Flux-Limiter Schemes for the Convective Acceleration

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Abstract- A 3D numerical model MECCA (Model of Estuarine and Coastal Circulation Assessment) to simulate coastal flow coupled with some physical processes (sediment transport) is available at our laboratory. This model uses the finites differences method to discretize a set of the gouverning equations (3D shallow water equations). The Upwind scheme was chosen to approximate the advective terms. This scheme is characterised by a large numerical diffusion especially in the vicinity of discontinuities regions. To eliminate this unrealistic effect, in this paper we introduce some limiters schemes initialy used for gas dynamics such : Minmod, Superbee, MC and Van Leer. Our prupose is the use this kind of schemes to aproximate the all advective terms (hydrodynamic and scalar transport equation (S.T.E)). Throught the numerical tests, in this study we conclude that for the coupled model and in order to reduce the numerical diffusion, it's necessary to use the limiters schemes for all terms advective. However, for the Saint-Venant equations with a regular initial and boundaries condition, it's suggested also to use the limiters schemes at most to give the true physical effect (turbulence effect) of the viscosity coefficient, because generaly this physical coefficient became a calibration parameter in the ocean modelling.

Keywords--- Flux-limiter, numerical diffusion, High resolution schemes, Total variation diminishing Schemes.

1. Introduction

This study investigates several flux-limiter schemes, that have been chosen because they satisfy many of the requirements of a good advection scheme, they are Total Variation Diminishing (TVD), mass conservative and less diffusive than the simpler schemes. It is well known that the scalar transport is the combination of different physical processes: (i) advection, by the statistical mean of velocity in which all scalar quantities are transported in the direction of the flow without deforming their initial distribution, (ii) diffusion, due to turbulent velocity fluctuation where scalar shape may be smoothed. The numerical approximation of the advective process in the oceanographic and meteorological modelling requires important choices and compromises to be made when flow simulations are carried out. One example of these compromises is the numerical treatment of the advective terms in the transport equation. These compromises are necessary, to minimize both artificial numerical diffusion and dispersion. The resulting numerical diffusion may severely damp the flow, producing exaggerated inaccurate results, whereas the artificial numerical dispersion may introduce non-physical oscillations called wiggles.

Traditionally the large scale oceanography and meteorology models use the upwind and central difference scheme to solve the advective terms present in the flow equation and passive scalar equations, for example the Princeton Ocean Model (POM) [1] and the Model of Estuarine and Coastal Circulation Assessment (MECCA) [2]. However, these schemes give rise to numerical instability or exaggerated numerical diffusion.

This present work relates to an improvement of the space discretization of the three-dimensional equations of model MECCA [3], [4], this model use the Upwind scheme to approximate the advective terms, This scheme is characterized by a large numerical diffusion especially in the vicinity of discontinuities regions. To eliminate this unrealistic effect, in this paper we introduce some limiters schemes initialy used for gas dynamics such: Minmod, Superbee, MC and Van Leer. Smaoui and al. [5], [6] have implemented the limiters schemes patterns to identify fresh water /salt water generated by the flow in Somme estuary in the Eastern Channel (Northern of France) . In this paper and to the difference of previous work [5], [6], we implement the flux-limiters schemes to approach the convective terms of the equations of momentum and transport scalar. The question which thus arises and which justifies this work is the following:"With the fluxl imiters schemes in the equations of the momentum, what one limits of advantage the numerical diffusion attached to the numerical solution of the equation of concentration ?" To answer to this question, we carried out two numerical tests covering some applications in geophysics flows. These tests have been devoted to solve numerically the equation of Burger coupled with the scalar transport equation. Let us note that we chose the equation of Burger to represent the hydrodynamics, because under certain initial conditions and in extreme cases, this equation offers an exact solution. Consequently, we will be able to quantify the errors due to the numerical approximations. This work is organized in the following way : The second section briefly describes model MECCA. To illustrate the principle of operation of the flux-limiter methods, we present in the third section the discretization of a convective equation 1D by this kind of method. The fourth and the fifth section will relate to the results and simulation undertaken in this work. Finally, we finish by a conclusion in which we summarize the whole of the got results.

2. Brief description of the MECCA model

The MECCA model (Model of Estuarine and Coastal Circulation Assessment), initially developed by Hess [2], uses finite-difference approximations to solve the discretized 3D equations governing conservation of momentum, mass, heat and salt on a beta plane, subject to the hydrostatic and Boussinesq approximations. It is able to simulate time-varying water cur- rents, salinities and temperatures in shallow-water domains at time scales ranging from a few minutes to several months, and space scales stretching from a few kilometres to a few hundred kilometres. The mode is designed to simulate circulation driven by tides, wind, water density gradients as well as atmospheric pressure gradients. The MECCA model was applied successfully to simulate a tidal flow and the sediment transport at English Channel [4], Chesapeake Bay [2], and the Mediterranean Sea [3].

