

Application of Wavelet Transform on Images: A Review

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Abstract- Due to rise in the digital technology the use of digital images have also been increased. There are various techniques used for transformation of image being image compression, image denoising, image fragmentation etc. the compression technique is basically used so that the storage space used by the images can be minimized in this regard the wavelet plays a very important role. This paper focuses on the work that has been done in the field of image transformation using wavelets.

Keywords- Fourier Transform, Haar, PSNR, MSE, DWT, DCT, ROI

INTRODUCTION

Wavelet transform is basically an improved version of the Fourier transform where Fourier transform is a powerful tool that is used for analyzing the components of a stationary signal. Wavelet analysis is basically introduced to improve seismic signal analysis by switching from short time Fourier analysis to new better algorithm to detect and analyze abrupt changes in Daubechies[2,3], Mallet[4]. The major drawback encountered with Fourier transform is that it does not contain any local information.

'Wavelet' is a term introduced by Jean Morlet in the year 1982 which means small wave. The detailed mathematical study and applications of wavelet transform is given by Morlet & Grossman [5]. Wavelet analysis is basically used to solve difficult problems in the fields like mathematics, signal processing, pattern recognition, image processing, and computer graphics and in medical imaging.

WAVELET TRANSFORM

The word wavelet corresponds to a small wave (the sinusoids used in Fourier analysis are big waves) and in brief, a wavelet can be expressed as an oscillation that decays quickly.

The mathematical conditions for wavelets are:

$$\int_{-\infty}^{\infty} |\varphi(t)|^2 dt < \infty \quad (1)$$

$$\int_{-\infty}^{\infty} |\varphi(t)| dt = 0 \quad (2)$$

$$\int_{-\infty}^{\infty} \frac{|\varphi(w)|}{|w|} dw < \infty \quad (3)$$

The equation (3) is known as the admissibility condition.

The condition of the mother wavelet introduced by Morlet is given by:

$$\varphi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \varphi\left(\frac{t-b}{a}\right), a, b \in R, a \neq 0 \quad (4)$$

Parameter a here is the scaling parameter or scale, which measures the degree of compression. The parameter b is the translation parameter which determines the time location of the wavelet. If $|a| < 1$, then the wavelet in equation (4) is the compressed version (smaller support in time-domain) of the mother wavelet and it corresponds mainly to the higher frequencies. On the other hand, when $|a| > 1$, then $\varphi_{a,b}(t)$ has a larger time-width than $\varphi(t)$ and corresponds to lower frequencies. Thus, wavelets have time-widths adapted to their frequencies. This is the main reason for the success of the Morlet wavelets in signal processing and time-frequency signal analysis.

The main advantage of using the wavelets is that they offer a simultaneous localization in time and frequency domain. The second main advantage of wavelets is that, using fast wavelet transform, it is computationally very fast. Wavelets even have the great advantage of being able to separate the fine details in a signal. Very small wavelets can be used to isolate very fine details in a signal, while very large wavelets can identify coarse details. Wavelet theory is capable of revealing aspects of data that other signal analysis

techniques miss the aspects like trends, breakdown points, and discontinuities in higher derivatives and self-similarity. It can often compress or de-noise a signal without appreciable degradation [18].

APPLICATIONS

K. Grochenig and W.R. Madych on their paper proposed the notion of multiresolution analysis and formulated a very interesting relation between the theory of compactly supported wavelet basis and the theory of similar tiling for construction [1].

Francois G. Meyer, Amir Averbuch and Jan- Olov Stromberg developed a new fast 2-D convolution- decimation algorithm with factorized non- separable 2-D filters to demonstrate that an advantage can be gained by constructing a basis adapted to target image. The algorithm proposed by them is four times faster than the standard convolution- decimation and produces great textures when applied on Barbara and fingerprint images. The block diagram for the wavelet packet compression algo is shown in fig.2 where the compression consists of three parts(a)selection of the best basis, and calculation of the coefficients of the image, (b)quantization of the coefficients, and (c) entropy coding [6].

