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# Bursting and Chaotic Activities in the Nonlinear Dynamics of FitzHugh-Rinzel Neuron Model

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**Abstract**— Selection of an appropriate neuron model for neuroscience studies is a crucial task for researchers. Some of the neuron models are too simple to exhibit the complex dynamics of the neuron and others are very complex and cannot be used in a network as they result in computationally expensive analysis. Study of chaotic behavior and bursting phenomenon of biophysical neuron models is an important step towards analyzing the overall functioning of the brain. In-depth analysis of bursting and chaotic behavior of FitzHugh- Rinzel neuron model has been made in this paper work.

**Keywords**— Biological Neuron, Time Response, Phase Portrait, FitzHugh-Rinzel Model, Nonlinear Dynamics, Bursting, Chaotic Behavior.

## INTRODUCTION

The brain is one of the most complex objects in the universe. Although many attempts have been made to investigate and model the functionalities of the brain, the exact working of it is still unknown. The research in the field of computational neuroscience is aimed to know about the brain with more intricacy and to develop more realistic models of its constituents. These models are important tools for characterizing what nervous systems do, determining how they function, and understanding why they operate in particular ways. As most of these models are dynamical in nature, theory of dynamical systems is useful in gaining new insights into the operation of nervous system. The primary step for understanding the brain dynamics is to understand the dynamical behavior of mathematical models of individual neurons. The most important part of this study is the bifurcation analysis of the neurons and their networks. Certain bifurcations in the membrane potential result in neural excitability, spiking, and bursting. Revealing these bifurcations in neuron models helps in knowing various functions of the brain. Such types of studies include the analysis of chaotic behavior of neural systems. These neural systems can be individual neurons or their interconnections. The ongoing research in this regard is to examine the role of chaos in learning. Exploring dynamics of biological neuron models is helpful not only in neuroscience studies but also in neural network applications.

In literature, different dynamical models are proposed to represent bio- physical activities of neurons. Commonly used models for the study of spiking and bursting behaviors of neurons include integrate-and-fire model and its variants [5, 6, 18, 30], FitzHugh-Nagumo model [7], Hindmarsh-Rose model [15, 10], Hodgkin-Huxley model [13, 11] and Morris-Lecar model [25]. A short review of these models is provided by Rinzel in [26, 27, 28]. An excel- lent comparison of more than twenty neurocomputational properties of the most popular spiking and bursting models have been made in [16]. Bifurcation phenomena in individual neuron models including the Hodgkin-Huxley, Morris-Lecar and FitzHugh-Nagumo models have been investigated in the literature [15, 27, 4]. Rinzel and Ermentrout [27] studied bifurcations in the Morris-Lecar model by treating the externally applied direct current as a bifurcation parameter.

From various experiments, it has been well established that neuronal activities show many characteristics of chaotic behavior. Some researchers believe that this sort of behavior is necessary for the brain to engage in continual learning – categorizing a novel input into a novel category rather than trying to fit it into an existing category [29, 9, 1]. Freeman developed a mathematical model for EEG signals generated by the olfactory system in rabbits [9]. He suggested that the learning and recognition of novel odors, as well as the recall of familiar odors can be explained through chaotic dynamics of the olfactory cortex. Attempts have been made to represent the neurodynamics of biological neural networks in terms of artificial neural network type of structures with some extent to their intricacies. Chaotic dynamics based neural networks have also been proposed to capture some of the characteristics of learning in the brain [2]. Nonlinear dynamics of various neuronal models has been investigated in [24, 19, 20]. Chaos in firing rate recurrent neural network models have been investigated in [22]. The effect of synaptic bombardment has been explored in the dynamics of various biological neuron models [21]. Nonlinear dynamical analysis on coupled modified FitzHugh-Nagumo neuron model has also been performed [23].

The brain takes the incoming sensory data as input, encodes them into various biophysical variables and performs a number of computations on these variables to extract relevant features from the input. Biophysical mechanisms responsible for these computations are dynamical in nature and lead to various types of learning. Therefore, there are two closely related issues in computational neuroscience – nonlinear dynamics of different constituents of nervous system and the roles of various biophysical activities in learning. There have been many evidences in literature to experimentally explore the chaotic behavior of neuronal activities. Study of chaos and other phenomena of nonlinear dynamics in various levels of brain modeling can provide a significant help in investigating the learning mechanism.

Nonlinear dynamical analysis of FitzHugh-Rinzel neuron models is carried out to investigate different bifurcations and chaos. The same analysis has been carried out also for a firing-rate recurrent neural network of three neurons and it is observed that its dynamical behavior becomes chaotic at some set of parametric values. This study supports the role of chaos in continual learning – categorizing a novel input into a novel category rather than trying to fit it into an existent category.

## NEURON MODELS

Neurons or nerve cells are the fundamental building blocks of the nervous system. These cells form the basis of the brain. Therefore, a sufficient in-depth knowledge of neurons is necessary for study of the brain. A typical human brain consists of approximately 100 billion neurons, each neuron having at least 10,000 connections with other neurons. A typical neuron has three major regions: the soma, the axon and the dendrites. Dendrites form a dendritic tree which is a very fine bush of thin fibers around body of the neuron. Dendrites receive information from neurons through axons– long fibers that serve as transmission lines. An axon is a long cylindrical connection that carries impulses from the neuron. The end part of an axon splits into a fine arborization which terminates in a small end-bulb almost touching the dendrites of neighboring neurons. The connections between the ends of axons and the dendrites or cell bodies of other neurons are specialized structures called synapses. Electrochemical signals flow in neurons, originating at the dendrites or cell body in response to stimulation from other neurons and propagating along axon branches which terminate on the dendrites or cell bodies of thousands of other neurons [3].

