Response of Structures Composed of Random Visco-elastic Material Properties

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Abstract— Visco-elasticity is the property of materials that exhibit both viscous and elastic characteristics when undergoing deformation due to applied external load. Visco-elastic materials (VEM) properties depend both on temperature and time. To mathematically characterize the properties of visco-elastic materials, Prony series and Laplace Transform are utilized. In general, these materials exhibit certain amount of scatter in their properties due to the dispersion in the values of material properties and applied external load. For design purposes, it is essential to know the potential variations in the structural response due to the system or external randomness. In the present work, the static and dynamic response of structures, where certain components are composed of visco-elastic materials are considered. Two cases are studied. The first consists of studying the response of solid rocket propellant grain and the second study is on vibration isolators for a PCB with aerospace application. The study considers the random response due to presence of inherent scatter in the values of applied loads and material properties. For sensitive applications where design margins are narrow, it is important to consider the dispersions in design parameters for optimal and safe design based on rational decisions than arbitrary safety factors.

Keywords— Visco-elastic Materials, Finite element method, Prony series, Equivalent modulus

NOMENCLATURE

D	Displacement
E	Young's modulus
P	Load
C_X	Coefficient of variation in the random variable X
δ	Deflection
E_e, ρ_i, E_i, D_q, τ_j, D_j,	Degrees of freedom of the Prony series
E_q,D_q	Independent terms of the Prony series
E_i , D_j	Dependent terms of the Prony series
$E(t)$	Relaxation modulus
G	Shear
$g(t)$	Shear relaxation modulus as a function of time
G	Acceleration due to gravity
G_∞	Long term modulus or Equilibrium modulus

INTRODUCTION

The mechanical properties of crystalline materials, at constant temperature, can be described in terms of stress and strain, the mathematical description of visco-elastic materials involves the introduction of a new variable time. Visco-elasticity and related phenomena are of great importance in the study of biological materials. Just as strain can be measured in more than one way, so the related rate of strain (i.e., the amount of strain per unit time) can be measured in a number of different ways [1]. A visco-elastic material will return to its original shape after any deforming force has been removed (i.e., it will show an elastic response) even though it will take time to do so (i.e., it will have a viscous component to the response). Visco-elasticity is the property of materials that exhibit both viscous (dashpot-like) and elastic (spring-like) characteristics when undergoing deformation. Food, synthetic polymers, wood, soil and biological soft tissue as well as metals at high temperature display significant visco-elastic effects. The term elasticity refers to solid materials, and in these cases the stress is a linear function of the strain. Visco-elastic material (VEM) properties depending on both temperature and time. When unloaded, it eventually returns to the original, undeformed state. [1].

Arvin. H [1] studied comparing molecular theories that describe VEMs behavior and models based on fractional derivatives. Their work demonstrated that, from a reduced number of parameters, it is possible to predict with some precision the dynamic behavior of those materials. A methodology was established to perform the modeling in finite beam elements and rectangular plates with a VEM layer, in order to attenuate the effect of vibration on structures. Ayatac Arikoglu [2] studied the vibration and thermal buckling behaviour of sandwich beams with composite facings and visco-elastic core in comparison with sandwich plate. They examined the effects of fiber angle, aspect ratio and the core thickness on the performance of the beam and plate elements. They concluded that there is a considerable difference between the predictions of mentioned elements for some fiber angles. Prony series is used in an attempt to apply an efficient numerical method in the time domain to relate relaxation and creep functions of VEMs, which were tested using experimental data from a few polymeric materials. Carlos Alberto Bavastri [4] discussed about the method for determining the Prony Series coefficients of a visco-elastic relaxation modulus using load versus time data for different sequences of load ratio adjusted to the convolution integrals of tested materials. For lightweight and flexible structures, the sandwich with a visco-elastic core is very effective in controlling and reducing vibration responses. Jayakumar. K et.al, [5] discussed about the Multivariate method to examine the response statistics to account scatter in the material properties.