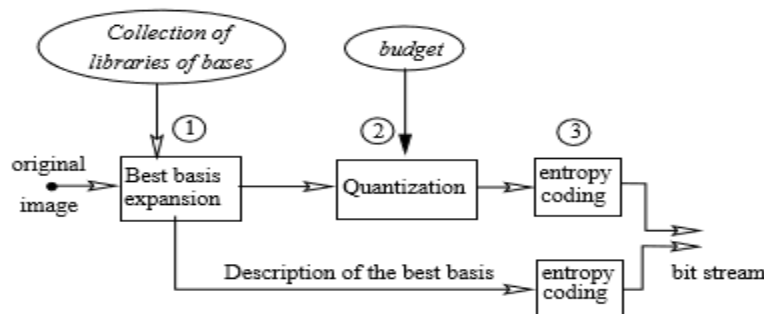


Fig. 2 Block diagram of wavelet packet compression algorithm [6]

Adrian Munteanu, Jan Cornelis, Geert Van Der Auwera, & Paul Cristea introduced a new wavelet based embedded compression technique that efficiently exploits the intraband dependencies and make use of the quad-tree approach to encode the significance maps [7].

Tim Flaherty, Yang Wang in their paper generated the result of Grochenig and W.R. Madych where they showed that a Haar- type wavelet basis of $L^2(\mathbb{R}^n)$ can be constructed from the characteristic function X_Ω of a compact set Ω , if and only if Ω is an integral affine tile of Lebesgue measure [1] one to the multi-wavelet settings [8].

Edmund Y. Lam & Joseph W. Goodman offered a comprehensive mathematical analysis of the DCT coefficient distribution of natural images to show that by using a doubly stochastic model, the Laplacian distribution of the coefficients can be determined [9].

Charilaos Christopoulos, Athanassios Skodras & Touradj Ebrahimi proposed that JPEG 2000 is a superior technique to all the existing still image compression standards on the basis of their comparative study. The reconstructed images compared at 1 bpp by means of JPEG and JPEG 2000 is shown in fig.2 [10].

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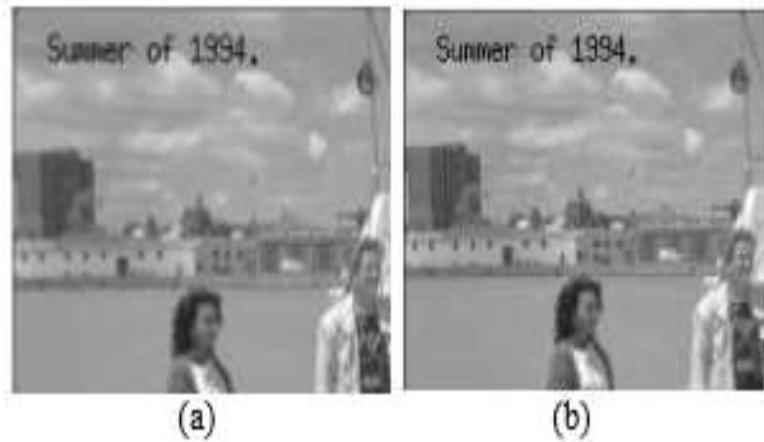


Fig. 2 Reconstructed images compressed at 1 bpp by means of: (a) JPEG (b) JPEG 2000 [10]