3. Transport equations and their discretization

Let us start by solving the transport equation that describes the purely advective concentration C in a flow field. This equation is described by:

$$\frac{\partial C}{\partial t} + \nabla.(\vec{V}C) = 0 \qquad (1)$$

where $\vec{V}$ is the velocity vector. For simplicity we illustrate the methods under study by considering the one dimensional scalar advection equation:

$$\frac{\partial C}{\partial t} + \frac{\partial uC}{\partial x} = 0 \qquad (2)$$

The flux of the concentration C is denoted by F, the regular grid spacing byΔx, and the time step by Δt. Hence discretization of Eq. (2) is obtained in the flux form on a staggered grid by integration over the finite volume $\Omega_i = [x_{i-1/2}, x_{i+1/2}]$ and can be expressed explicitly in time as:

$$C_i^{n+1} = C_i^n - \frac{\Delta t}{\Delta x}\left[(uC)_{i+1/2}^n - (uC)_{i-1/2}^n\right] \qquad (3)$$

Where n is the time level $t_n = n\Delta t$, $C_i = C(x_i, t_n)$ and $F_{i+1/2}^n$, $F_{i-1/2}^n$ are the C fluxes (called also numerical fluxes) through the right and left boundaries of the grid cell respectively. The accuracy of finite volume discretization is mainly related to the computation of the cell-face fluxes. Many methods have been proposed [7-8] for example, the piecewise polynomial interpolation suggested by van Leer [9]. The basic idea of this so-called k-interpolation is that linear and quadratic approximations of the solution on each cell lead respectively to second and third order space discretizations [10]. The second order schemes have been successful in eliminating the numerical diffusion, but they give rise to non-physical oscillations near regions of large gradients. In the next section we shall present a method to obtain higher order schemes without oscillations.

3.1. Flux-limiter methods

The most straightforward approximation to $C_{i+1/2}^n$ is certainly $C_{i+1/2}^n = 0.5(C_{i+1} + C_i)$. This expression gives an approximation of the partial derivative $\frac{\partial uC}{\partial t}$ by central differences. Unfortunately, this method leads to the appearance of spurious oscillations in the numerical solution. A free-oscillations solution is provided, by the use of the upstream approach (also called upwind scheme),

$$\left(F_{i+1/2}^n\right)_{Up} = \begin{cases} u_{i+1/2}^n C_i^n & if \quad u_{i+1/2}^n \geq 0, \\ u_{i+1/2}^n C_{i+1}^n & if \quad u_{i+1/2}^n < 0 \end{cases} \qquad (4)$$

which can be written by a single formula :

$$\left(F_{i+1/2}^n\right)_{Up} = \frac{1}{2}\left[\left(u_{i+1/2}^n + \left|u_{i+1/2}^n\right|\right)C_i^n + \left(u_{i+1/2}^n - \left|u_{i+1/2}^n\right|\right)C_{i+1}^n\right] \qquad (5)$$

This amounts to replacing $\left(\frac{\partial uC}{\partial t}\right)$ by the so called upwind differences, which avoids non-physical oscillations, but is unfortunately affected by excessive numerical diffusion. One strategy to avoid non- physical oscillations and excessive numerical diffusion is the hybrid method which uses the second order numerical flux in smooth regions and limits the solution in vicinity of discontinuities by

using the monotonic upwind method in these regions. This procedure is carried out by introducing a slope-limiter (also called flux-limiter) based on the local gradient of the solution (4). We write the interface value $C_{i+1/2}^{n}$ as the sum of the diffusive first order upwind term and an ”anti-diffusive”
one. The higher order anti- diffusive part is multiplied by the flux limiter, which depends locally on the nature of the solution by means of the non-linear function $\theta_{i+1/2}$. This function is expressed by the slopes ratio at the
neighborhood of the interfaces in the upwind direction (4).

$$\theta_{i+1/2} = \begin{cases} \frac{C_i^n - C_{i-1}^n}{C_{i+1}^n - C_i^n} = \theta_{i+1/2}^+ & if \quad u_{i+1/2}^n \geq 0, \\ \frac{C_{i+2}^n - C_{i+1}^n}{C_{i+1}^n - C_i^n} = \theta_{i+1/2}^- & if \quad u_{i+1/2}^n < 0 \end{cases} \quad (6)$$

