Nonlinear analysis of cortical bone using Finite Element Method and algorithms developed in MATLAB

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Abstract— The fabrication of patient-specific engineering Implant is highly appreciated which requires prior determination of porous and mechanical characteristics [1, 8]. This is because of, the increase in porosity decreases the mechanical properties and increases the damping factor which leads to subsequent bone loss upon healing or remodeling [2, 8]. The hypothesis is that the patient specific scaffold Implants should provide a biomimetic mechanical environment for initial function and appropriate remodeling of regenerating tissue while concurrently providing sufficient strength in compression and porosity for cell migration and cell delivery [6]. The reason behind this approach is to improve the host-implant response with a better Osseo-integration compared to classical approach [4]. This work primarily aims at study of Non-Linear [23, 24] behavior of patient specific hydroxyapatite (HAP-Cortical Bone) which can be fabricated as Implant using algorithms developed in MATLAB [3], a non-invasive polymer cranial fracture implant with biocompatible approach.

Keywords— Non-linear, Bio-mechanics, MATLAB, Mindlin-Reissner theory, Cranial Implants, Finite Element Method, Three-Point bending, Explicit loading, Cortical Bone.

INTRODUCTION

The ability to precisely design and manufacture biomaterial scaffold Implants is needed since traditional methods for scaffold design and fabrication cannot provide the control over scaffold Implants architecture design to achieve specified properties within fixed limits on porosity [1, 2, 8]. The theoretical formulation for polymeric implants approach provides a conducive environment with host compared to traditional implants in terms of non-carcinogenic, radio-opaque, Osseo-integration and non-toxic [2]. The study of this HAP implant is carried out by considering a rectangular plate of specified dimensions, which can be used for Implants using MATLAB [3], with the algorithms developed based on the theories matching to the behavior of the cadaveric cortical bone for non-linear analysis.

This work is focused mainly on study of the nonlinear behavior of cortical bone-HAP, as an alternative for the titanium Implants. This is because large dimensions of cranial bone defects and sometimes rather bad conditions of adjacent soft tissues, such as craniectomy defects do not only require reconstruction for the protection of the intracranial structures, but also requires creation of Bio-compatible environment with better Osseo-integration and less stress shielding effect, Host-Implant response and harmonic contours with long-term stability, reduction of patient complaints, easy handling of the implants, and the possibility of oncological follow-up with qualified imaging methods of cranial Implants [4].

This work is carried out by implementing the Mindlin-Reissner theory [25] for a rectangular plates of specified dimensions which can be used as an implant by considering the properties similar to that of a human bone and studying its behavior with human bone using the algorithms developed in MATLAB [3]. The analysis, specifically meshing of the rectangular plate with iso-p quadrilateral elements (plate elements), the boundary conditions and the explicit loading conditions were implemented using the algorithms developed in MATLAB [3] and the resulting displacement were studied.

Next, the real cadaveric bone specimen of HAP is prepared with proper dimensions. Also, the fixture for conducting the three-point bending test and tensile test is prepared. The prepared specimen is then subjected to three-point bending and tensile test. The obtained results from tensile test is used to determine the Young's modulus of cadaveric cortical bone, which is made used in the properties for the algorithms developed in MATLAB. Next, the displacements from the experimental data and finite element method in MATLAB are compared for best custom cranial implants and HAP for various parameters such as stress shielding, effective strength, stiffness, host-implant response, Osseo-Integration

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METHODOLOGY

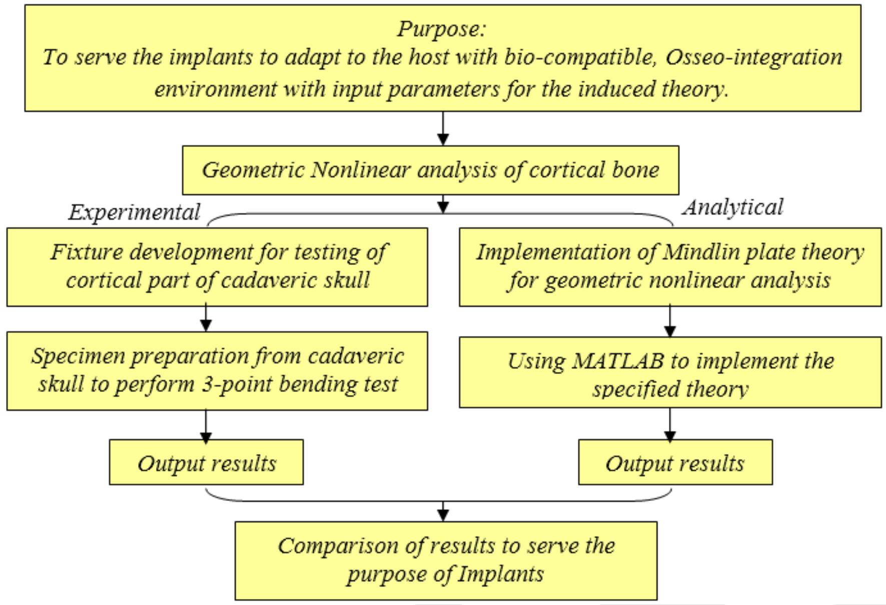

Figure 1: Overall procedure for the Non-linear analysis of cortical bone

Theoretical Formulations:

