

# Construction of low complexity Array based Quasi Cyclic Low density parity check (QC-LDPC) codes with low error floor

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**Abstract**— Low Density Parity Check (LDPC) codes are class of linear block error correcting codes and are very popular due to their Shannon capacity approaching performance. Due to their low decoding complexity they outperform the existing convolutional and turbo codes and find use in many wire and wireless applications. This paper gives an overview about construction of Quasi Cyclic regular LDPC based on shifting identity matrix. The algorithm produces Parity check matrix which is free of cycle 4 and hence giving girth of at least 6. Simulation shows that with sum product decoder we can achieve good Bit Error rate (BER) and also gain the advantage of memory efficient circuit due to quasi cyclic structure of parity check matrix.

**Keywords**— Quasi Cyclic-LDPC codes, message passing, regular LDPC, irregular LDPC, circulant permutation matrix, Tanner Graph, girth.

## INTRODUCTION

Low-Density Parity-Check (LDPC) codes were first invented by Gallager in 1962 [1]. Initially these codes were ignored but recently they were rediscovered and shown to perform near Shannon's limit [2],[3],[4],[5],[6].

A regular LDPC code is described by parity check matrix  $H$  having following properties: (1) each row has  $\alpha$  1's ; (2) each column has  $\beta$  1's ; (3) number of 1's common between any two rows denoted by  $\lambda$  is no greater than 1; (4)  $\alpha$  and  $\beta$  are small compared to code length  $n$ . Fourth property ensures that number of 1's.  $\alpha$  and  $\beta$  are also called as row weight and column weight respectively. Third property ensures that there is no cycle of length 4 in Tanner graph [7]. From fourth property it is clear that number of 1's are small or low compared to total entries in  $H$  matrix and hence the name low density parity check code.

LDPC codes can be represented in two ways. One by using parity check matrix and second by using Tanner graph or *bipartite graph*. A *bipartite graph* is one in which the nodes can be partitioned into two classes, and there is no edge connection between two nodes of the same class. A *Tanner graph* for an LDPC code is a bipartite graph such that: (1) In the first class of nodes, there is one node for each of the  $n$  bits in the codeword and are also referred as bit nodes. (2) In the second class of nodes, there is one node for each of the  $m$  parity checks which is equal to number of rows of matrix  $H$  and are also called as check nodes. (3) Connection of edge is made from bit node to a check node if the bit is included in the parity check matrix [8]. Loop in the Tanner graph is called as cycle and the number of edges in cycle is called as girth. Girth 4 can affect the performance of decoding LDPC code and hence removal of girth 4 is necessary. If the column or rows weights of  $H$  matrix are not constant then it is called as irregular LDPC.

Consider the following regular LDPC  $H$  matrix having constant row and column weight. Tanner graph for given matrix is shown in figure 1.

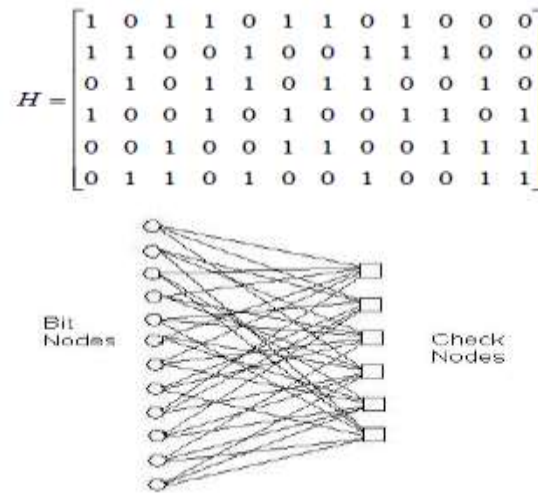


Fig. 1. A parity check matrix H and its corresponding Tanner graph

### ARRAY BASED QUASI CYCLIC LDPC CODES

The major disadvantage of LDPC code is that more amount of memory is needed to store parity check matrix. This problem can be solved by Quasi Cyclic LDPC (QC-LDPC) codes [11]-[13] since their parity check matrices include circulant matrices and hence can be encoded with the shift registers. The codes are quasi cyclic because we can obtain another codeword by simply cyclically shifting a codeword within each block of circulant.

A  $(r,t)$  regular quasi cyclic matrix H can be represented based on cyclic shifting  $\rho \times \rho$  identity matrix as :

$$H = \begin{bmatrix} I_{\alpha_{11}} & I_{\alpha_{12}} & \cdot & \cdot & \cdot & I_{\alpha_{1t}} \\ I_{\alpha_{21}} & I_{\alpha_{22}} & \cdot & \cdot & \cdot & I_{\alpha_{2t}} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ I_{\alpha_{r1}} & I_{\alpha_{r2}} & \cdot & \cdot & \cdot & I_{\alpha_{rt}} \end{bmatrix} \quad (1)$$

where  $\alpha_{ij} \in \{0,1,\dots,\rho-1\}$  and  $I_{\alpha_{ij}}$  ( $1 \leq i \leq r, 1 \leq j \leq t$ ) represents  $\rho \times \rho$  matrix obtained by cyclically shifting identity matrix to the left by  $\alpha_{ij}$  times. From above structure it is clear that H matrix has t ones in each row and r ones in each column.

