqrt{\frac{2\mu}{M\rho}}\,\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}\;-\;\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}\;+\;\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial y}\right]$$
$$+\frac{\partial}{\partial x}\left[-\frac{h}{M}+\sqrt{\frac{2\mu}{M\rho}}\,\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}\;+\;\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}\;+\;\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial y}\right]-\frac{\partial}{\partial y}\left[-\frac{h}{M}+\sqrt{\frac{2\mu}{M\rho}}\,\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}\;+\;\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}\;+\;\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial x}\right]$$
$$=-\frac{U}{2}\frac{\partial}{\partial x}\left[\rho\sqrt{\frac{2\mu}{M\rho}}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}\;+\;\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}\;+\;\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\right]$$
$$-\frac{U}{2}\frac{\partial}{\partial y}\left[-\rho\sqrt{\frac{2\mu}{M\rho}}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}-\;\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}-\;\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\right]-\rho W^{*} \tag{2.1}$$

Where x, y and z are coordinates, μ is the viscosity, U is the sliding velocity, P is the pressure, ρ is the fluid density, and W^{*} is fluid velocity in z-direction. The Extended Generalized Reynolds Equation for the second order rotatory theory of hydrodynamic lubrication, in ascending powers of rotation number M can be written as equation (2.2). For the case of pure sliding $W^{*}=0$, so we have the equation as given:

$$\frac{\partial}{\partial x}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial x}\right]+\frac{\partial}{\partial y}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right]+\frac{\partial}{\partial x}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial y}\right]$$
$$-\frac{\partial}{\partial y}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial x}\right]$$
$$=-\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\}\right]\quad-\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\}\right] \tag{2.2}$$

$$\frac{\partial}{\partial y}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right]+\frac{\partial}{\partial x}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial y}\right]$$
$$=-\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\left\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\right\}\right]\quad-\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\left\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right\}\right] \tag{2.3}$$

Taking $h=h(y)$, $U=+U$, $P=P(y)$ (2.4)

$$\frac{d}{dy}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{dP}{dy}\right]=-\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\left\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right\}\right] \quad (2.5)$$

For the determination of pressure distribution in the positive regions, the boundary conditions are as follows:
(i) *P=0 at* $h=h_0$ *or P=0 at y=0 or P=0 at* $\theta=0$
(ii) $P=dP/d\theta=0$ *at* $y=y_1$ *or* $\theta=\gamma$ (say) (2.6)
Where is determined by putting $\theta=\gamma$ and P=0 in the equation of pressure.

3. Pressure Equation:

The solution of the differential equation under the boundary conditions imposed is given by

$$P=-\sqrt{\frac{Rh_0}{2}}\,M\rho U\left[\frac{17\,M^2\rho^2{h_0}^4}{1680\mu^2}\,F(\theta)-\tan\theta\,F(\gamma)\right] \quad (3.1)$$

Where $F(\theta)$ is given by

$$F(\theta)=\tan\theta\left[\frac{1}{9}\,sec^8\theta+\frac{8}{63}\,sec^6\theta+\frac{48}{315}\,sec^4\theta+\frac{192}{945}\,sec^2\theta+\frac{384}{945}\right] \quad (3.2)$$

4. Load Capacity:

The load capacity is given by

$$W=\int_{\gamma}^{0}P\,dy \quad (4.1)$$

$$=\int_{\gamma}^{0}P\,sec^2\theta\,\sqrt{2Rh_0}\,d\theta$$

$$=\sqrt{2Rh_0}\int_{\gamma}^{0}P\,sec^2\theta\,\,d\theta$$

$$W=Rh_0M\rho U\left[\frac{17M^2\rho^2{h_0}^4}{1680\mu^2}\int_{\gamma}^{0}F(\theta)sec^2\theta\,\,d\theta-\int_{\gamma}^{0}\tan\theta\,\,sec^2\theta\,\,d\theta\right] \quad (4.2)$$

$$W=-\frac{17M^3\rho^3{h_0}^3UR}{1680\mu^2}\left(\frac{1}{10}+\frac{1}{90}sec^{10}\gamma+\frac{1}{63}sec^8\gamma+\frac{8}{315}sec^6\gamma+\frac{16}{315}sec^4\gamma+\frac{192}{945}tan^2\gamma\right)-\frac{Rh_0M\rho U}{2}\,tan^2\gamma \quad (4.3)$$