In the present work, to generate the equivalent modulus of visco-elastic materials relaxation modulus versus time curve is used. This is utilized to carry out frequency and transient response analysis of structures composed of visco-elastic materials. The material randomness is considered i.e., material properties are not deterministic. Finite element method based software PreWin/ FEAST^{SMT} is used for analysis. PreWin/ FEAST^{SMT} is indigenous software continuously being developed at Vikram Sarabhai Space Centre/Indian Space Research Organization.

PRONY SERIES

For visco-elastic materials it is necessary to consider the time-dependent response. The time-dependent uniaxial stress $\boldsymbol{\sigma(t)}$ and strain $\boldsymbol{\varepsilon(t)}$ can be obtained by equation (1a and 1b) considering this material under isothermal, nonaging linear conditions [4].

$$\boldsymbol{\sigma(t) = \int_{\tau=0}^{\tau=t} E(t-\tau)\frac{\partial\varepsilon(t)}{\partial\tau}\,d\tau \qquad (1a)}\ , \qquad \boldsymbol{\varepsilon(t) = \int_{\tau=0}^{\tau=t} D(t-\tau)\frac{\partial\sigma(t)}{\partial\tau}\,d\tau \qquad (1b)}$$

where t is the time-like integration variable and $E(t)$and $D(t)$are experimentally obtained relaxation modulus and creep compliance, respectively.

Even though these material functions are obtained from experimental observation, it is necessary to represent them by mathematical functions in order to perform stress analysis on visco-elastic materials and to inter convert these visco-elastic functions. Among all analytical representations available, the Prony Series is one of the most used due to computational efficiency associated with its exponential basis function. The analytical description of relaxation modulus $E(t)$ and creep compliance $D(t)$ by Prony series is expressed in equation (2a and 2b)

$$E(t) = E_e + \sum_{i=1}^{M} E_i e^{-\frac{t}{\tau_i}} \qquad (2a), \qquad D(t) = D_g + \sum_{i=1}^{N} D_j\left(1 - e^{-\frac{t}{\tau_j}}\right) \qquad (2b)$$

where, E_e, ρ_i, E_i, and D_g, τ_j, D_j, are degrees of freedom of the Prony series. Equations (2a and 2b) are obtained representing the viscoelastic material by a mechanical model consisting of linear springs and dashpots. The terms E_e and D_g are called independent terms. The exponential terms ρ_i and τ_j are known as time constants because they appear in association with the time variable t. The set of terms E_i and D_j are the dependent terms of the Prony series and the number of terms used (M and N, respectively) is determined accordingly to the experimental data. Usually around 8 to 16 terms must be used in Prony series equation (2a and 2b) to have a satisfactory mathematical model to be fitted from experimental data [3]. The Prony series for the shear relaxation is

$$g(t) = G_\infty + \sum_{i=1}^{N} G_i e^{-\frac{t}{\tau_i}} \qquad (3)$$

where, $g(t)$ is the shear relaxation modulus, G_∞ is the long term modulus or equilibrium modulus, G_i is the constants, t is the time, τ_i is the relaxation times. The advantage of the Prony series is that its Laplace transform can be easily obtained.

PRONY SERIES IN LAPLACE TRANSFORM

Using the equation (3) for shear relaxation, when t = 0, shear relaxation modulus will be

$$g(t=0) = G_0 = G_\infty + \sum_{i=1}^{N} G_i \qquad (4)$$

From eq. (4) we can find G_∞, and substituting this in equation (3), we get

$$g(t) = G_0 - \sum_{i=1}^{N} G_i \left(1 - e^{-\frac{t}{\tau_i}}\right) \qquad (5)$$

Applying Laplace transform to eq. (5)

$$G(S) = \frac{G_0 - \sum_{i=1}^{N} G_i}{S} + \sum_{i=1}^{N} \frac{G_i}{S + \frac{1}{\tau_i}} \qquad (6)$$

Substituting the values of G_i, τ_i and $S = \frac{1}{2t}$, to determine, G_0

$$G_0 = \frac{E}{2(1+\nu)} \qquad (7)$$

where, E = Young's modulus (1594.11), ν = Poisson's ratio (0.4924). From this we can find the value of $g(t)$ and $G(S)$ for each time.