Various mathematical models for biological neurons have been proposed in literature [7, 15, 13, 25, 26, 16] to represent their biological activities. As it is generally believed that neurons communicate with each other via action potentials, almost all of these models represent neuronal behavior in terms of membrane potential and action potential. Some most popular models are Hodgkin-Huxley [13], integrate-and-fire [18], FitzHugh-Nagumo (FHN) [7], FitzHugh-Rinzel (FHR) [26], Morris-Lecar [25], Wilson-Cowan [31, 14], Izhikevich [16] and Hindmarsh-Rose [10] models. These neuron models represent the characteristics of the responses of different types of real neurons with different levels of biological plausibility.

Some widely used models of spiking and bursting neurons [7, 15, 13, 25, 26, 16] are reviewed in the remaining part of this section. These models can be expressed in the form of ordinary differential equations (ODE). It is discussed whether these models have biophysically meaningful and measurable parameters and can exhibit bursting and chaotic activities.

## FITZHUGH-RINZEL MODEL

Dendrites receive electrical signals through dendritic spines that consist of a spherical head and a stem connected to the dendrite. FitzHugh-Rinzel (FHR) model describes the dynamics of dendritic spines [28]. This model shows how the potential difference between the spine head and its surrounding medium vacillates between the silent phase and the active phase or bursting. The system switches phases depending on the strength of the slowly changing drift current in the dendrite. This model is a modification of the FHN model and incorporates a third state variable to model the bursting behavior of neurons. This model is represented by a set of three coupled nonlinear differential equations. It takes the following form

$$\frac{dv}{dt} = v - \frac{v^3}{3} - w + y + I \tag{1}$$

$$\frac{dw}{dt} = \delta(a + v - bw) \tag{2}$$

$$\frac{dv}{dt} = \mu(c - v - dy) \tag{3}$$

Variable $v$ represents the potential difference between the dendritic spine head and its surrounding medium, $w$ is recovery variable and $y$ represents the slowly moving current in the dendrite. In this model, $v$ and $w$ together make up a fast subsystem relative to $y$.

## NONLINEAR DYNAMICS OF FITZHUGH-RINZEL MODEL

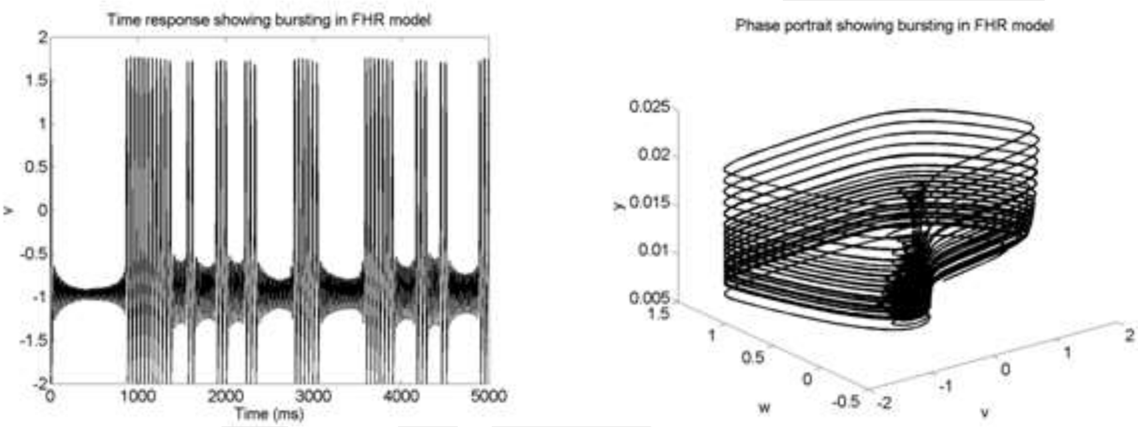
FitzHugh-Rinzel (FHR) model is a three dimensional model derived from the FHN model to incorporate bursting phenomena of nerve cells. It takes the following form

$$\frac{dv}{dt} = v - \frac{v^3}{3} - w + y + I \tag{4}$$

$$\frac{dw}{dt} = \delta\,(a + v\,-bw) \tag{5}$$

$$\frac{dy}{dt} = \mu\,(c\,-\,v\,-\,dy) \tag{6}$$

This model exhibits spiking as well as bursting phenomenon of the neuron. Figure 1 shows time response and phase portrait of this system at $I = 0.3125$, $a = 0.7$, $b = 0.8$, $c = -0.775$, $d = 1$, $\delta = 0.08$, and $\mu = 0.0001$. It is observed that the model exhibits bursting at these values of parameters.




(a) Time Response (b) Phase Portrait

Figure 1: Time response and phase portrait of FHR model at $I = 0.3125$, $a = 0.7$, $b = 0.8$, $c = -0.775$, $d = 1$, $\delta = 0.08$ and $\mu = 0.0001$. The model exhibits bursting at these values of parameters.