Introduction of this new parameter namely (θ) and the limiter function φ, leads to the flux limiter version of the hybrid scheme as :

$$C_{i+1/2}^n = \begin{cases} C_i^n + \frac{1}{2}(C_{i+1}^n - C_i^n)\phi(\theta_{i+1/2}^+) & si \quad u_{i+1/2}^n \geq 0 \\ C_{i+1}^n - \frac{1}{2}(C_{i+1}^n - C_i^n)\phi(\theta_{i+1/2}^-) & sinon \end{cases} \quad (7)$$

The interface value $C_{i-1/2}^{n}$ is obtained from $C_{i+1/2}^{n}$ by substituting the induce i by i−1. Note that expression (6) is the inverse of the definition used in [12], but the same expression can be obtained if the limiter function φ is symmetric, i.e.

$$\phi(\theta) - \theta\phi\left(\frac{1}{\theta}\right) \quad (8)$$

From Eq. (7), one can see that if φ = 0 once again we find the upwind scheme, and if φ = 1 the scheme is reduced to the centred one. The limiting procedure must be carried out under some constraints to ensure stability of the scheme. A well-known criterion, as proposed by Gastell and Lau [11], is the so-called convection boundless criterion (CBC). In this paper, the flux-limiter must be built to satisfy the total variation diminishing (TVD) concept due to [12], following [13].

3.2. Total variation diminishing concepts

The theory of the TVD concept and slope limiter was introduced by Harten [12], as the criterion to combine accuracy and monotonicity properties of the higher order schemes used to solve the scalar conservation equation. With this concept, during the time evolution no new under- or overshoots could be created. We recall the main results from this concept. The total variation of the discrete solution is defined as:

$$TV(C^n) = \sum_{i=-\infty}^{i=+\infty} \left|C_i^n - C_{i-1}^n\right| \quad (9)$$

A scheme is said to satisfy the TVD constraints if

$$TV(C^{n+1}) \leq TV(C^n) \quad (10)$$

For our purposes, we assume that the scheme Eq. (3) can be written in an incremental form:

$$C_i^{n+1} = C_i^n - A_{i-1/2}^n(C_i^n - C_{i-1}^n) + B_{i+1/2}^n(C_{i+1}^n - C_i^n) \quad (11)$$

With

$$A_{i-1/2}^n = A\left(C_{i-2}^n, C_{i-1}^n, C_i^n, C_{i+1}^n\right) \quad and \quad B_{i+1/2}^n = B\left(C_{i-1}^n, C_i^n, C_{i+1}^n, C_{i+2}^n\right) \quad (12)$$

The discrete solution C_i^n called monotonic if for each i

$$\min(C_{i-1}^n, C_{i+1}^n) \leq C_i^n \leq \max(C_{i-1}^n, C_{i+1}^n) \quad (13)$$

Scheme Eq. (11) is called monotonicity preserving if C_{i+1}^n remains monotonic when C_i^n is monotonic. Hence, no new minima or maxima are created when time evolves. The following theorem due to [12] gives a sufficient condition to ensure monotonicity preservation of the numerical scheme.

Theorem 1. ***Total Variation Diminishing scheme is monotonicity preserving***

The main interest of the incremental form Eq. (11) for scheme Eq. (3) is that sufficient conditions can be derived in order to achieve the TVD property of the family of approximate solutions. In this way Harten [15] proves the following lemma.

Lemma 1.
If $A_{i+1/2} \geq 0$, $B_{i+1/2} \geq 0$ and $A_{i+1/2} + B_{i+1/2} \leq 1$ *then the scheme Eq.(11) is TVD* In order to satisfy the TVD condition, the Harten's lemma implies that the flux-limiter must satisfy certain constraints. For example with an uniform positive velocity u, a nd the symmetric property of the limiter i (i.e. $(F_{i+1/2})U_p = uC_i^n$), the substitution of (7) in (3) gives :

$$C_i^{n+1} = C_i^n - u\frac{\Delta t}{\Delta x}\left[1 + \frac{1}{2}\frac{\phi(\theta^+_{i+1/2})}{\theta^+_{i+1/2}} - \frac{1}{2}\phi(\theta^+_{i+1/2})\right](C_i^n - C_{i-1}^n) \quad (14)$$