PROBABILISTIC ANALYSIS

Probabilistic composite mechanics and probabilistic composite structural analysis are formal methods, which are used to quantify the scatter that is observed in composite material properties and structural response [6]. Probabilistic structural response estimate for different random input parameters the following methods have widely been applied: Monte-Carlo simulation, Convex method, Multi-variate method.

In this present work, Multi- variate method is used for determining the statistical measures, mean and standard deviation. The sensitivity of the responses due to scatter in the values of system parameters are expressed using their respective (non-dimensional) coefficient of variation.

The dependent variable $\boldsymbol{\psi}$ and displacement $\boldsymbol{\delta}$ is functionally related to the basic random variables E , $\boldsymbol{\mu}$, $\boldsymbol{t}$, $\boldsymbol{l}$ and ρ. For mathematical simplicity these variables are denoted by X_1, X_2,…,X_5 ,respectively. It is assumed that these basic variables have small dispersion about their mean values $\boldsymbol{X_1^*}$, $\boldsymbol{X_2^*}$,…, $\boldsymbol{X_5^*}$, respectively. This implies a good quality control during the manufacturing process, which is true for the most aerospace applications. Under this assumption, $\boldsymbol{\psi}$ lends itself to Taylor series expansion about the mean values [5]. Keeping only up to second power of small quantities, $\boldsymbol{\psi}$ can be expressed as:

$$\boldsymbol{\psi(X_1, X_2, \ldots\ldots, X_3)} \\ = \boldsymbol{\psi\left(X_1^*, X_2^*, \ldots\ldots X_5^*\right) + \sum_{i=0}^{5} \frac{\partial\psi}{\partial X_i}(X_i - X_i^*) + \frac{1}{2}\sum_{i=1}^{5}\sum_{j=1}^{5} \frac{\partial^2\psi}{\partial X_i \partial X_j}(X_i - X_i^{\ *})\left(X_j - X_j^{\ *}\right)} \qquad (8)$$

All derivatives of $\boldsymbol{\psi}$ are evaluated at the mean values of the basic random variables [5]. The mean and the variance of the independent variable $\boldsymbol{\psi}$ are obtained from equation (8).

$$Var(\Psi) = E\left[(\Psi - \mu_X)^2\right]$$

$$\cong \frac{1}{2}\sum_{i=0}^{5} Var(X_i)\left(\frac{\partial \Psi}{\partial X_i}\right)^2 + \frac{1}{4}\sum_{i=1}^{5}\sum_{j=1}^{5}\frac{\partial^2\Psi}{\partial {X_i}^2}\frac{\partial^2\Psi}{\partial {X_j}^2} Var(X_i)Var(X_j) \qquad (9)$$

The positive square root of variance is the standard deviation.

A. Mean

The mean value μ_x is simply the average value. It is also known as the expected value. If X is continuous, probability density function of X is f_X (x): Therefore, the mean is

$$E[X] = \mu_x = \int_{-\infty}^{+\infty} \mathrm{x}\, \mathrm{f_X}\,(\mathrm{x})\mathrm{dx} \qquad (10)$$

B. Variance

The variance of $\boldsymbol{\sigma^2}_{\mathbf{X}}$ is the second moment of the PDF about the mean, expressed as Var(X):

$$\mathbf{Var(X)} = \int_{-\infty}^{+\infty} (\boldsymbol{\eta} - \boldsymbol{\mu}_{\mathbf{x}})^2\ .\mathbf{f_X}\,(\boldsymbol{\eta})\mathbf{d}\boldsymbol{\eta} \qquad (11)$$

