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# LS- Sasakian Manifold with Semi-symmetric Non-metric F-connection

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**Abstract**— Hayden [1] introduced the idea of metric connection with torsion tensor in a Riemannian manifold. In 1975, Golab [2] studied quarter-symmetric connection in a differentiable manifold. T.Imai [3] discussed on hypersurfaces of a Riemannian manifold with semi-symmetric metric connection. In 1980, R. S. Mishra and S. N. Pandey [4] discussed on quarter-symmetric metric F-connection and in 1970, K. Yano [9] studied on semi symmetric metric connections and their curvature tensors. In 1992, Nirmala S. Agashe and Mangala R. Chafle [5] studied semi-symmetric non-metric connection in a Riemannian manifold. Symmetric connections are also studied by K. Yano and T. Imai [10], A.Sharfuddin and S.I.Husain [8], R. N. Singh and S. K. Pandey [7] and many others. The purpose of this paper is to introduce a semi-symmetric non-metric F-connection in Lorentzian Special Sasakian manifold.

**Keywords**— Lorentzian Special Sasakian manifold, semi-symmetric non-metric F-connection, Nijenhuis tensor.

## 1. INTRODUCTION

An n-dimensional differentiable manifold $M_n$, on which there are defined a tensor field $F$of type (1, 1), a vector field $T$, a 1-form $A$ and a Lorentzian metric $g$, satisfying for arbitrary vector fields $X, Y, Z, \ldots$

(1.1) $\overline{\overline{X}} = -X - A(X)T,$

(1.2) $A(T) = -1,$

(1.3) $g(\overline{X}, \overline{Y}) = g(X, Y) + A(X)A(Y)$, where $A(X) = g(X, T)$, $\overline{X} \stackrel{\text{def}}{=} FX,$

(1.4) (a) $(D_X F)(Y) + A(Y)\overline{X} - `F(X,Y)T = 0 \Leftrightarrow$

(b) $(D_X `F)(Y,Z) - A(Y)`F(Z,X) - A(Z)`F(X,Y) = 0$, where $`F(X,Y) \stackrel{\text{def}}{=} g(\overline{X}, Y)$

(1.5) $D_X T = \overline{\overline{X}},$

Then $M_n$ is called a Lorentzian special Sasakian manifold (an LS-Sasakian manifold).

In an LS-Sasakian manifold, it can be easily seen that

(1.6) $A(\overline{X}) = 0,$

(1.7) rank $F = n - 1.$

(1.8) $`F(X,Y) = -`F(Y,X),$

(1.9) (a) $(D_X A)(\overline{Y}) = `F(X,Y) \Leftrightarrow$ (b) $(D_X A)(Y) = -g(\overline{X}, \overline{Y})$

Nijenhuis tensor in an L-Contact manifold [6] is given by

(1.10) $`N(X,Y,Z) = (D_{\overline{X}}`F)(Y,Z) + (D_{\overline{Y}}`F)(Z,X) + (D_X`F)(Y,\overline{Z}) + (D_Y`F)(\overline{Z},X)$

Where

$$`N(X,Y,Z) \stackrel{\text{def}}{=} g(N(X,Y),Z)$$

## 2. SEMI-SYMMETRIC NON-METRIC F-CONNECTION IN AN LS-SASAKIAN MANIFOLD

An affine connection B is called non-metric connection, if

(2.1) $B_X g \neq 0$

An affine connection B is called F- connection, if

(2.2) $B_X F = 0$

Let us consider non-metric F-connection having torsion tensor $S$ of the form

(2.3) $S(X,Y) = A(Y)X - A(X)Y,$

Where $S$ is the torsion tensor of the connection $B$.

Therefore,

**Definition 2.1** A linear connection satisfying (2.1), (2.2) and (2.3) is called a semi-symmetric non-metric F-connection.