This is a scheme of the incremental form (11) with

$$A_{i-1/2}^n = \frac{\Delta t}{\Delta x}\left[1 + \frac{1}{2}\frac{\phi(\theta_{i+1/2})}{\theta_{i+1/2}} - \frac{1}{2}\phi(\theta_{i+1/2})\right] \textit{and} \qquad B_{i+1/2}^n = 0 \quad (15)$$

applying the Harten's lemma, the scheme is TVD if

$$\left|\frac{\phi(r)}{r} - (s)\right| \leq 2 \quad (16)$$

Sweby [13] also specifies that φ(r) > 0 and that φ(r) = 0 for r ≤ 0. Under these additional restrictions the condition (16) becomes

$$0 < \frac{\phi(r)}{r} \leq 2 \qquad et \qquad 0 \leq \phi(r) \leq 2 \quad (17)$$

The region defined by (17) is shown in Fig. 1 along with the limiter corresponding to the centred diffferences. Since this scheme is known to produce spurious wiggles in the solution with strongly varying gradients, it is not surprising that this scheme is not uniformly within the TVD region. Among the pro- posals, which have been discussed in Refs. [14, 12] the following limiter functions used in this study satisfy these constraints [14] including the following :
- Limiter Minmod : φ(θ) = max(0,min(1, θ))

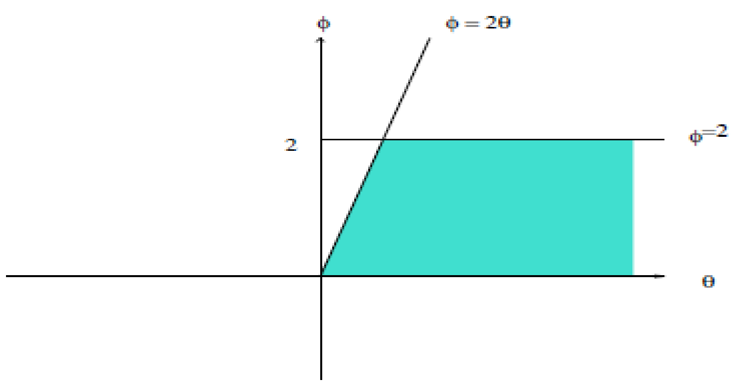


Fig. 1 – Total Variation Diminishing region for pure convection.

- Limiter Superbee : φ(θ) = max(0,min(1, 2θ),min(2, θ))
- Limiter de Van Leer : φ(θ) = (θ + |θ|)/(1 + |θ|)
- Limiteur MC : φ(θ) = max[0,min((1 + θ)/2, 2θ)]

The Minmod and Superbee limiters have been introduced by Roe in [15] and [16]. The Van Leer limiter was introduced in [17] and the MC limiter was also introduced by Van Leer in a later paper [18]. Note that it is easy to verify that the limiter functions proposed above are symmetric.

3.2.1 Two-dimensional case

The generalization of the numerical schemes to the multi-dimensional case is not always an easy task in the numerical computation. For example, in the 2D case the Lax-Wendroff scheme introduces the cross derivative terms. These terms must be included in spatial discretization if the second order approximation in time is desired. Besides accuracy problems of the schemes, sometimes also the mathematical properties are not carried over directly from one dimension to two dimensions. For example Spekreijse [19] shows that for explicit schemes, a 2D monotonic scheme is not necessarily TVD. However, numerical experiments [19] have shown that 2D schemes using the splitting technics (1D second order accurate TVD scheme in each direction perpendicular to the cell face) give accurate results, with no oscillations. In this section we present the 2D algorithm to solve the two-dimensional purely advective scalar equation given by

$$\frac{\partial C}{\partial t} + \frac{\partial uC}{\partial x} + \frac{\partial vC}{\partial y} = 0 \qquad (18)$$

Where u and v are the velocity components in x- and y-directions respectively. Discretization of Eq. (18) in the incremental form gives :

$$C_{i,j}^{n+1} = C_{i,j}^{n} - \alpha_{i-1/2,j}^{n}(C_{i,j}^{n} - C_{i-1,j}^{n}) + \beta_{i+1/2,j}^{n}(C_{i+1,j}^{n} - C_{i,j}^{n})$$
$$-\lambda_{i,j-1/2}^{n}(C_{i,j}^{n} - C_{i,j-1}^{n}) + \mu_{i,j+1/2}^{n}(C_{i,j+1}^{n} - C_{i,j}^{n}) \qquad (19)$$

With

$$\alpha_{i-1/2,j}^{n} = \alpha(C_{i-2,j}^{n}, C_{i-1,j}^{n}, C_{i,j}^{n}, C_{i+1,j}^{n}) \qquad ,\beta_{i+1/2,j}^{n} = \alpha(C_{i-1,j}^{n}, C_{i,j}^{n}, C_{i+1,j}^{n}, C_{i+2,j}^{n})$$
$$\lambda_{i,j-1/2}^{n} = \lambda(C_{i,j-2}^{n}, C_{i,j-1}^{n}, C_{i,j}^{n}, C_{i,j+1}^{n}) \qquad ,\mu_{i,j+1/2}^{n} = \mu(C_{i,j-1}^{n}, C_{i,j}^{n}, C_{i,j+1}^{n}, C_{i,j+2}^{n}) \qquad (20)$$

In Ref. [19], a class of explicit schemes in two dimensions is considered, a new definition of monotonicity is also introduced.