C. Standard deviation

The positive square root of the variance is known as the standard deviation σ_X :

$$\boldsymbol{\sigma}_{\mathbf{X}} = \sqrt{\mathbf{Var\ (X)}} \qquad (12)$$

D. Coefficient of variation

Coefficient of variation C_X a non-dimensional term expressed in %, is the ratio of the standard deviation and the mean:

$$\mathbf{C_X} = \frac{\boldsymbol{\sigma}_{\mathbf{x}}}{\boldsymbol{\mu}_{\mathbf{x}}} \times \mathbf{100} \qquad (13)$$

Analysis

A. Static analysis for solid propellant rocket

A solid propellant rocket is the simplest form of chemical propulsion. The fuel and the oxidizer are both incorporated in a single solid called the propellant grain, located inside a container called the motor casing. A solid propellant is combustive matter, stable at room temperature, which once ignited, releases gas continuously at elevated temperature. A grain must hold its shape over an extended temperature range, and must withstand the stresses and strains imposed on it during handling, ignition, and firing in a rocket. A schematic illustration of the typical solid propellant rocket motor is shown in Figure 1.

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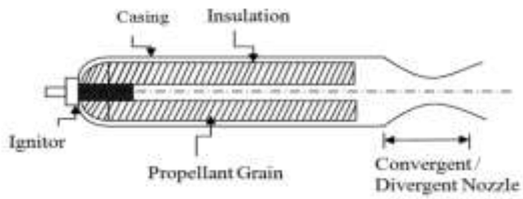

Figure 1: Typical Solid Propellant Motor

An axisymmetric cylindrical shaped solid propellant grain is modeled for structural analysis. Insulation as well as steel casing is provided around propellant grain. The typical material properties are considered.

The Young's modulus, pressure, Poisson's ratio are considered to have scatter about their mean, which are the random independent variables. The deflections are obtained for a range of dispersion in the random variables and are expressed as coefficient of variation.

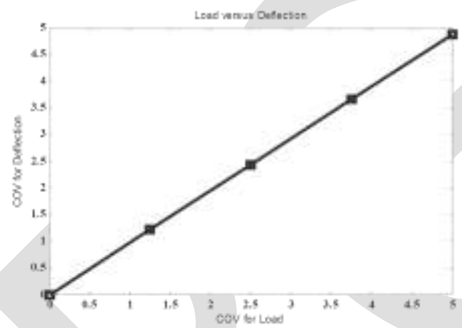

Figure 2: Scatter in deflection due to dispersion in pressure P

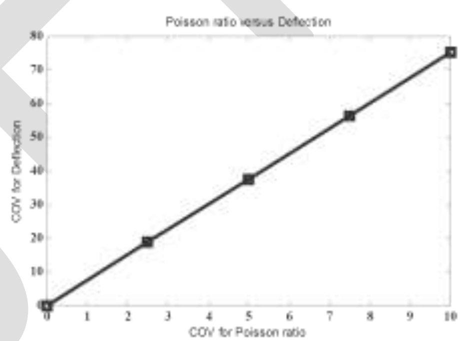

Figure 3: Scatter in deflection due to dispersion in Poisson's Ratio μ

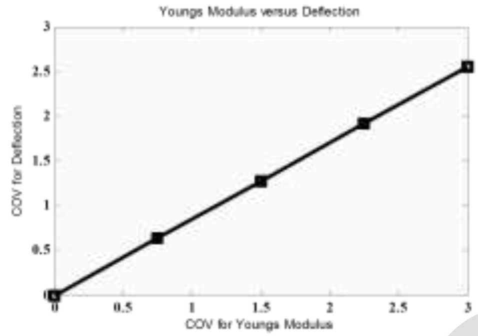

Figure 4: Scatter in deflection due to dispersion in Young's Modulus E

From the analysis it is found that the deformation of solid propellant is more sensitive to change in the values of pressure and Poisson's ratio than Young's modulus shown in Figure 2, 3, 4.

