REVIEW OF STABILITY ANALYSIS OF NON-LINEAR CONTROL SYSTEMS

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Abstract- The importance of control system in mechanical, electrical and electronics has been enhanced to a great extent in the recent years. This paper provides an evaluation of several stability analysis strategies to the non-linear control systems. Nonlinear control systems pertain with nonlinear systems that are time-variant. As control system design in general aims to satisfy certain performance objectives, such as stability, accurate input tracking, disturbance rejection, and robustness or insensitivity to parameter uncertainty, the dissimilar nature of nonlinear systems essentially demands for a diversity of design methods of different nature. The major considerations of this study are transfer functions along with stability and steady state error. To improve the stability of the nonlinear systems, numerous researches have been proposed.

Keywords – Control systems, Non-linear control systems, Stability analysis, Lyapunov, Popov criterion, fuzzy controller, Takagi-Sugeno.

INTRODUCTION

A Control system is a device, or set of devices, that manages, commands, directs or regulates the behavior of other devices or systems. Industrial control systems are used in industrial production for controlling equipment or machines. The main feature of control system is, there should be a clear mathematical relation between input and output of the system. When the relation between input and output of the system can be represented by a linear proportionality, the system is called linear control system. Again when the relation between input and output cannot be represented by single linear proportionality, rather the input and output are related by some non-linear relation, the system is referred as non-linear control system.

There are two common classes of control systems, open loop control systems and closed loop control systems. In open loop control systems output is generated based on inputs. In closed loop control systems current output is taken into consideration and corrections are made based on feedback. A closed loop system is also called a feedback control system. The controlling mechanism is classified into linear control system and non-linear control system. Most of the theories and practices focus on feedback control. A typical layout of a feedback control system is shown in Figure 1 [1].

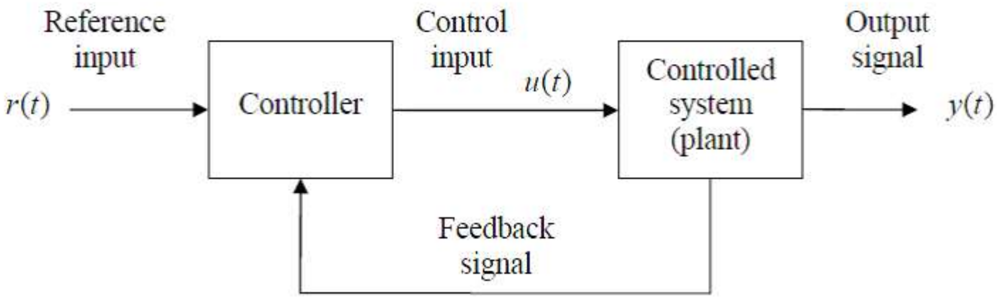

Figure 1: Basic Feedback control system

Nonlinear control systems are the systems that are nonlinear, time-variant, or both. Control theory is an interdisciplinary branch of engineering and mathematics that is concerned with the behavior of dynamical systems with inputs, and how to modify the output by changes in the input using feedback. The system to be controlled is called the "plant". In order to make the output of a system follow a desired reference signal a controller is designed which compares the output of the plant to the desired output, and provides feedback to the plant to modify the output to bring it closer to the desired output.

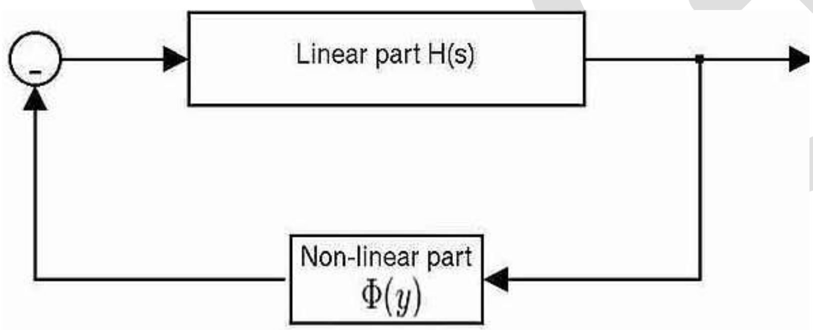

Figure 2: Nonlinear Control System

Nonlinear control systems are control systems in which nonlinearity plays a substantial role, either in the controlled process (plant) or in the controller itself. Nonlinear plants ascend definitely in numerous engineering and natural systems, including mechanical and biological systems, aerospace and automotive control, industrial process control, and many others.