Array codes [9] proposed by J. L. Fan are regular quasi cyclic in nature and they are free of cycle 4. For prime number  $\rho$  and for positive integer  $r \leq t$ , they are given by following form:

$$H = \begin{bmatrix} I & I & I & \dots & I \\ I & \alpha & \alpha^2 & \dots & \alpha^{t-1} \\ I & \alpha^2 & \alpha^{2 \times 2} & \dots & \alpha^{2 \times (t-1)} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ I & \alpha^{r-1} & \alpha^{2 \times (r-1)} & \dots & \alpha^{(r-1) \times (t-1)} \end{bmatrix} \quad (2)$$

where  $I$  is  $\rho \times \rho$  identity matrix and  $\alpha$  is  $\rho \times \rho$  left or right cyclic shift of identity matrix and  $\alpha^0 = I$ . For example, when  $\rho$  is 5

$$\alpha = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{or} \quad \alpha = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (3)$$

H matrix defined by above form has  $t$  ones in each row and  $r$  ones in each column. This has rank  $\rho r - r + 1$ .

E. Eleftheriou proposed modified array LDPC [10] having following form:

$$H = \begin{bmatrix} I & I & I & \dots & I & I & \dots & I \\ O & I & \alpha & \dots & \alpha^{j-2} & \alpha^{j-1} & \dots & \alpha^{k-2} \\ O & O & I & \dots & \alpha^{2(j-3)} & \alpha^{2(j-2)} & \dots & \alpha^{2(k-3)} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ O & O & \dots & O & I & \alpha^{j-1} & \dots & \alpha^{(j-1)(k-1)} \end{bmatrix} \quad (4)$$

where  $I$  and  $O$  are  $\rho \times \rho$  identity matrix and null matrix respectively.  $k$  and  $j$  are positive integers such  $k, j \leq \rho$  and  $\alpha$  is  $\rho \times \rho$  left or right cyclic shift of identity matrix. Here H is full rank matrix and upper triangular nature of matrix helps in encoding in linear time. Clearly above matrix gives irregular QC-LDPC code.

### Error performance of regular and irregular QC-LDPC codes

The decoding procedure can be described in terms of message passing or sum-product algorithm (SPA) algorithm [8] in which all variable nodes and all check nodes iteratively pass messages along their edges. The values of the code bits are updated accordingly with each iteration. The algorithm continues until a valid codeword is generated or until the completion of a specified number of iterations.

Simulations are done for both regular and irregular QC-LDPC codes. Figure 2 shows the BER for regular QC-LDPC and Figure 3 shows the BER of irregular LDPC codes. For regular QC-LDPC BER of  $0.0028 (10^{-2})$  for BPSK modulated code length of 184 bits over additive white Gaussian noise at 4 dB Signal-to-Noise ratio (SNR) is obtained. Similar BER of  $10^{-2}$  at 4dB SNR is obtained for irregular QC-LDPC codes.

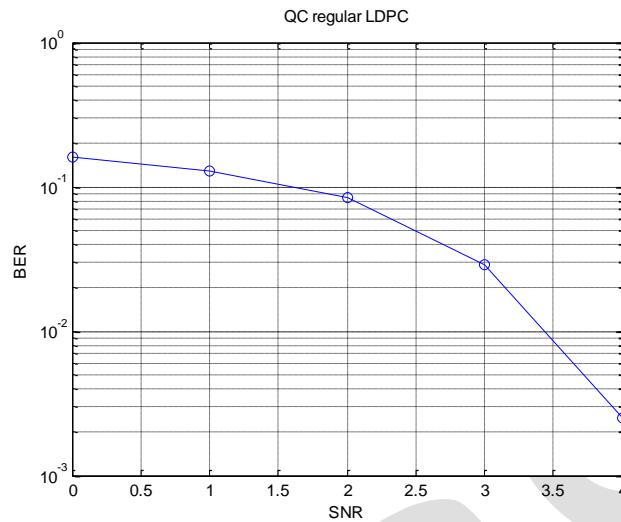


Fig. 2. BER for Regular QC-LDPC code

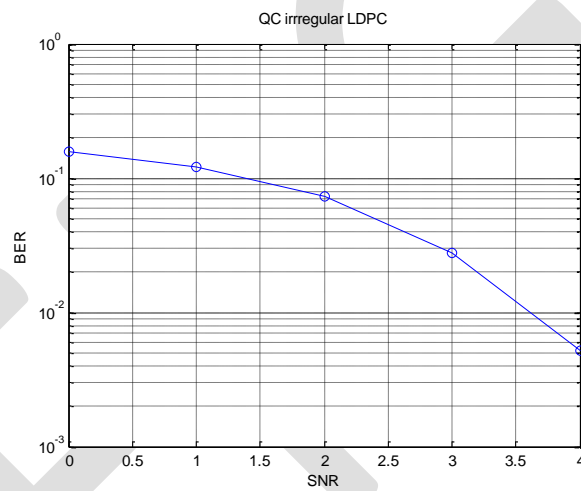


Fig. 3. BER for Irregular QC-LDPC code

## CONCLUSION

Array based QC-LDPC codes, obtained by shifting identity matrices have been simulated. Rank and BER property of LDPC codes resulting from this are studied. Simulation shows that given different parameters  $r, t, \rho, k$  etc. parity-check matrix  $H$  can be easily constructed for regular and irregular QC-LDPC codes with low complexity as compared to random and algebraic construction of LDPC codes. The advantage of this construction method is that parity-check matrix  $H$  is free of cycle 4. Due to their low encoding complexity and low error floor they are good competitors to other LDPC codes constructed from finite geometry and other methods. Another advantage is because of quasi cyclic structure decoding can be performed by using shift registers giving memory efficient circuit.

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