Definition 1. Scheme Eq. (19) is called monotonic if

$$\alpha_{i-1/2,j}^{n} \geq 0 \quad , \quad \beta_{i+1/2,j}^{n} \geq 0 \quad , \quad \lambda_{i,j-1/2}^{n} \geq 0 \quad , \quad \mu_{i,j+1/2}^{n} \geq 0 \qquad (21)$$

As mentioned above and for description simplicity, we use in this paper the 2D extension of fluxlimiting schemes as splitting mode in each one-dimensional direction. Application of this to (18) with the cell-centred finite volume discretization gives the semi-discrete equation :

$$\frac{dC_i}{dt}|\Delta x \Delta y = -[uC]_{i-1/2}^{i+1/2} - [vC]_{j-1/2}^{j+1/2} \qquad (22)$$

In the one-dimensional case, we give the complete expressions of the interface values $C_{i+1/2,j}^{n}$, $C_{i,j+1/2}^{n}$ for an arbitrary velocity field :

$$C_{i+1/2,j}^{n} = \begin{cases} C_{i,j}^{n} + \frac{1}{2}(C_{i+1,j}^{n} - C_{i,j}^{n})\phi(\theta_{i+1/2,j}^{+}) & si \quad u_{i+1/2,j}^{n} \geq 0 \\ C_{i+1,j}^{n} - \frac{1}{2}(C_{i+1,j}^{n} - C_{i,j}^{n})\phi(\theta_{i+1/2,j}^{-}) & sinon \end{cases} \qquad (23)$$

$$C_{i,j+\frac{1}{2},j}^{n} = \begin{cases} C_{i,j}^{n} + \frac{1}{2}(C_{i,j+1}^{n} - C_{i,j}^{n})\phi(\theta_{i,j+1/2}^{+}) & si \quad v_{i,j+1/2}^{n} \geq 0 \\ C_{i,j+1}^{n} - \frac{1}{2}(C_{i,j+1}^{n} - C_{i,j}^{n})\phi(\theta_{i,j+1/2}^{-}) & sinon \end{cases} \qquad (24)$$

With

$$\theta^{+}_{i+1/2,j} = \frac{C^n_{i,j} - C^n_{i-1,j}}{C^n_{i+1,j} - C^n_{i,j}} \quad , \quad \theta^{-}_{i+1/2,j} = \frac{C^n_{i+2,j} - C^n_{i+1,j}}{C^n_{i+1,j} - C^n_{i,j}}$$

$$\theta^{+}_{i,j+1/2} = \frac{C^n_{i,j} - C^n_{i,j-1}}{C^n_{i,j+1} - C^n_{i,j}} \quad , \quad \theta^{-}_{i,j+1/2} = \frac{C^n_{i,j+2} - C^n_{i,j+1}}{C^n_{i,j+1} - C^n_{i,j}}$$

Note that if the limiter function φ satisfies the conditions (17), then the scheme given by Eq. (23) and (24) is monotonic. This is a direct consequence of Theorem 1 and Definition 1.

4. Results and discussion

For to test the performance of the numerical schemesr, we realize some numerical tests. In these tests, we consider the Burgers equation with a transport equation scalar. To achieve the objective of this paper, we carry out a first series of tests which we discretize the Burger equation with the Upwind scheme and the transport equation with all other schemes. The second series will use the Superbee limiter to the equation Burger and other schemes for the transport equation.