## BIFURCATION ANALYSIS OF FITZHUGH-RINZEL (FHR) MODEL

Bifurcation analysis of FHR model has been carried out with respect to different parameters.

### BIFURCATION ANALYSIS WITH $\mu$ AS A BIFURCATION PARAMETER

Jacobian matrix, $J$ of this model at the equilibrium point $(v_e, w_e, y_e)$ for $I = 0.3125$, $a = 0.7$, $b = 0.8$, $c = -0.775$, $d = 1$ and $\delta = 0.08$, in terms of the bifurcation parameter $\mu$ is gives as

$$J = \begin{bmatrix} 1 - v_e^2 & -1 & 1 \\ \frac{2}{25} & -\frac{8}{125} & 0 \\ -\mu & 0 & -\mu \end{bmatrix}$$

Eigenvalues of the linearized system are computed by solving $|\lambda I - J| = 0$ for $\lambda$. Thus, eigenvalues $\lambda_1$, $\lambda_2$, and $\lambda_3$ are the roots of following characteristic equation of the linearized FHR system.

$$\lambda^3 + \lambda^2\mu - \frac{117}{125}\lambda^2 + \frac{8}{125}\lambda\mu + \frac{2}{125}\lambda + v_e^2\lambda^2 + v_e^2\lambda\mu + \frac{8}{125}v_e^2\lambda + \frac{8}{125}v_e^2\mu = 0$$

This characteristic equation can be written as

$$\lambda^3 + A\lambda^2 + B\lambda + C = 0$$

where

$$A = \mu - \frac{117}{125} + v_e^2$$

$$B = \frac{8}{125}\mu + \frac{2}{125} + v_e^2\mu + \frac{8}{125}v_e^2$$

$$C = \frac{2}{25}\mu + \frac{8}{125}v_e^2\mu$$

Applying Routh-Hurwitz stability criteria, we get the following array

| | | |
|---|---|---|
| $\lambda^3$ | $1$ | $B$ |
| $\lambda^2$ | $A$ | $C$ |
| $\lambda^1$ | $\frac{AB - C}{A}$ | $0$ |
| $\lambda^0$ | $C$ | $0$ |

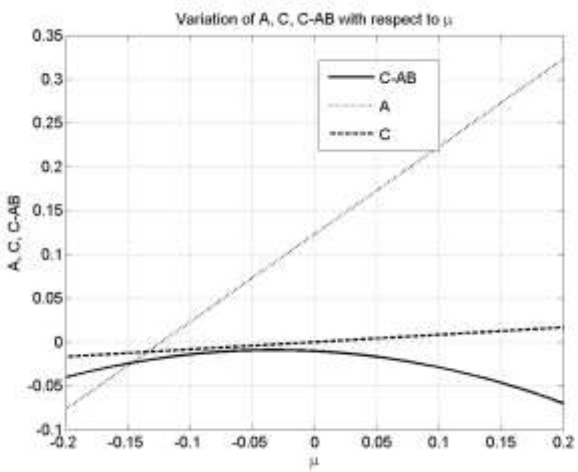



Figure 2: Plots of $A$, $C$ and $C - AB$ with respect to $\mu$. $A$, $B$, and $C$ are the coefficients of the characteristic equation of FHR model. $C - AB$ is always negative for all values of bifurcation parameter $\mu$. $A$ and $C$ crosses zero value at $\mu = -0.13$ and $\mu = -0.02$, respectively. Therefore, Hopf bifurcation takes place at these values of $\mu$.

It is found that there are possibilities of Hopf bifurcations at those values of parameter $\mu$ for which $A = 0$, $C = 0$ or $C - AB = 0$. At the equilibrium point $(-1.0292, -0.4116, 0.2542)$, $C - AB$, in terms of $\mu$ comes out to be

$$C - AB = -1.1233\mu^2 - 0.0744\mu - 0.0103$$

Thus, $C - AB$ is always negative for all values of bifurcation parameter $\mu$. Plots of $A$, $C$, and $C - AB$ with respect to $\mu$ are shown in Figure 2. It is clear from this figure that $A$ and $C$ crosses zero value at $\mu = -0.02$ and $\mu = -0.13$ and therefore Hopf bifurcation takes place at these values of $\mu$. However, these values do not lie in the practical range of $\mu$. To reveal the bifurcation phenomenon in the practical range of $\mu$, the bifurcation diagram and largest Lyapunov exponent are plotted. Figure 3 shows the bifurcation diagram with $\mu$ as bifurcation parameter. Plot of largest Lyapunov exponent with respect to $\mu$ is shown in Figure 4. Positiveness of the largest Lyapunov exponent for some values of $\mu$ indicates its chaotic behavior which is also observed in the bifurcation diagram. Time response and phase portrait are plotted for $\mu = 0.0002$ in Figures 5(a) and 5(b), respectively. These plots show the chaotic behavior.