B. TRANSIENT RESPONSE ANALYSIS FOR PCB BOX

A Printed Circuit Board (PCB) mechanically supports and electrically connects electronic components using conductive tracks, pads and other features etched from copper sheets laminated onto a non-conductive substrate. Advanced PCBs may contain components - capacitors, resistors or active devices - embedded in the substrate.

A PCB box to applied base excitation is the modeled for structural analysis. This PCB box is mounted on visco-elastic pads for vibration isolation. The Young's modulus is considered to have scatter about their mean, which are the random independent variable. The displacement, velocity and acceleration are obtained for different base excitation for a range of dispersion in the random variable.

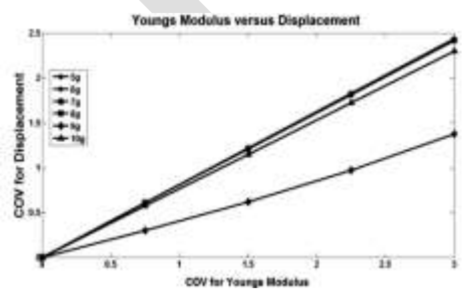

Figure 5: Scatter in displacement due to dispersion in Young's Modulus

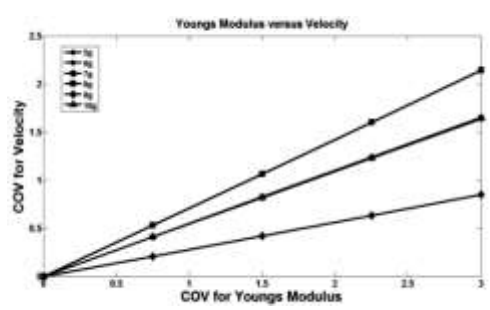

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Figure 6: Scatter in velocity due to dispersion in Young's Modulus

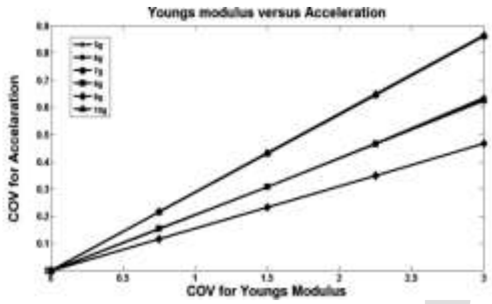

Figure 7: Scatter in acceleration due to Dispersion in Young's Modulus

The frequency response of PCB to applied base excitation is examined for the given configuration. Generally, aerospace structures are designed for high natural frequencies. This is to ensure that the excitation frequencies do not cause resonance in the system.

The PCB box mentioned in section B is considered for transient response analysis.

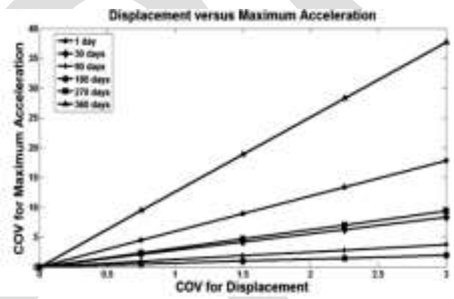

Figure 8: Scatter in maximum acceleration due to dispersion in the initial displacement

From the analysis, it is inferred that the maximum acceleration response increases due to poor vibration isolation by the visco-elastic pads shown in Figure 8.

CONCLUSION

In this paper, structures composed of VEM are examined, for static load and base excitation case. The response statistics due to dispersions in the applied load and system properties are evaluated. This aids in identifying the most sensitive parameter or load which influence its behavior.

The static response of the structure is sensitive to the Young's modulus and applied load. The Young's modulus dispersion does not cause large dispersions in the acceleration response, but the initial displacement does. For low margin design, such studies are useful for optimal and safe design.

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