4.1. Test 1 : Coupling Burger/Transport

This test solves the monodimensional convection of a discontinous profile of a concentration transported by a velocity field calculated by the Burger equation on the domain [−L, L] or L is an arbitrary coefficient in R :

$$\begin{cases} \frac{\partial u}{\partial t} + \frac{\partial (u^2/2)}{\partial x} = 0 & for \quad x \in [-L, L[\\ \frac{\partial c}{\partial t} + \frac{\partial uc}{\partial x} = 0 & for \quad x \in [-L, L[\\ c(x,0) = c_0(x) \quad et \quad u(x,0) = u_0(x) & \\ \left(\frac{\partial c}{\partial x}\right)_{x=L} = 0 \quad et \quad \left(\frac{\partial u}{\partial x}\right)_{x=L} = 0 & \end{cases} \qquad (25)$$

Where the initial conditions u_0 and c_0 are taken with discontinuities:

$$c(x,0) = c_o(x) = \begin{cases} 1 & si \quad a \le x \le b \\ 0 & sinon \end{cases} \qquad (26)$$

with a and b are parameters to choose in [−L,L].

$$u(x,0) = u_o(x) = \begin{cases} 0, & si \quad x \in [-L, 0] \\ 1, & sinon \end{cases} \qquad (27)$$

In fact, it is this model that will allow us to answer the question that arises. for this, we have split this application into two numerical test. In the first, one approach the convective acceleration by the Upwind scheme, while in the second it is discretized by the best performing slope limiter (Superbee) [6]. For both tests, the transport equation will be tested by all schemes used in this work. We recall that for this test, we comment only the results of the transport equation.

• Test 1.1 (Burger : Upwind/ transport : all schemes)

The results of this simulation are presented in figures 2 et 3 where the curves are plotted in simple line. The examination of table (I) shows a perfect conservation of the mass from all schemes (Rmas $= \left|\frac{\sum_{i=1}^{Nx} u_{i,num}}{\sum_{i=1}^{Nx} u_{i,initial}}\right| = 1$) In figure (2.a), the upwind scheme shows an excessive spreading out of the profil of the concentration and a loss of de 42% of the maximum value of the solution. We report the non-monotonic character of the schemes centred and Second upwind which appears bye the violation of the extremums of the solution $C_{min} \leq 0, C_{max} \geq 1$ (figure 2.b, figure 3.c). As for slope limiters, we observe the superiority of Superbee limiter on other limiters (Table 1). All figures (3.d, 3.e, 3.f, 3.g) shows that the limiter Minmod diffuse more than the other limiters, but much less than the Upwind scheme. Also, we note the Superbee limiter keeps the best shape of the concentration profile (Conservation the mass). Finally, we note that the MC limiter is located in performance between the limiter Van Leer and Superbee limiter.

• Test 1.2 (Burger :Superbee / transport :all schemes)

The results of this simulation are presented in Figures 2 and 3 where the curves are shown by a solid line.
This is certainly the key test of this study, these results will help answer the question that was asked at the beginning. It allows to assess the contribution of the approximation of the acceleration convective by a high-order scheme on the coupling in transport equation.
The comparison figures (2.a) for Upwind scheme shows a spread of the solution less important in ”test 1.2” than ”test 1.1”. Note also that the loss of the maximum value of the solution is less than in ”test 1.2” that ”test 1.1”, this report is also also observed for all other limiters. Figures (2b) and (3c) show a contrary to other schemes a degradation in the performance which results in an amplification of the parasitic oscillations. Table (II) shows that the approximation of convective acceleration by the Superbee limiter reduced advantage of numerical diffusion generated by the discretization of the convective term in the transport equation. This reduction is quatified by a gain to the maximum of the solution compared to the results of ” test 1.1”. And given the percentage gain for different schemes.
10.29% for centred scheme.
09.43% for Mimnod limiter.
09.06% for Upwind scheme.
08.97% for Van Leer limiter.
07.68% for M.C. limiter
06.98% for Superbee limiter.

We note that the Superbee limiter has the lowest rate of gain over other schemes. This report is not surprising, since this is the limiter is most powerful that the others and therefore any improvement will have a lesser effect on this limiter (in other essential term of the improvement is already in the definition itself of the limiter). Finally, the comparison for example of the figures (3.e) makes it possible to deduce that the limitation from the accelaration convective also allowd a significant improvement in the conservation of the form. This remark also applies to all schemes except the centred and Second Upwind schemes.