The control system which contains at least one nonlinear factor is called as nonlinear control system. The nonlinear factor is the static characteristic between the input and output that cannot satisfy the linear relationship. The subject of nonlinear control system includes all of the mathematical relationship except the linear relationship. Thus there is no common design method for nonlinear systems.

Properties of nonlinear control systems are:

- They do not follow the principle of superposition (linearity and homogeneity).
- They may have multiple isolated equilibrium points.
- They may exhibit properties such as limit cycle, bifurcation, chaos.
- Finite escape time: Solutions of nonlinear systems may not exist for all times.

Nonlinear control systems are classified into two types: Smooth nonlinearities and non-smooth nonlinearities. Smooth nonlinearities are theoretically easier to deal with than non-smooth nonlinearities. The nonlinearities are, in a sense, invertible. Indeed, there exists a class of smooth nonlinear problems which are “close to” linear in the sense that they can be converted, via feedback, into linear problems. The associated design strategy is known as “feedback linearization”.

Non-smooth nonlinearities are typically more challenging as they lack a globally definable inverse. One of the most common forms of non-smooth nonlinearity is that of actuator amplitude and/or slew rate limits. Approaches to this problem can be classified as being unexpected, cautious, evolutionary, and tactical. In the serendipitous approach, one allows occasional violation of the constraints but no particular precautions are taken to mitigate the consequences of the constraints [2].

Stability analysis plays a major role in control systems. Several techniques and methodologies exist for the analysis and design of nonlinear control systems. A brief and generic explanation of some prominent ones is presented in the literature review.

LITERATURE REVIEW

Stability of a system covers a wider class of systems that do not obey the superposition principle, and applies to more real-world systems, because all real control systems are nonlinear. These systems are often governed by nonlinear differential equations. Advanced mathematical concepts like limit cycle theory, Poincaré maps, Lyapunov stability theorem, and describing functions are described in this section.

The problem of absolute stability of an indirect control system with one non-linearity was analysed by Popov [3] by using a special method which differs from the second method of Liapounoff. In the obtained criteria of absolute stability, main condition is expressed by means of the transfer function of the linear part of the system. It is proved that by plotting Liapounoff function of the general type, a quadratic form plus an integral from non-linearity, it is impossible to obtain stability area for the analysed problem (in the parameter space) wider than the area resulting from application of the suggested criteria.

Billings [4] proposed a study dealing with the identification of block-oriented and bilinear systems, selection of input signals; structure detection, parameter estimation and results from catastrophe theory were included. The limitations, relationships and applicability of the methods were discussed throughout. Although the identification time using these inputs was reduced by a factor of 70, compared with a Gaussian white input, anomalies appeared in the fourth-order autocorrelation functions.

Qu [5] presented the definitive treatment of stability analysis and robust control design for nonlinear uncertain systems. This was the first work to tackle robust control design for nonlinear entities as power systems, robotics, and more. It shows how to build high performance and better control into systems that are too complex to be modelled accurately. A unique feature of this research is that, Lyapunov-based approach to control design is employed, which is the only universal approach for nonlinear systems.

Cao and Lin [6] have described Takagi-Sugeno (TS) fuzzy models as an effective representation of complex nonlinear systems in terms of fuzzy sets and fuzzy reasoning applied to a set of linear input-output sub-models. In this study, the TS fuzzy modeling approach is utilized to carry out the stability analysis and control design for nonlinear systems with actuator saturation. The TS fuzzy representation of a nonlinear system subject to actuator saturation is obtained and the modeling error is also captured by norm-bounded uncertainties. A set invariance condition for the system in the TS fuzzy representation is first established. Based on this set invariance condition, the problem of estimating the domain of attraction of a TS fuzzy system under a constant state feedback law is formulated and solved as a linear matrix inequality (LMI) optimization problem. By viewing the state feedback gain as an extra free parameter in the LMI optimization problem, we arrive at a method for designing state feedback gain that maximizes the domain of attraction. A fuzzy scheduling control design method is also presented to further broaden the domain of attraction. An inverted pendulum is used to show the effectiveness of the proposed fuzzy controller.