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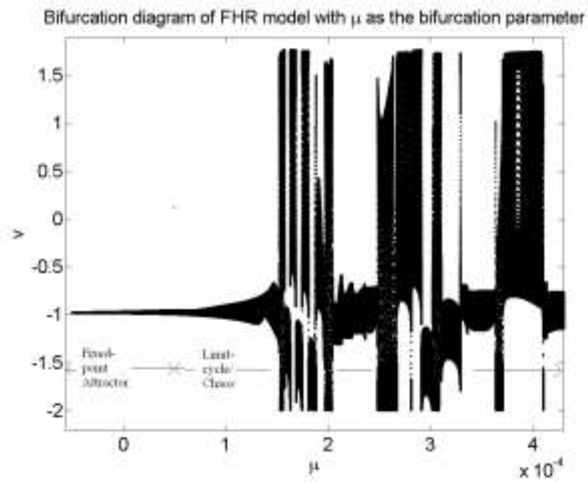



Figure 3: Bifurcation diagram of FHR model with $\mu$ as bifurcation parameter. This bifurcation diagram shows that the minimum value of $\mu$ for either spiking or bursting or chaotic response is of the order of $1 \times 10^{-4}$. $y$-axis of this plot shows the values of variable $v$ at different time instants after transients.

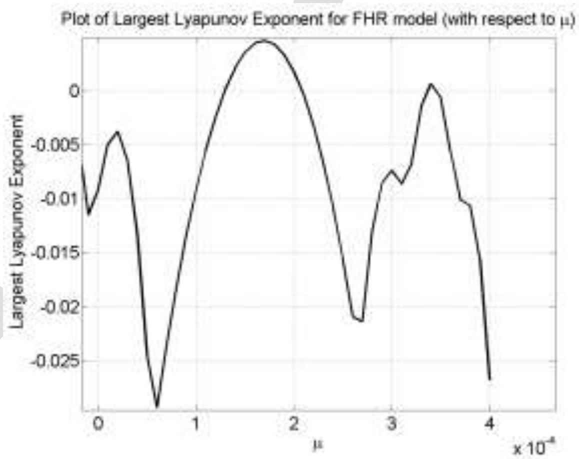



Figure 4: Plot of the largest Lyapunov exponent of FHR model with respect to $\mu$. It is positive for $1.4 \times 10^{-4} < \mu < 2.2 \times 10^{-4}$ and $3.4 \times 10^{-4} < \mu < 3.43 \times 10^{-4}$. Therefore, the model exhibits chaotic response for these ranges of $\mu$.

## BIFURCATION ANALYSIS WITH $I$ AS A BIFURCATION PARAMETER

Bifurcation analysis, with current ($I$) as a bifurcation parameter, is carried out in order to investigate the effect of stimulus on the spiking behavior of FHR model. For this, the Jacobian matrix, $J$ of this model at the equilibrium point ($v_e$, $w_e$, $y_e$) for $\mu = 0.0001$, $a = 0.7$, $b = 0.8$, $c = -0.775$, $d = 1$, and $\delta = 0.08$, in terms of the bifurcation parameter $I$ is given as

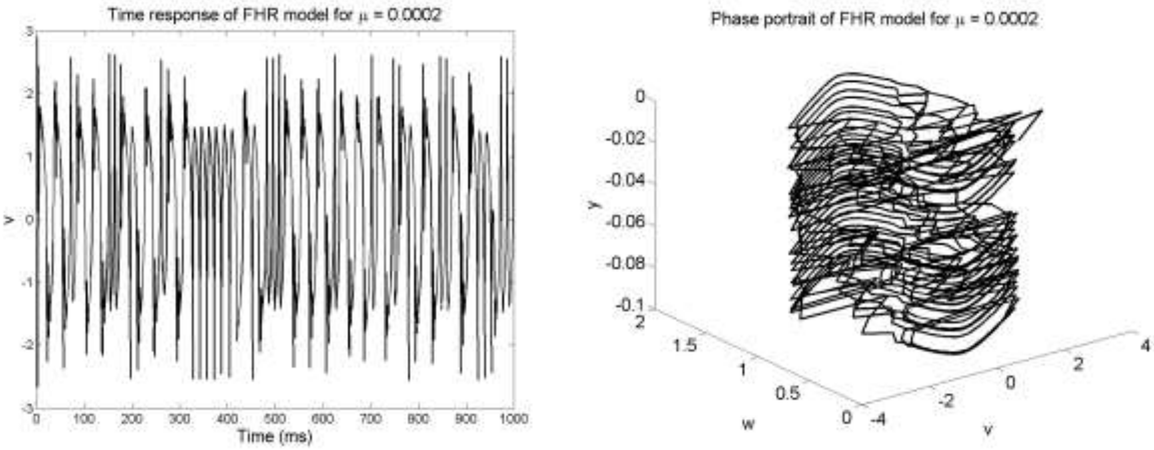
$$J = \begin{bmatrix} 1 - v_e^2 & -1 & 1 \\ \frac{2}{25} & -\frac{8}{125} & 0 \\ -\frac{1}{10000} & 0 & -\frac{1}{10000} \end{bmatrix}$$

For the above values of parameters, there exists one and only one equilibrium point for all values of $I$. Figure 6 shows the location of this equilibrium point as a function of $I$. Characteristic equation, in terms of $v_e$, comes out to be

$$\lambda^3 - \frac{9359}{10000}\lambda^2 + \frac{2501}{156250}\lambda + \frac{1}{125000} + v_e^2\lambda^2 + \frac{641}{10000}v_e^2\lambda + \frac{2501}{156250}v_e^2 = 0$$

$$v_e = \frac{1}{10}\sqrt[3]{\sigma} - \frac{25}{2}\frac{1}{\sqrt[3]{\sigma}}$$

$\sigma = -2475 + 1500I + 25\sqrt{12926 - 11880I + 3600I^2}$




(a) Time Response (b) Phase Portrait

Figure 5: Time response and phase portrait of FHR model for μ = 0.0002. The model exhibits chaotic response for this value of $\mu$.