Geometric non linearity [25] is considered because change in geometry as the structure deforms is taken into account in setting up the strain displacement and equilibrium equations. Strain-displacement equations e=Du. The operator D is nonlinear when finite strains are expressed in terms of displacements.

In all the geometric nonlinear problems we solve the following equation which is called the "force residual".

$$r(u,\Lambda)=0$$

This vector includes all the discrete equilibrium equations encountered in nonlinear static analysis formulated by the displacement method. Here u is the state vector containing unrestrained degrees of freedom that characterize the configuration of the structure, r is the residual vector that contains out-of-balance forces conjugate to u, and Λ is an array of assignable control parameters. Now, varying the vector r with respect to the components of u while keeping Λ fixed yields the Jacobian matrix.

Mindlin Plate Theory

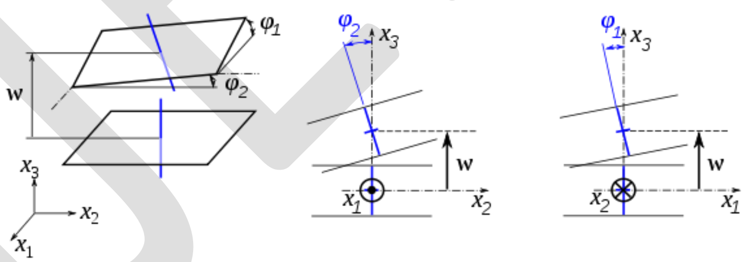

Figure 2: Out of plane deformation according to Mindlin plate theory.

The non-linear static analysis and only geometric non-linearity is considered where the material is considered to be homogeneous, isotropic and linear elastic. Dynamical effects are neglected and deformations are large to use finite deformation theory (Geometric non-linearity). This plate theory includes transverse shear deformations. Here we look at the elements with 5 degrees of freedom three translations (u, v, w) and three rotations (θ_x, θ_y, θ_z) with $\theta_z = 0$, indicating zero stiffness.

$$\bar{u} = u - z\theta_x$$

$$\bar{v} = v - z\theta_y \ and \ \bar{w} = w$$

Finite element discretization

For finite element analysis, we considered a rectangular plate of dimension from a real cadaveric skull specimen which was subjected to experimentation and used the same material properties determined experimentally being, Young's modulus is 3020 MPa and Poisson ratio is 0.3. With these parameters the plate is divided into 80 mesh elements along length and 25 mesh elements along the width to generate the mesh.

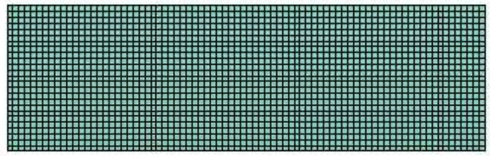

Figure 3: Rectangular plate with mesh elements

Quadrilateral iso-parametric shell element is used for discretization with five degree of freedom per node with $\theta_z = 0$.

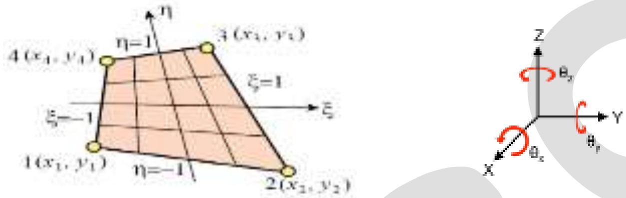

Figure 4: Figure representing Iso-parametric 4 noded Quadrilateral element and Five Degree of freedom at each node with $\theta_z = 0$.

The first variation of potential energy is written as

$$\delta\Pi = \delta U - \delta W = 0$$

$$\delta U = \delta W$$

The variation of strain energy of the plate is written as summation of in-plane bending strain energy and transverse shear strain energy.

$$\delta U = \int_A d\epsilon_{ib}^T \, \hat{\sigma}_{ib} dA \; + \int_A d\epsilon_s^T \, \hat{\sigma}_s dA$$

For non-linear system, the element equilibrium equation is

$$R = \int_A B_{ib}^T \, \hat{\sigma}_{ib} dA \; + \int_A B_s^T \, \hat{\sigma}_s dA - F = 0$$

where R is residual and F is a generalized forces comes from variation of external work done.