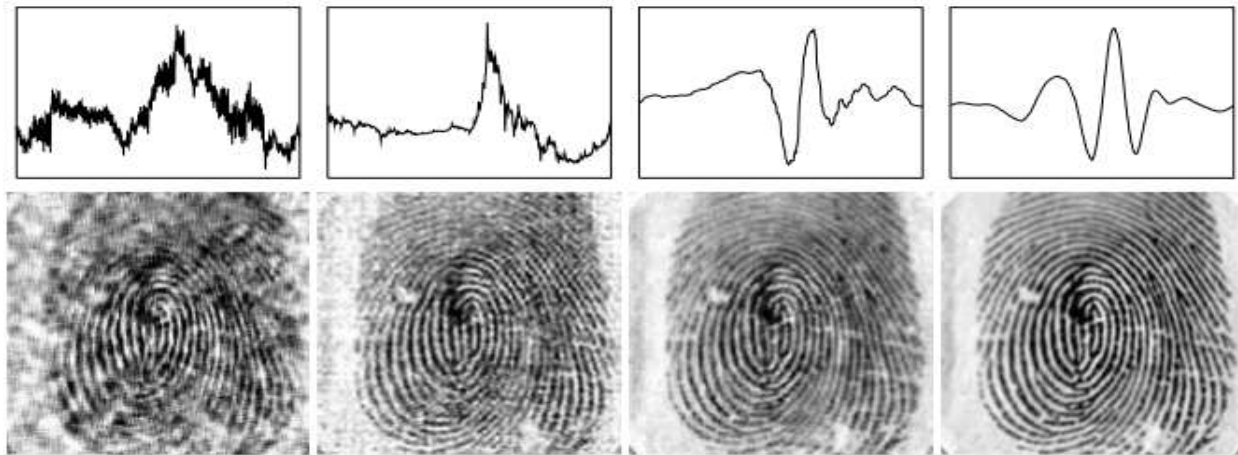
Albert Cohen & Basarab Matei introduced new multiscale representations for images which incorporates the L^2 error approach method in which the BW method gave the best results while the EA method provides the best results in case of visibility [11].

Sonja Grgic, Mislav Grgic and Zovko- Cihlar presented comparative study of different wavelet-based image compression system by examining wavelets on the basis of different wavelet functions, filter orders, no. of decompositions, compression ratio and image contents for implementation in a still image compression system [12].

Pitor Porwik, Agnieszka Lisowska presented graphic dependencies between the parts of Haar and wavelet spectra and also compared the wavelets in the 2-D space. The work show the graphical way of presentation of decomposition levels for both the Haar matrix based methods & wavelets [13].

R. Sudhakar, Ms R Karthiga, S. Jayaraman, proposed that the latest techniques such as EBCOT, ASWDR perform better than its predecessors such as EZW, WDR. They also produced some of the lowest errors per compression rate & highest perceptual quality by using wavelet coding techniques such as EZW, SPIHT, SPECK, WDR algorithm and ASWDR algorithm [14].

Uli Grasmann, Risto Mikkulainen, proposed a method based on the coevolutionary genetic algorithm used in [15] to evolve specialized wavelets for fingerprint images to show how the evolved wavelets outperform the hand-designed wavelet techniques. The progress of evolution during a typical run is shown at generations 1, 10, 20 and 50 in the figure 3. The top row in the figure shows the winner wavelets, and the bottom rows show the resulting compressed test image at 16:1. From the experiment it is concluded that the first generation produces a more or less random wavelet that performs poorly whereas, over the next generations, both image quality and the smoothness of the wavelets increase sharply and the performance keeps increasing after generation 50, although the differences are less obvious [16].



Generation 1 Generation 10 Generation 20 Generation 50

Fig.3 The progress of evolution during a typical run is shown at generations 1, 10, 20 and 50 [16]

Agnieszka Lisowska introduced theory of geometrical wavelets called wedgelets to code images with edges in a very efficient way. The theory of extended wedgelets was also presented to represent images in an efficient & sparse way then in case of known wedgelets and it produce a better visual effects along with higher compression ratio [17].

M. Sifuzzaman, M.R. Islam and M.Z. Ali presented the advantages of using wavelet transform compared to Fourier transform. The examination of the fingerprint performed by them is shown in fig 4 [18].



Fig .4 The fingerprint compressed image using wavelet transform reconstructed by the mathematician using software [18]

Anuj Bhardwaj and Rashid Ali used MFHWT algorithm for image compression to show that this algorithm produce better results than those obtained by any other methods on an average [19]. The result obtained by this method is shown in Fig.5