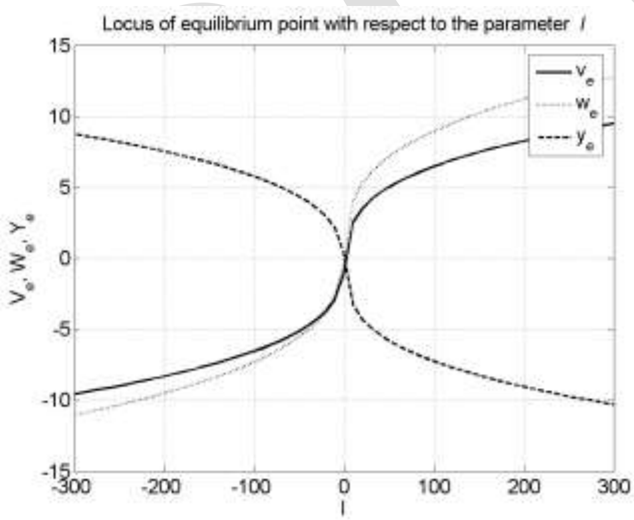



Figure 6: Locus of equilibrium point of FHR model as a function of *I*. There is one equilibrium point for every value of *I*.

This characteristic equation can be written as

$\lambda^3 + A\lambda^2 + B\lambda + C = 0$

where

$$A = v_e^2 - \frac{9359}{10000}$$

$$B = \frac{641}{10000}v_e^2 - \frac{2501}{156250}$$

$$C = \frac{1}{156250}v_e^2 - \frac{1}{125000}$$

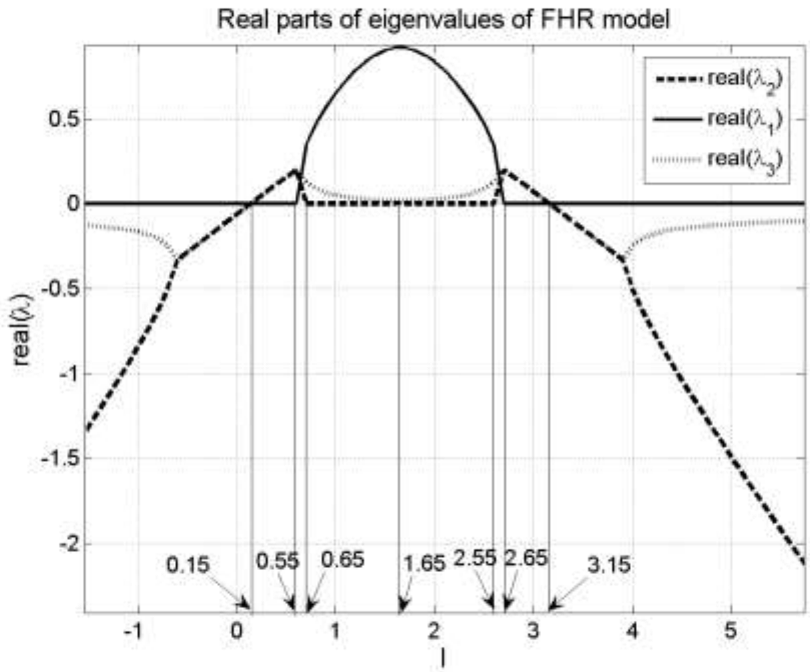



Figure 7: Real parts of eigenvalues of the FHR model. This plot indicates the values of *I* for which the real parts of eigenvalues vanish.

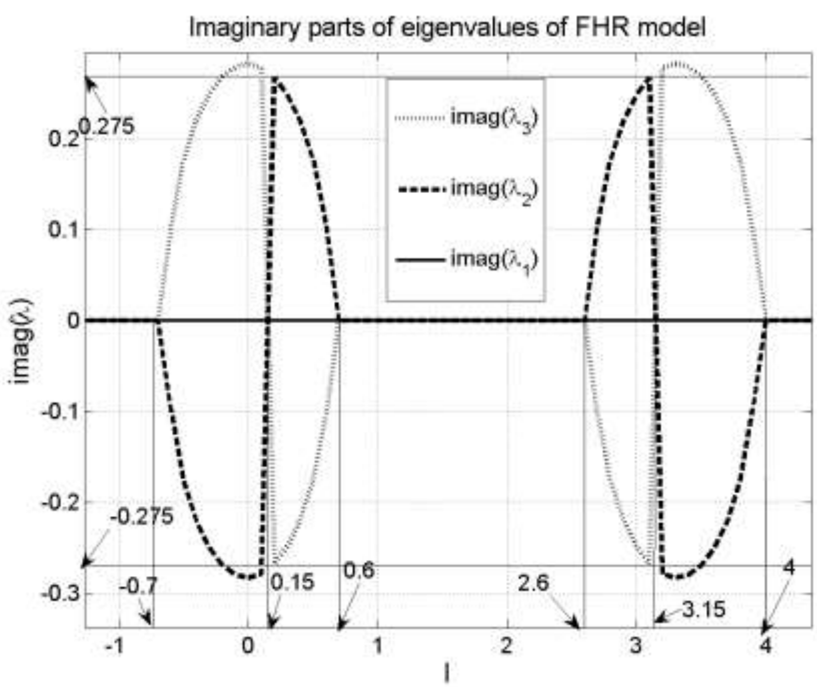



Figure 8: Imaginary parts of eigenvalues of the FHR model. This plot indicates the range of *I* for which there exist complex eigenvalues.