Schemes	Rmass	Cmin	Cmax
Upwind	0.9998	0	0.5892
centred	1.0000	-0.2803	1.3060
Second Upwind	1.0000	-0.2803	1.3060
Minmod	1.0000	0	0.8360
Superbee	1.0000	0	0.9337
Van Leer	0.9999	0	0.9076
MC	0.9999	0	0.9273

Tab. I – *Comparing the convection schemes for simulation of a concentration equation with Upwind Scheme for the velocity.* Δt = 1 , Δx = 1, *at t =25*

Schemes	Rmass	Cmin	Cmax
Upwind	0.9998	0	0.6426
centred	1.0000	-0.3391	1.4404
Second Upwind	1.0000	-0.3391	1.4404
Minmod	1.0000	0	0.9149
Superbee	1.0000	0	0.9989
Van Leer	0.9999	0	0.9891
MC	1	0	0.9986

Tab. II – *Comparing the convection schemes for simualtion of a concentration equation with Superbee scheme for the velocity.* Δt = 1 , Δx = 1, *at t =25*

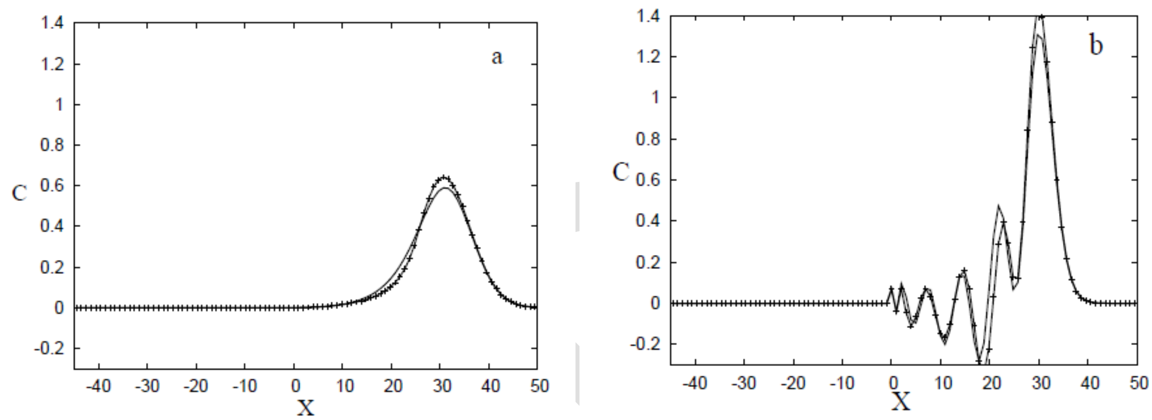
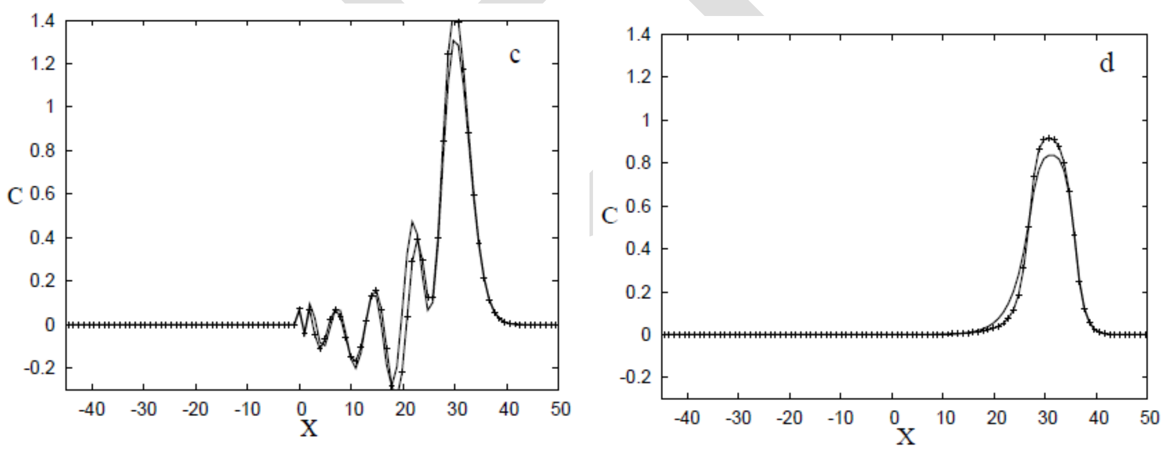


Fig. 2 – The concentration profile C obtained with a scheme : a) Upwind ; b)centred. solid line : Burger approched by Superbee, discontinous line :Burger approched by Upwind