The solution algorithm for the assembled nonlinear equilibrium equations is based on Newton-Raphson method which need linearization of equations at equilibrium point. Here linearized equations are of the form

$$K_T d\alpha_{n+1}^i = R_{n+1}^i$$

where R is residual and i denotes iteration number.The residual can be written as

$$R = K_S\alpha - F_{n+1}$$

where F and $K_S\alpha$ are external and internal forces respectively.

The element In-plane, bending and the shear stiffness matrix are evaluated into global stiffness matrix. Implementing the Boundary condition and the explicit Loading condition, assemblage of element stiffness matrix and force vector into global level is carried out. The results obtained from these formulations and algorithms developed in MATLAB [3] are as shown below.

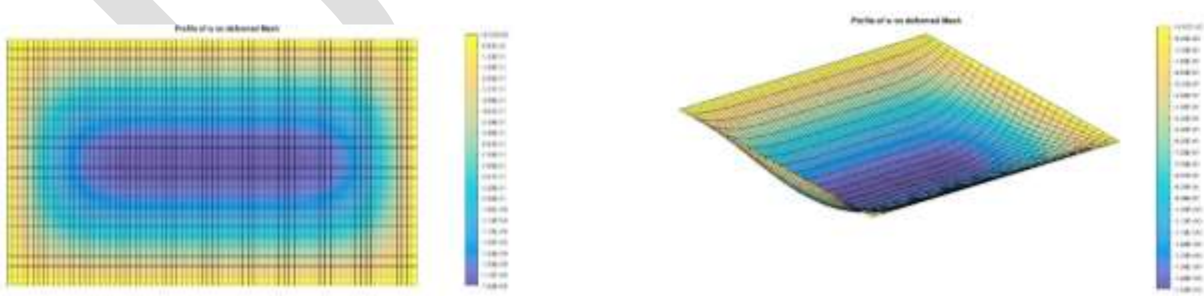

Figure 5: Output mesh of Transverse displacement of rectangular plate in MATLAB.

From the theoretical formulations and analysis in MATLAB [3] we have found that the transverse displacement of a rectangular plate according to Mindlin plate theory subjected to a transverse load of 360N has a transverse displacement of 1.524 mm which almost matches to the experimental results [14, 16, 22].

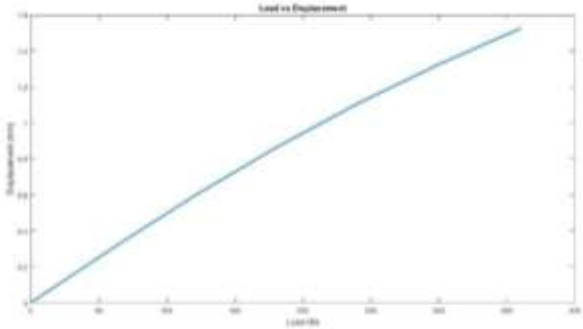

Figure 6: Theoretical plot of displacement vs. load in MATLAB.

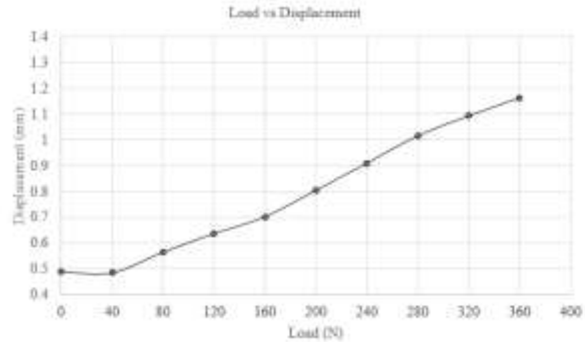

Figure7: Experimental Plot of displacement vs. load

CONCLUSION

This study has focused on the development of improved non-linear theoretical and analytical methods for predicting the geometric nonlinear static response of cranial implants with human skull. The present report is limited to the study of non-linear static response of cortical bone and validation with experimental results. An extensive study on the material nonlinearity and dynamic response of these implants will be presented in a future work.

A new approach in describing the geometric nonlinearity of implants for cortical bone has been proposed. This approach consists of considering transverse loads including transverse shear deformations, this is because, an optimal fixation implant is one which maximizes the stability of the fracture by keeping the relative shear movement of the fragments to a minimum while allowing for sufficient compression across the fracture site, thereby promoting healing and reducing stress shielding effect [7].

The properties of these elements (Implants) can be derived from tensile test of cadaveric skull or from the journal references. The basic principles or refined finite element models including step loading and boundary condition have been simulated in MATLAB [3] for the geometric nonlinearity. Such a scheme was developed in this study within the framework of a special purpose analysis program for the nonlinear static analysis of cranial implants.

To establish the validity of the proposed models correlation studies of analytical predictions with experimental evidence of the load-displacement response of cadaveric skull specimen under static load reversals and three point bending test were conducted.

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