Routh-Hurwitz stability criterion shows that there are possibilities of Hopf bifurcations at those values of parameter $I$ for which $A = 0$, $C = 0$ or $C - AB = 0$. First condition, i.e., $A = 0$, is equivalent to $v_e = 0.2532$ and the second condition, i.e., $C = 0$, will not be satisfied for any real value of $v_e$. Third condition, i.e., $C - AB = 0$, can be expressed as

$$-0.641v_e^4 + 0.584v_e^2 + 0.015 = 0$$

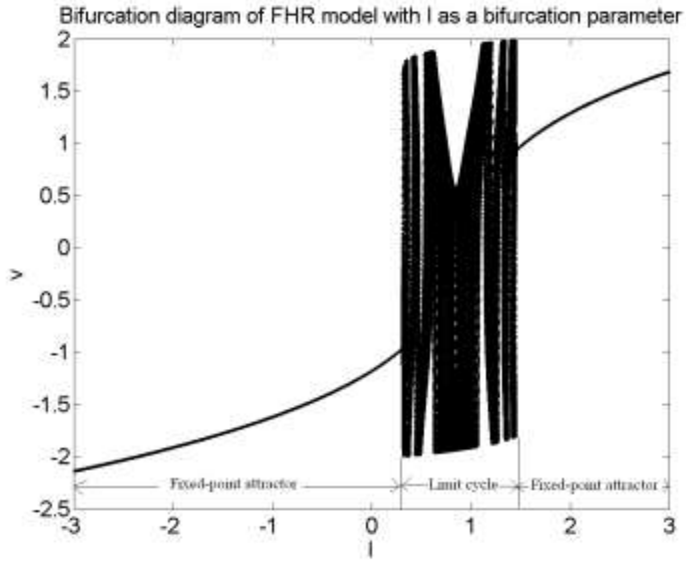
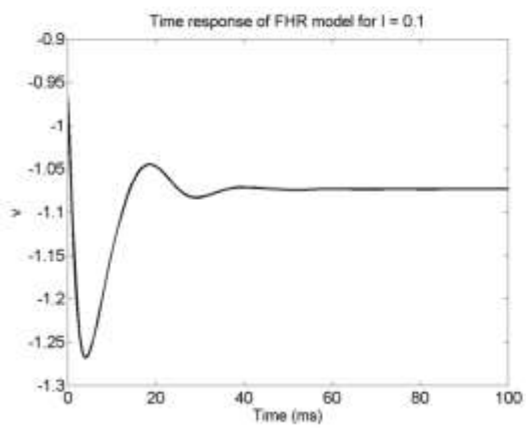
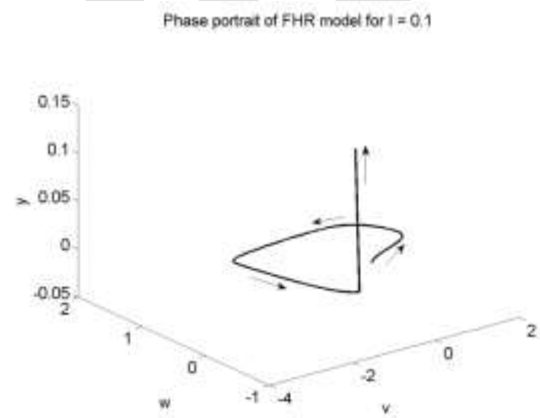



Figure 9: Bifurcation diagram of FHR model with *I* as a bifurcation parameter. Qualitative change in the dynamical behavior of the model takes place as the parameter *I* is changed. The dynamics changes from converging to periodic and again from periodic to converging as *I* is increased. *y*-axis of this plot shows the values of variable *v* at different time instants after transients.

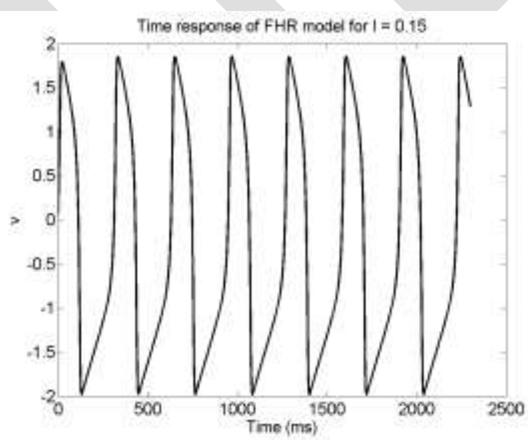



(a) Time Response

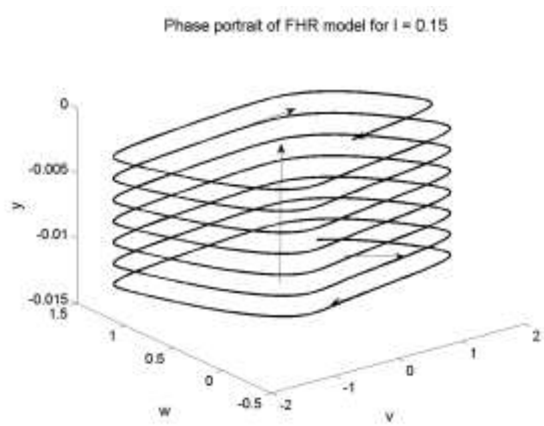



(b) Phase Portrait

Figure 10: Time response and phase portrait of FHR model for $I = 0.1$. The model exhibits a fixed-point attractor for this value of *I*.