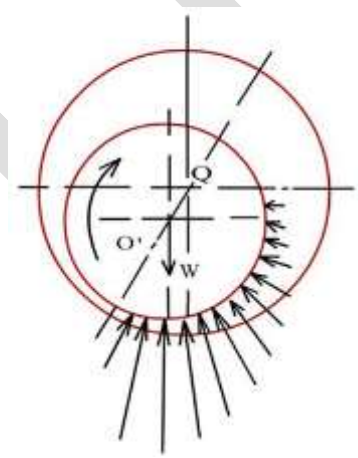

Figure-4.1 (Load capacity [10])

5. Calculation tables and graphs:

By taking the values of different mathematical terms in <u>C.G.S.</u> system the calculated tables and graphical representations are as follows:

ISSN 2091-2730

5.1 Table:

$U = 80,\ \rho = 1.0,\ R = 3.35,\ h_o = 0.0167,\ \mu = 0.0002,\ \theta = 30^0,\ \gamma = 60^0$

S.NO.	M	W
1.	0.1	6.2844159
2.	0.2	46.2472872
3.	0.3	153.5670693
4.	0.4	361.9222176
5.	0.5	704.9911875
6.	0.6	1216.452434
7.	0.7	1929.984414
8.	0.8	2879.265581
9.	0.9	4097.974391
10.	1.0	5619.7893

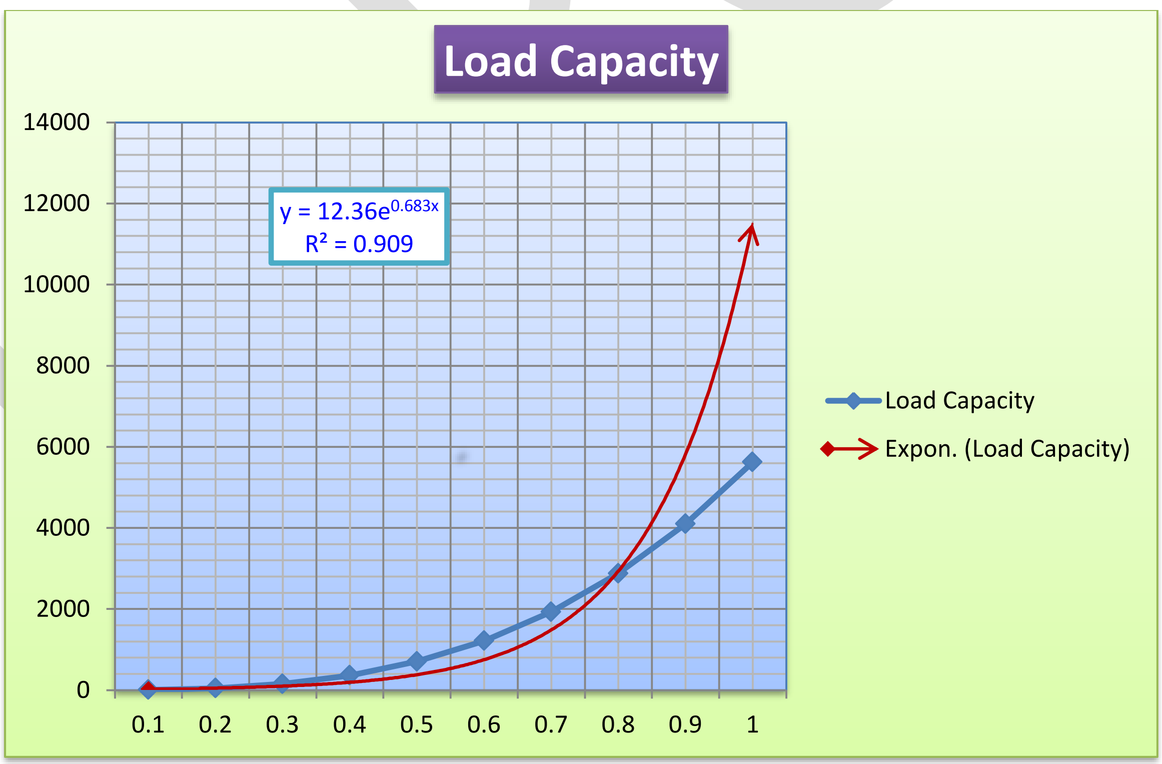

Figure-5.1 (Variation of load capacity with respect to rotation number M with exponential trendline)