Tomescu [7] presented a stability analysis method for nonlinear processes with Takagi-Sugeno (T-S) fuzzy logic controllers (FLCs). The design of the FLCs was established on heuristic fuzzy rules. The stability analysis of the fuzzy control systems were executed using LaSalle's invariant set principle with non-quadratic Lyapunov candidate function. This study explained that if the derivative of Lyapunov function is negative semi-definite in the active region of each fuzzy rule, then the overall system will be asymptotically stable in the sense of Lyapunov (ISL). The stability theorem suggested in the paper ensures sufficient stability conditions for fuzzy control systems controlling a class of nonlinear processes. Here the controller implements an (often heuristic) set of logical (or discrete) rules for synthesizing the control signal based on the observed outputs. Defuzzification and fuzzification procedures are used to obtain a smooth control law from discrete rules. The stability analysis algorithm ensuring the stability of the class of fuzzy logic control systems considered is based on their theorem.

Another research considers the delay-dependent stability analysis and controller design for uncertain T–S fuzzy system with time-varying delay. A new method is provided by introducing some free-weighting matrices and employing the lower bound of time-varying delay. Based on the Lyapunov–Krasovskii functional method, sufficient condition for the asymptotical stability of the system is obtained. By constructing the Lyapunov–Krasovskii functional appropriately, the researchers could avoid the supplementary requirement that the time-derivative of time-varying delay must be smaller than one. The fuzzy state feedback gain was derived through the numerical solution of a set of linear matrix inequalities (LMIs). The upper bound of time-delay was obtained by using convex optimization such that the system could be stabilized for all time-delays [8].

Marinosson [9] proposed a system where the stability properties of equilibrium points of general continuous autonomous nonlinear systems were analyzed via linear programming. The novel methods presented did not require the nonlinear system to be of any specific type, like piecewise affine, and show that it is possible to extract a lot of non-trivial information regarding stability by using linear constraints. The Linear Program LP1 gives explicit linear programs for dynamical systems such that if there is no feasible solution of the program, then the equilibrium of the system under consideration cannot be a constant. The bounds of the derivatives of the Lyapunov function are equally good and it hardly seems possible to improve them, without making major restrictions concerning the system in question. Linear Program LP2 gives explicit linear programs for dynamical systems too. Such a linear program, generated for a particular system, has the property that a feasible solution of it defines a continuous piecewise affine (CPWA) Lyapunov function for the system. In both cases a lower bound of the region of attraction is provided by the preimages of the Lyapunov or Lyapunov-like function.

Cho and Lee [10] present an adaptive control approach using a model matching technique for 3-DOF nonlinear crane systems. The proposed control is linearly composed of two control frameworks: nominal PD control and corrective control. A nonlinear crane model is approximated by means of feedback linearization to design nominal PD control avoiding perturbation. A corrective control to compensate system error feasibly occurring due to perturbation, which is derived by using Lyapunov stability theory with bound of perturbation, is proposed. Additionally, stability analysis is achieved for the proposed crane control system and they analytically derived sufficient stability condition with respect to its perturbation.

Sahu, Gupta and Subudhi [11] explained a new method to analyze the condition and region of stability of nonlinear time-varying systems by introducing the notions of dynamic poles and dynamic-Routh's stability method. The stability analysis is carried out in a special type of complex plane called g(t)-plane which is similar to the traditional s-plane where both real and imaginary axis have time dependent parameters. The theorem is established and numerical examples from literature are solved with simulation results to show the efficacy and accuracy of the proposed method. The phase plane curve is presented for showing the stability condition of the nonlinear time varying systems. Their theorem comprises of the following steps:

(i) The necessary condition for the nonlinear system to be stable is; all the elements of the first column of dynamic-Routh's array must have positive values.

(ii) If a zero is present on the first column of dynamic Routh's array, then the corresponding dynamic pole will oscillate on the imaginary axis.

(iii) The number of times of sign change of the elements of first column of dynamic-Routh's array is the number of dynamic poles on the right hand side of g-plane which force the nonlinear system to be unstable.