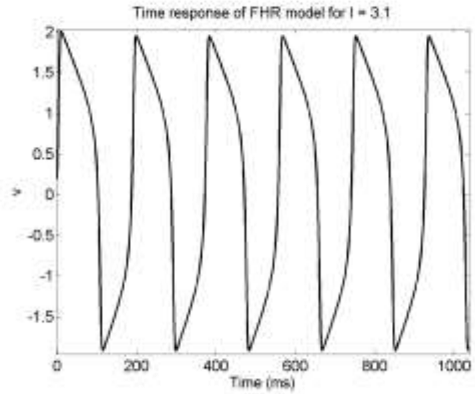



(a) Time Response

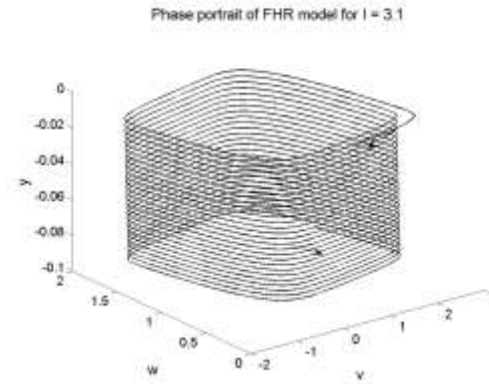



(b) Phase Portrait

Figure 11: Time response and phase portrait of FHR model for $I = 0.15$. The model exhibits a limit-cycle attractor for this value of *I*.

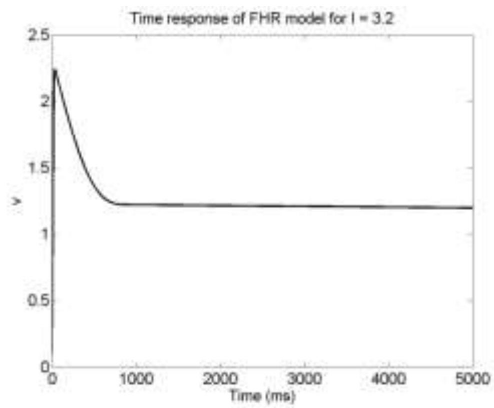



(a) Time Response

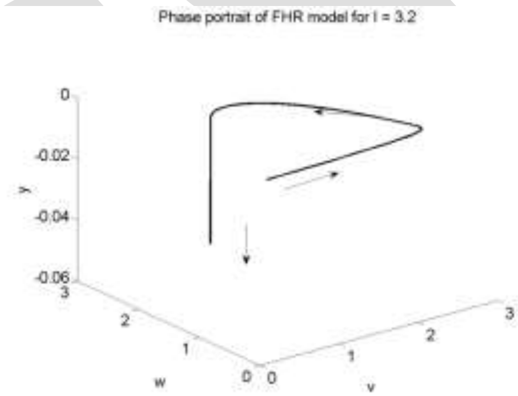



(b) Phase Portrait

Figure 12: Time response and phase portrait of FHR model for $I = 3.1$. The model exhibits a limit-cycle attractor for this value of $I$.




(a) Time Response




(b) Phase Portrait

Figure 13: Time response and phase portrait of FHR model for $I = 3.2$. The model exhibits a fixed-point attractor for this value of $I$.

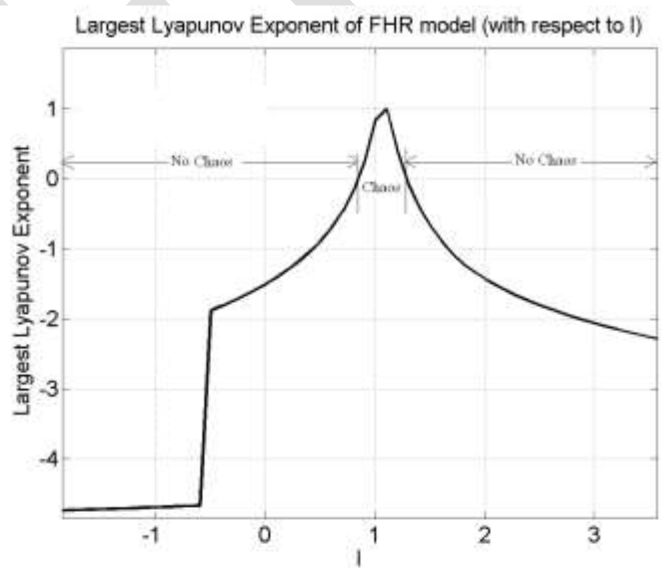
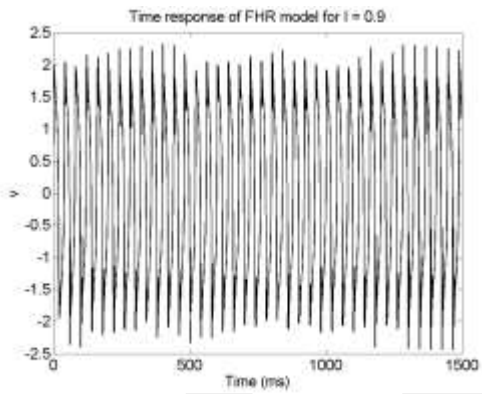
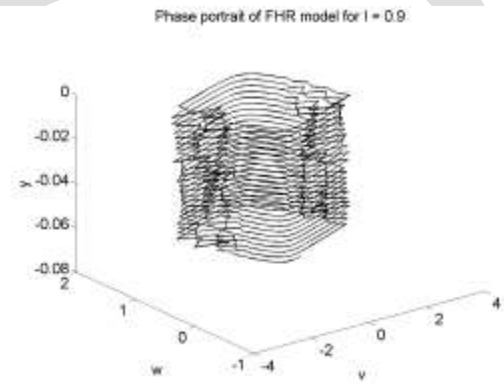