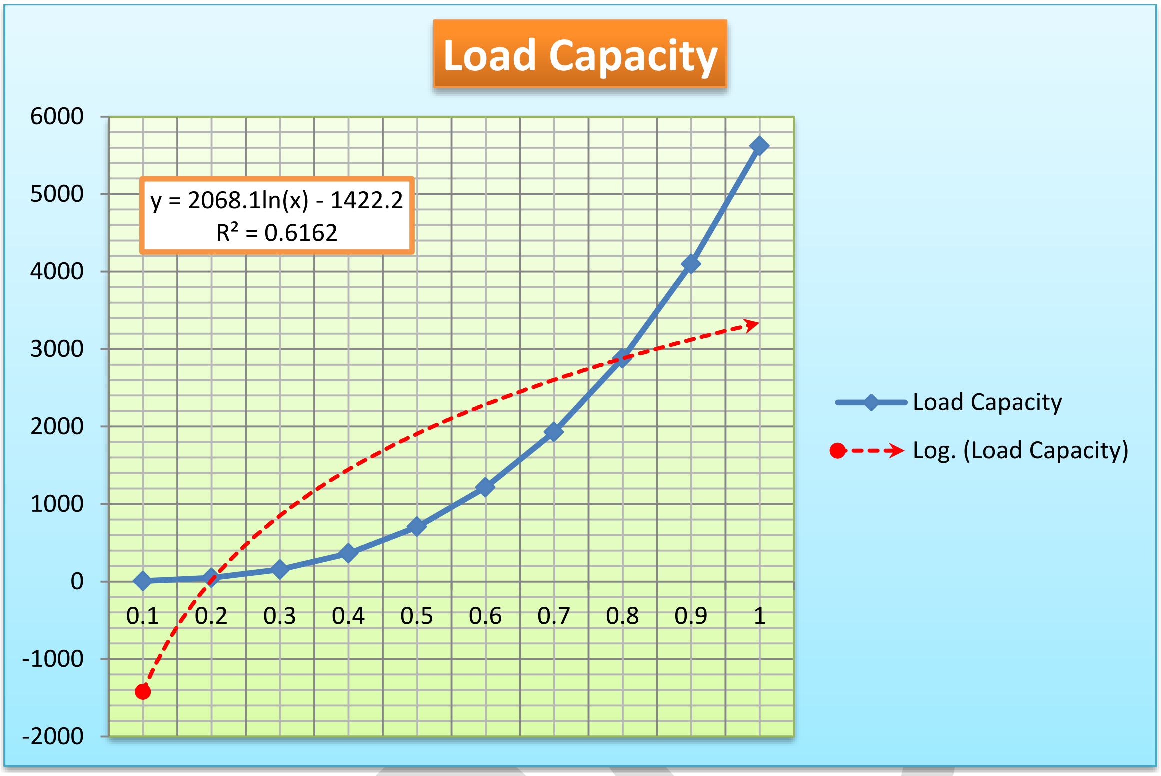

Figure-5.2 (Variation of load capacity with respect to rotation number M with logarithmic trendline)