Yadav, Choudhary and Thirunavukarasu [12] discussed a new class of PID controller where the system to be controlled is assumed to be modeled or approximated by second-order transfer functions. The design is done by using sigmoidal function, which is represented as a nonlinear function. The controller design by nonlinear method presented consists of a nonlinear gain in cascade with a linear constant gain PID controller. The sigmoidal function represents nonlinear gain k as the function of the error e, as shown below.

$$k = k_0 + k_1 \left\{ \frac{2}{1 + \exp(-k_2 e)} - 1 \right\}$$

Where k_0, k_1 and k_2 are user-defined positive constants, the gain k is lower-bounded by $k_{min}=k_0-k_1$ when $e=-\infty$, that is upper-bounded by $k_{max}=k_0+k_1$ when $e=+\infty$, that is $k_{min}<k<k_{max}$, and furthermore $k=k_0$ when $e = 0$. Thus k defines the central value of k, k1. Linear PID controller for magnetic levitation gives better response, quick setting time with high precision and high stability. But the system response is up to certain limit. In real world applications process won't be stable. Definite uncertainties will be affecting the system throughout the process. So for such kind of uncertainties classical PID doesn't hold good.

Jankovic [13] illustrated a particular application where the disturbance decoupling paradigm is used to design a controller that coordinates the electronic throttle and variable cam timing actuators to achieve a desired transient engine performance. The problem of coordinating the electronically controlled throttle (ETC) and variable cam timing (VCT) actuators in controlling the torque of gasoline (spark-ignited) engines has been considered. The main challenge is due to the multivariable and nonlinear nature of the plant under control. To illustrate this, the steady state air intake in VCT engines where both actuators (the throttle and the VCT) control the amount of air and, in turn, engine torque are reflected.

Forbes [14] describes the input-output stability theory as in how inputs map to outputs through an operator that represents a system to be controlled or the controller itself. Within this framework, the Small Gain, Passivity, and Conic Sector Stability theorems have been used to assess the stability of a negative feedback interconnection involving two systems that each have specific input-output properties. Firstly, characterization of the input-output properties of Linear Time-Varying (LTV) systems are taken into consideration for which various theorems are presented, that ensure that a LTV system has finite gain, is passive, or is conic. Next, the stability of various negative feedback interconnections is examined. Motivated by the robust nature of passivity-based control, we consider how to overcome passivity violations. This investigation leads to the hybrid conic systems framework whereby systems are described in terms of multiple conic bounds over different operating ranges. A special case relevant to systems that experience a passivity violation is the hybrid passive/finite gain framework. Sufficient conditions are derived that ensure the negative feedback interconnection of two hybrid conic systems is stable.

Svarc [15] described the Popov criterion for the stability of nonlinear control systems. The Popov criterion gives sufficient conditions for stability of nonlinear systems in the frequency domain. It has a direct graphical interpretation and is convenient for both design and analysis. In the article presented, a table of transfer functions of linear parts of nonlinear systems is constructed. The tables include frequency response functions and offers solutions to the stability of the given systems. The table makes a direct stability analysis of selected nonlinear systems possible. The stability analysis is solved analytically and graphically. Then it is easy to find out if the nonlinear system is or is not stable.

A framework for real-time, full-state feedback, unconstrained, nonlinear model predictive control that combines trajectory optimization and tracking control in a single, unified approach has been examined. The proposed method uses an iterative optimal control algorithm, namely Sequential Linear Quadratic (SLQ), in a Model Predictive Control (MPC) setting to solve the underlying nonlinear control stability problem and simultaneously derive the optimal feedforward and feedback terms. The customized solver can generate trajectories of multiple seconds within only a few milliseconds. The performance of the approach is validated on two different hardware platforms, an AscTec Firefly hexacopter and the ball balancing robot Rezero [16].

Jake Bouvrie and Boumediene Hamzi explained briefly about the balanced reduction of the load in Nonlinear control systems that behaves linearly when lifted into a high (or infinite) dimensional feature space where balanced truncation may be carried out implicitly. A system expressed in the coordinates where each state is equally controllable and observable is called its balanced realization. This method uses samples of the impulse response of a linear system to construct empirical controllability and observability [17].

CONCLUSION

Control system design has reached a degree of significant development and there exists extensive understanding of “standard” stability problems of the nonlinear systems. The aim of this review is to briefly assess recent developments in nonlinear control and look past the various solutions to innovate a novel technique to stabilize the nonlinear control systems. The literature review describes several stability analysis techniques for the nonlinear control systems. We assume a general perspective in the solutions studied and propose an alternate solution for this work that could be established by reducing the nonlinear system into a linear system using trajectory optimization techniques.

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