**Theorem 2.1** In an LS-Sasakian manifold, the connection $B$ defined by

(2.4) $B_X Y = D_X Y - A(Y)X + g(X,Y)T - 2A(X)Y$

is a semi-symmetric non-metric connection, whose metric is given by

(2.5) $(B_X g)(Y,Z) = 4A(X)g(Y,Z)$

**Proof.** Put

(2.6) $B_X Y = D_X Y + H(X,Y)$

Where $H$ is a tensor field of type (1, 2), given by

(2.7) $H(X,Y) = \alpha A(Y)X + \beta g(X,Y)T - 2A(X)Y,$

Where α and $\beta$ are constants to be determined.

From (2.6) and (2.7), we get

(2.8) $B_X Y = D_X Y + \alpha A(Y)X + \beta g(X,Y)T - 2A(X)Y$

Then, torsion tensor of the connection $B$ is given by

(2.9) $S(X,Y) = H(X,Y) - H(Y,X)$

Therefore

(2.10) $`H(X,Y,Z) = \alpha A(Y)g(X,Z) + \beta A(Z)g(X,Y) - 2A(X)g(Y,Z)$ and

(2.11) $`S(X,Y,Z) = `H(X,Y,Z) - `H(Y,X,Z)$

Where

(2.12) $`H(X,Y,Z) \stackrel{\text{def}}{=} g(H(X,Y),Z)$

(2.13) $`S(X,Y,Z) \stackrel{\text{def}}{=} g(S(X,Y),Z)$

From (2.2) and (2.8), we get

(2.14) $(D_X F)(Y) - \alpha A(Y)\overline{X} - \beta `F(X,Y)\mathrm{T} = 0$

In an LS-Sasakian manifold, we have

(2.15) $(D_X F)(Y) + A(Y)\,\overline{X} - `F(X,Y)\mathrm{T} = 0$

From (2.14) and (2.15), we get, $\alpha = -1,\ \beta = 1$

Putting these values in (2.8), we obtain (2.4).

Also

(2.16) $X\big(g(Y,Z)\big) = (B_X g)(Y,Z) + g(\ B_X Y, Z) + g(Y,\ B_X Z) = g(\ D_X Y, Z) + g(Y,\ D_X Z)$

Using (2.4) in (2.16), we get (2.5).

**Theorem 2.2** In an LS-Sasakian manifold with semi-symmetric non-metric F-connection $B$, we have

(2.17) (a) $B_X T = 2X + 2\overline{\overline{X}}$

(b) $(B_X A)(Y) = (D_X A)(Y) + g(X,Y) + 3A(X)A(Y)$

(c) $(B_X `F)(Y,Z) = (D_X `F)(Y,Z) + 4A(X)`F(Y,Z) - A(Y)`F(Z,X) - A(Z)`F(X,Y)$

(d) $(B_X `F)\left(\overline{Y},\ \overline{\overline{Z}}\right) - (B_X `F)\left(\overline{\overline{Y}}, \overline{Z}\right) = 8A(X)g(\overline{Y},\ \overline{Z})$

(e) $(B_X `F)\left(\overline{\overline{Y}},\ \overline{\overline{Z}}\right) + (B_X `F)(\ \overline{Y},\ \overline{Z}\ ) = 8A(X)`F(Y,Z)$

**Theorem 2.3** Nijenhuis tensor with semi-symmetric non-metric F-connection $B$ is given by

(2.18) $`N(X,Y,Z) = \left(B_{\overline{X}} `F\right)(Y,Z) + \left(B_{\overline{Y}} `F\right)(Z,X) + (B_X `F)\left(Y,\overline{Z}\right) + (B_Y `F)\left(\overline{Z},X\right) -$

$4A(X)g(Y,Z) + 4A(Y)g(Z,X)$

**Proof.** (2.18) follows from (1.10) and (2.17) (c).

**Theorem 2.4** The connection induced on a submanifold of an LS- Sasakian manifold with a Semi-symmetric non-metric F-connection with respect to unit normal vectors $M$ and $N$ is also Semi- symmetric non-metric F-connection iff

(2.19) (a) $h\big(\hat{X},\hat{Y}\big) = \ p\big(\hat{X},\hat{Y}\big) + \rho\tilde{g}\big(\hat{X},\hat{Y}\big)$

(b) $k\big(\hat{X},\hat{Y}\big) = \ q\big(\hat{X},\hat{Y}\big) + \sigma\tilde{g}\big(\hat{X},\hat{Y}\big)$

**Proof.** Let $M_{2m-1}$ be submanifold of $M_{2m+1}$ and let $c : M_{2m-1} \to M_{2m+1}$ be the inclusion map such that

$$d \in M_{2m-1} \to cd \in M_{2m+1}\ ,$$

Where $c$ induces a Jacobian map (linear transformation) $J : T'_{2m-1} \to T'_{2m+1}$.