Figure 14: Plot of the largest Lyapunov exponent with respect to the bifurcation parameter $I$. The largest Lyapunov exponent is positive for $0.8 < I < 1.2$. Therefore, the model exhibits chaotic behavior in this range of $I$.

This equation has four roots among which two are real and are $v_e = \pm 0.9675$. Thus, Hopf bifurcation takes place at three values of $I$ corresponding to three values of $v_e$, i.e., $v_e = -0.9675, 0.2532, 0.9675$. The values of $I$ at which Hopf bifurcation takes place, are calculated by solving Equation 8 for above mentioned values of $v_e$. It is found that there are possibilities for Hopf bifurcation to take place for $I = 0.1387, 1.9719, 3.1612$. Eigenvalues at these values of $I$ are calculated by solving the characteristic equation and are given in Table 1. There is no Hopf bifurcation for $I = 1.9719$ as there is no pair of complex conjugate eigenvalues passing $j\omega$-axis for this value of $I$. Thus, Hopf bifurcation takes place for $I = 0.1387$ and $I = 3.1612$. Real and imaginary parts of eigenvalues of the FHR model linearized at its equilibrium points are plotted against $I$ in Figures 7 and 8, respectively. Figure 8 indicates the range of $I$ for which there are complex eigenvalues. There is no possibility of Hopf bifurcation at other values of $I$. Figure 7 indicates those values of $I$ at which real part vanishes and its rate of change with respect to $I$ is nonzero. It is observed from these plots that real part of eigenvalues vanishes without its imaginary part becoming zero at $I = 0.1387$ and $I = 3.1612$. It indicates the Hopf bifurcation at these points. These points are observed with a poor accuracy in the bifurcation diagram of Figure 9. Possible reasons of this inaccuracy are related to the limitations of numerical methods used for integration. Presence of Hopf bifurcation at $I = 0.1387$ as well as $I = 3.1612$ is verified by plotting time responses and phase portraits for I just before and just after these values. Figures 10, 11, 12, and 13 represent time responses and phase portraits for $I = 0.1, 0.15, 3.1,$ and $3.2$ respectively. It is observed that the response exhibits a fixed-point attractor at $I = 0.1$ and $3.2$ while it is oscillatory at $I = 0.15$ and $3.1$. Therefore, $I = 0.1387$ and $I = 3.1612$ are two Hopf bifurcation points. Lyapunov exponent analysis is performed for investigation of presence of chaos in FHR model when $I$ is a bifurcation parameter. Largest Lyapunov exponent is plotted in Figure 14, against the bifurcation parameter $I$ in order to detect the presence of chaotic attractors in this model. It is observed that the largest Lyapunov exponent is positive for some values of $I$. For these values of $I$, response of the FHR model is chaotic. Figure 15 represents time responses and phase portrait for $I = 0.9$. It is observed that the response exhibits chaos for this value of $I$.




(a) Time Response




(b) Phase Portrait

Figure 15: Time response and phase portrait of FHR model for $I = 0.9$. The model exhibits chaotic behavior for this value of $I$.

Table 1: Equilibrium points and eigenvalues for three different values of $I$

| $I$ | $v_e$ | $w_e$ | $y_e$ | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ |
|---|---|---|---|---|---|---|
| 0.1387 | -0.9675 | -0.3344 | 0.1925 | -0.0002 | 0 - j 0.2757 | 0 + j 0.2757 |
| 1.9719 | 0.2532 | 1.1915 | -1.0282 | 0.8481 | -0.0004 | 0.0241 |
| 3.1612 | 0.9675 | 2.0843 | -1.7425 | -0.0002 | 0 - j 0.2757 | 0 + j 0.2757 |

## CONCLUSIONS

One of the most effective approaches for the study of the nervous system is to look at its constituents as nonlinear dynamical systems. Dynamical analysis has been carried out on FHR neuron models. Eigenvalue analysis is performed for the detection of Hopf bifurcation and Lyapunov exponents are plotted for the study of chaos. Lyapunov exponent and eigenvalue analysis show that FitzHugh-Rinzel neuron model exhibits fixed points, limit cycles as well as chaotic (strange) attractors at various values of parameters. This study identifies FitzHugh-Rinzel neuron model as an appropriate model for investigating roles of bursting and chaos in continual learning. This research work can be extended in various directions. Dynamical analysis in other neuron models (including stochastic neuron models) and their interconnections can be performed in order to study various characteristics of brain signals (e.g., bursting, chaos, stochastic resonance, and threshold variability).

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