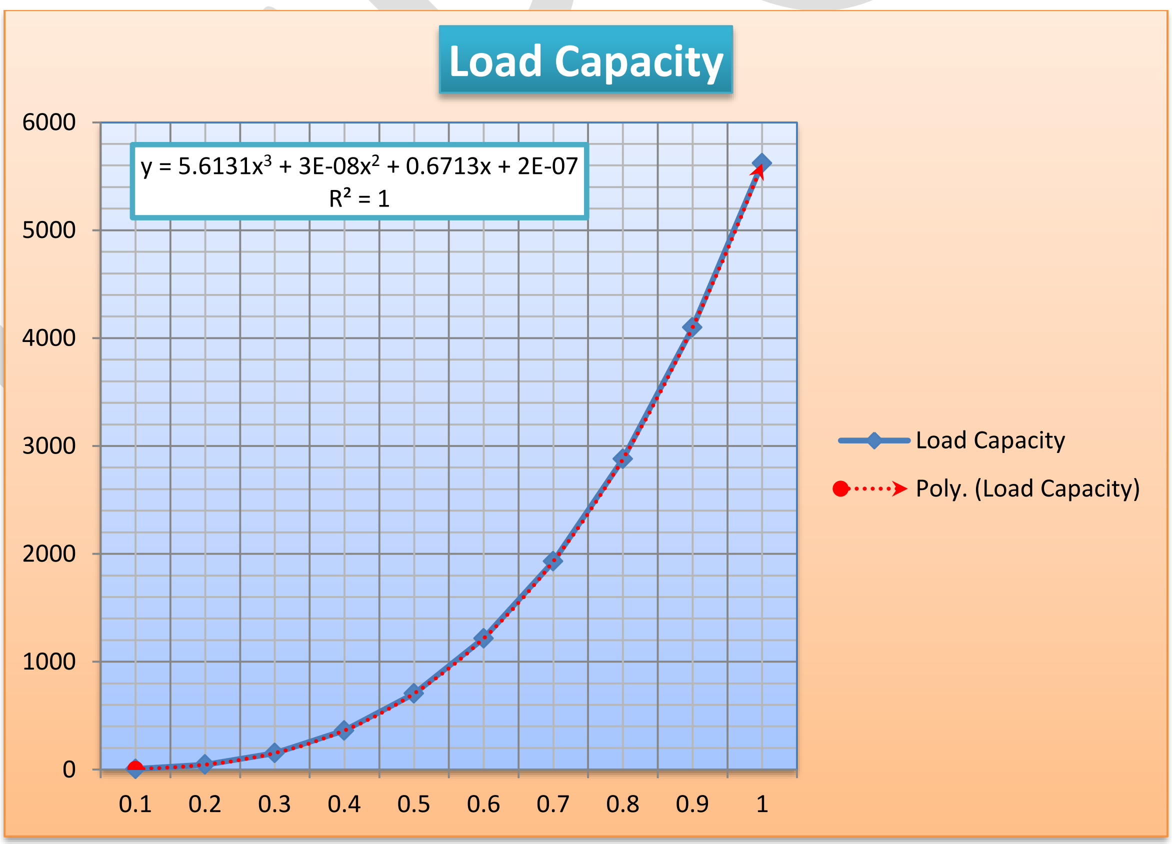

Figure-5.3 (Variation of load capacity with respect to rotation number M with third degree polynomial trendline)

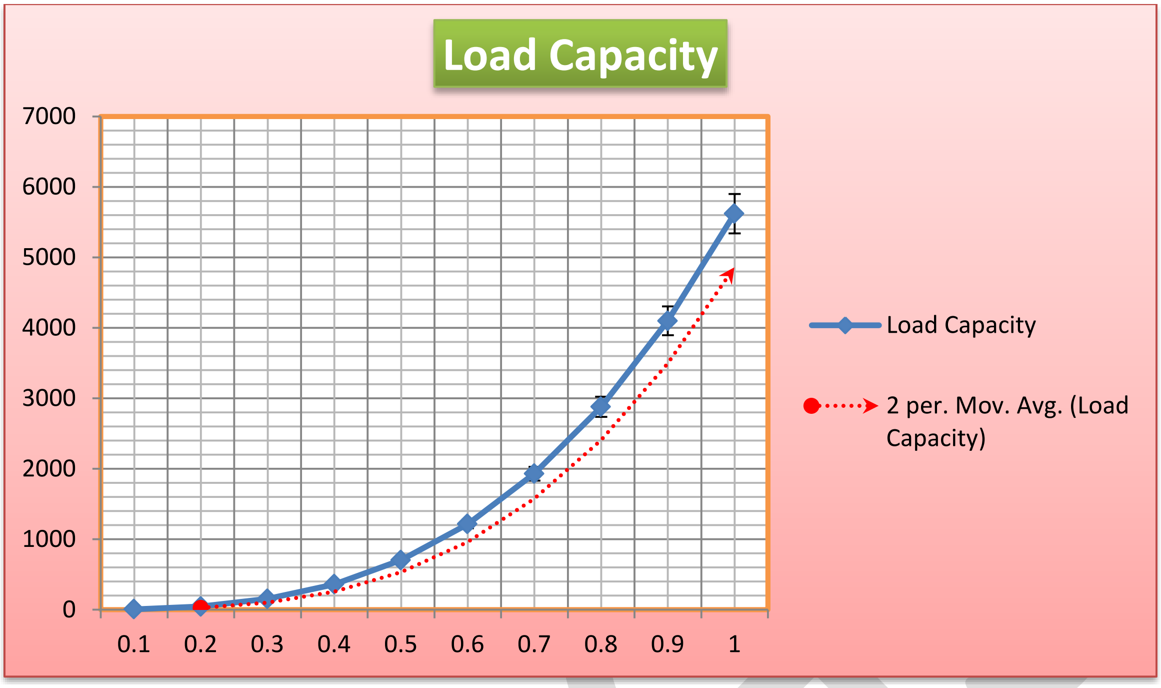

Figure-5.4 (Variation of load capacity with respect to rotation number M with 2-period moving average trendline with 5% error bars)