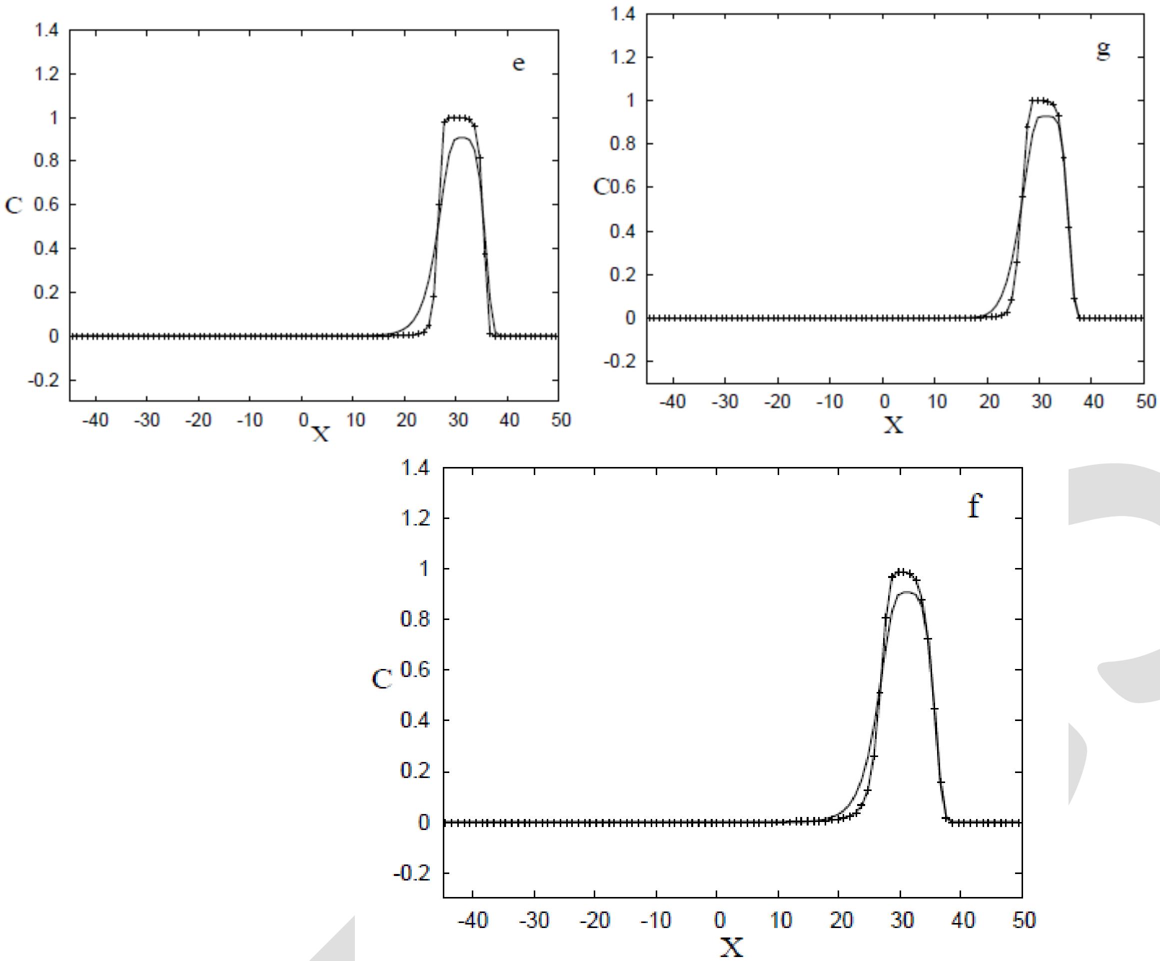


Fig. 3 – The concentration profile C obtained with a scheme : c)Second Upwind d)Minmod ; e)Superbee ; f)VanLeer ; g)M.C. solid line Burger approched by Superbee, discontinous line : Burger approched by Upwind

5. Conclusion

A series of slope limiter schemes was introduced and tested for monodimensional case. These schemes can be an alternative to advection schemes commonly used in ocean modeling. The Upwind and centred schemes were compared to four limiters slope (Minmod, Superbee Van Leer, MC) admitting the TVD property. These limiters Capable of reducing the numerical diffusion and limit the dispersion introduced by the terms of third order due to truncation errors. The schemes of order two in space are preferred oceanographers probably for the simplicity of their implementation and the absence of the numerical diffusion. However, these schemes are too dispersive, but this handicap can be absorbed by increasing the value of the coefficient of horizontal viscosity. This technique has also the disadvantage of returning this coefficient (resulting from the physical processes of turbulence) like a numerical parameter of chock. The tests presented in this study show that the slope limiters a total variation diminishing (TVD) can be implemented successfully in ocean modeling. In what we concerns, the slope limiters and in particular the Superbee limiter has considerably improves the characteristics of solution concentration when this one is coupled with the Burger equation. The tests undertaken here revealed that for a coupled problem, the numerical diffusion introduced in hydrodynamics is struck directly on the solution concentration (comparison test1.1 and test1.2).

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