$T'_{2m-1}$ is tangent space to $M_{2m-1}$ at point $d$ and $T'_{2m+1}$ is tangent space to $M_{2m+1}$ at point $cd$ such that

$$\hat{X} \text{ in } M_{2m-1} \text{ at } d \to J\hat{X} \text{ in } M_{2m+1} \text{ at } cd$$

Let $\tilde{g}$ be the induced metric tensor in $M_{2m-1}$ , then

(2.20) $\tilde{g}\big(\hat{X},\hat{Y}\big) = ((\ g(\ J\hat{X}, J\hat{Y}\ ))b$

Semi- symmetric non-metric F-connection $B$ in an LS- Sasakian manifold $M_n$ is given by

(2.21) $B_X Y = D_X Y - A(Y)X + g(X,Y)T - 2A(X)Y$

Where $X$ and $Y$ are arbitrary vector fields of $M_{2m+1}$. Let

(2.22) $T = Jt - \rho M - \sigma N,$

Where $t$ is $C^\infty$ vector fields in $M_{2m-1}$. $M, N$ are unit normal vectors to $M_{2m-1}$.

Denoting by $\grave{D}$ the connection induced on the submanifold from $D$.

Put

(2.23) $D_{JX} J\hat{Y} = J(\grave{D}_X \hat{Y}) - p(\hat{X},\hat{Y})M - q(\hat{X},\hat{Y})N$

Where $p$ and $q$ are symmetric bilinear functions in $M_{2m-1}$. Also

(2.24) $B_{JX} J\hat{Y} = J(\grave{B}_X \hat{Y}) - h(\hat{X},\hat{Y})M - k(\hat{X},\hat{Y})N$ ,

Where $\grave{B}$ is the connection induced on the submanifold from $B$ and $h, k$ are symmetric bilinear functions in $M_{2m-1}$.

Inconsequence of (2.21), we have

(2.25) $B_{JX} J\hat{Y} = D_{JX} J\hat{Y} - A(J\hat{Y})J\hat{X} + g(J\hat{X}, J\hat{Y})T - 2A(J\hat{X})J\hat{Y}$

Using (2.23), (2.24) and (2.25), we have

(2.26) $J(\grave{B}_X \hat{Y}) - h(\hat{X},\hat{Y})M - k(\hat{X},\hat{Y})N = J(\grave{D}_X \hat{Y}) - p(\hat{X},\hat{Y})M - q(\hat{X},\hat{Y})N - A(J\hat{Y})J\hat{X} + g(J\hat{X}, J\hat{Y})T - 2A(J\hat{X})J\hat{Y}$

Using (2.22), we get

(2.27) $J(\grave{B}_X \hat{Y}) - h(\hat{X},\hat{Y})M - k(\hat{X},\hat{Y})N = J(\grave{D}_X \hat{Y}) - p(\hat{X},\hat{Y})M - q(\hat{X},\hat{Y})N - a(\hat{Y})J\hat{X} +$

$$(Jt - \rho M - \sigma N)\tilde{g}(\hat{X},\hat{Y}) - 2a(\hat{X})J\hat{Y}$$

Where $\tilde{g}(\hat{Y}, t) \overset{\text{def}}{=} a(\hat{Y})$

Using (2.19) (a) and (2.19) (b), we get

(2.28) $\grave{B}_X \hat{Y} = \grave{D}_X \hat{Y} - a(\hat{Y})\hat{X} + \tilde{g}(\hat{X},\hat{Y})t - 2a(\hat{X})\hat{Y}$

This proves the theorem.

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