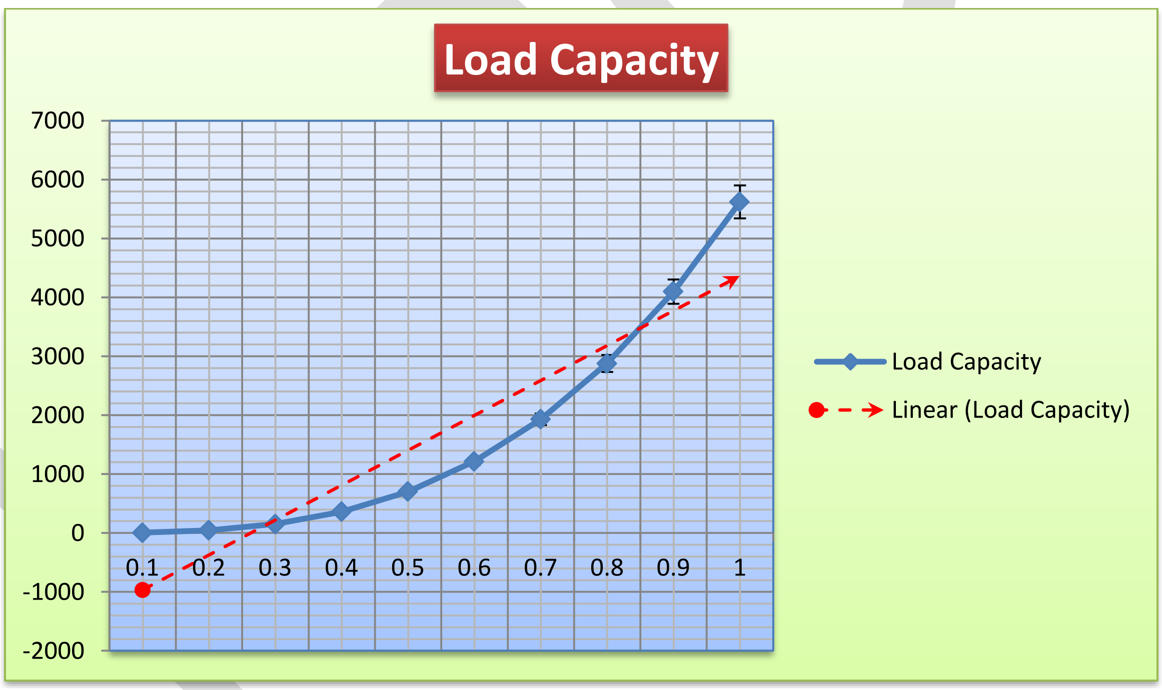

Figure-5.5 (Variation of load capacity with respect to rotation number M with linear trendline with 5% error bars)

6. Results and Discussion:

The variation of load capacity with respect to rotation number M is shown by the table and graphs. The figure-5.1 shows the exponential trendline by $y=12.36\ e^{\ 0.683x}$ with $R^2=0.909$. The figure-5.2 shows the logarithmic trendline by $y=2068\ log_e\, x\text{-}1422$ with $R^2=0.616$. The figure-5.3 shows the third degree polynomial trendline by $y=5.613\ x^3\text{-}3E\text{-}08X^2+0.671X+2E\text{-}07$ with $R^2=1$. The figure-5.4 shows the variation of load capacity with respect to rotation number M with two periods moving average trendline with 5% error bars. The figure-5.5 shows the variation of load capacity with respect to rotation number M with linear trendline with 5% error bars.

7. Conclusion:

The derived equation of load capacity is given by equation (4.3). The calculated values of the load capacity against rotation number M is shown in the table and graphical representation for the variation of load capacity is also shown by figure-5.1 to figure-5.5. The comparisons of the load capacity have been done with the help of geometrical figures, expressions, calculated tables and graphs for the lubricating discs in the second order rotatory theory of hydrodynamic lubrication. The analysis of equations for load capacity, tables and graphs show that load capacity is not independent of viscosity and increase with increasing values of rotation number.

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