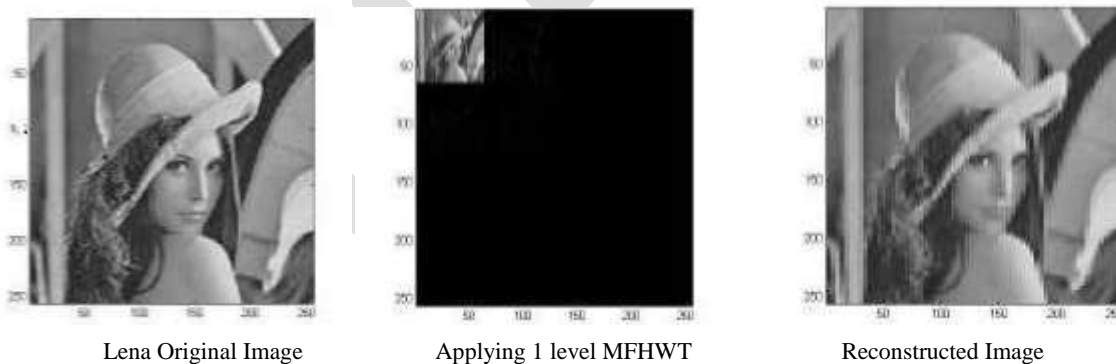


Fig.5 Operations performed on Lena.jpg Image [19]

Kamrul Hasan Talukder & Koichi Harada applied DWT to estimate the detail matrices from the information matrix to synthesize the reconstructed image using the estimated detail matrices and information matrix provided by the wavelet transform [20].

P. Raviraj & M.Y. Sanavullah developed computationally efficient & effective algorithm using Haar wavelet transform for compression of lossy images to minimize the computational requirements by applying various thresholding techniques to improve the quality of the reconstructed image. The experimental results for the cameraman image are shown in fig.6 [21].

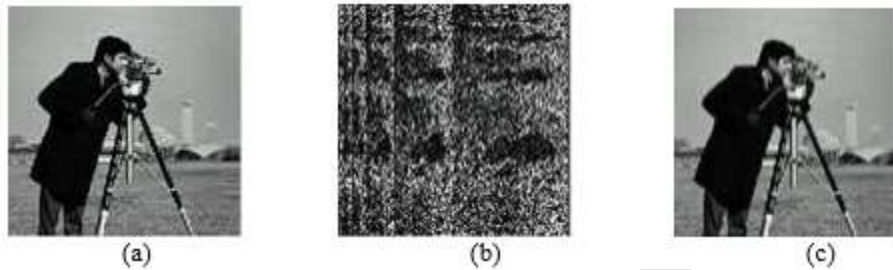


Fig.6. (a) Original, (b) Decomposed, (c) Reconstructed Images using Haar wavelet transform [21]

D. Vijendra Babu, Dr. N.R. Alamelu presented an enhanced image compression method using partial EZW algorithm. The proposed partial EZW algorithm on comparison with the EZW algo comes out to be more superior as it was capable of encoding each arbitrary shape ROI regions independently. This algorithm was also compared with other techniques such as SPIHT, modified SPIHT and embedded Zero Wavelet [22].

Anitha S presented the overview of various compression techniques based on DCT, DWT, and ROI & Neural Networks for the 2-D images [23].

CONCLUSION

In this paper we have reviewed how we can process the images using wavelet transform and found that the result obtained from wavelet transform using various wavelets and thresholding techniques are much more effective as compared to the Fourier transform. New advancements are being rapidly done in this area & the number of thresholding and denoising techniques are increasing day by day to improve the mean square error and the peak signal to noise ratio of the images without sacrificing the quality of the image. Therefore, wavelet analysis has proven to be a major area of research.

REFERENCES:

- [1] K.Grochenig, R.Madych, "Multiresolution Analysis, Haar Bases and Self-similar Tilings of \mathbb{R}^n ", IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. 38, NO.2, MARCH 1992.
- [2] Daubechies. I, "The wavelet transform, time-frequency localization and signal analysis", IEEE Transformation and Information Theory,36; 961- 1005, 1990.
- [3] Daubechies. I, "Ten Lectures on Wavelets", SIAM, Philadelphia, PA, 1992
- [4] Mallat.S, "A wavelet Tour of Signal Processing", Academi Press, New York, 1999.
- [5] Grossmann. A, Morlet. J, "Decomposition of Hardy functions into square integrable wavelets of constant shape", SIAM Journal of Analysis, 15: 723-736, 1984.
- [6] Francois G. Meyer, Amir Averbuch and Jan- Olov Stromberg, "Fast Adaptive Wavelet Packet Image Compression", IEEE Transactions on Image Processing, April1998.
- [7] Adrian Munteanu, Jan Cornelis, Geert Van der Auwera, and Paul Cristea, "Wavelet Image Compression— The Quadtree Coding Approach", IEEE TRANSACTIONS ON INFORMATION TECHNOLOGY IN BIOMEDICINE, VOL. 3, NO. 3, SEPTEMBER 1999
- [8] Tim Flaherty, Yang Wang, "HAAR TYPE MULTI-WAVELET BASES AND SELFAFFINE MULTITILES", ASIAN J. MATH Vol. 3, No.2, pp. 387400, June 1999.
- [9] Edmund Y. Lam, Joseph W. Goodman, "A Mathematical Analysis of the DCT Coefficient Distributions for Images", IEEE TRANSACTIONS ON IMAGE PROCESSING, VOL. 9, NO. 10, OCTOBER 2000.

- [10] Charilaos Christopoulos, Athanasios Skodras, Touradj Ebrahimi, "THE JPEG2000 STILL IMAGE CODING SYSTEM: AN OVERVIEW", IEEE Transactions on Consumer Electronics, Vol. 46, No. 4, pp. 1103-1127, November 2000.
- [11] Albert Cohen, Basarab Matei, "COMPACT REPRESENTATION OF IMAGES BY EDGE ADAPTED MULTISCALE TRANSFORM", IEEE 2001.
- [12] Sonja Grgic, Mislav Grgic, Zovko- Cihlar, "Performance Analysis of Image Compression Using Wavelets", IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS, VOL. 48, NO. 3, JUNE 2001.
- [13] Pitor Porwik, Agnieszka Lisowska, "The Haar-Wavelet Transform in Digital Image Processing: Its Status and Achievements", Machine Graphics & Vision, Vol. 13, n o. 1/2, 2004, pp .79-98.
- [14] R. Sudhakar, Ms R Karthiga, S. Jayaraman, "Image Compression using Coding of Wavelet Coefficients – A Survey", ICGST-GVIP Journal, Volume (5), Issue (6), June 2005.
- [15] U. Grasmann and R. Miikkulainen, "Evolving wavelets using a coevolutionary genetic algorithm and lifting", Genetic and Evolutionary Computation Conference, volume II, pages 969–980, New York, NY, 2004, Springer.
- [16] Uli Grasmann, Risto Mikkulainen, "Effective Image Compression using Evolved Wavelet", GECCO'05, June 25–29, 2005, Washington, DC, USA.
- [17] Agnieszka Lisowska, "Extended Wedgelets — Geometrical Wavelets in Efficient Image Coding", Machine GRAPHICS & VISION, Vol.0, no.0, 0000, pp.
- [18] M. Sifuzzaman, M.R. Islam, M.Z. Ali, "Application of Wavelet Transform and its Advantages Compared to Fourier Transform", Journal of Physical Sciences, Vol. 13, 2009.
- [19] Anuj Bhardwaj, Rashid Ali, "Image Compression Using Modified Fast Haar Wavelet Transform", World Applied Sciences Journal 7 (5): 647-653, 2009.
- [20] Kamrul Hasan Talukder & Koichi Harada, " Haar Wavelet Based Approach for Image Compression and Quality Assessment of Compressed Image", IAENG International Journal of Applied Mathematics, 36:1, IJAM_36_1_9.
- [21] P. Raviraj, M.Y. Sanavullah, "The Modified 2D-Haar Wavelet Transformation in Image Compression", Middle-East Journal of Scientific Research 2 (2): 73-78, 2007.
- [22] D. Vijendra Babu, Dr. N.R. Alamelu, " Wavelet Based Medical Image Compression Using ROI EZW", Int. J. of Recent Trends in Engineering and Technology, Vol. 1, No. 3, Nov 2009.
- [23] Anitha S, "2D image compression technique-A survey", International Journal of Scientific & Engineering Research ,Volume 2, Issue 7, July-2011.