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Solving Fuzzy Differential Equations using Runge-Kutta third order method with modified contra-harmonic mean weights

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Abstract— In this paper an attempt has been made to determine a numerical solution for the first order fuzzy differential equations by using Runge-kutta third order method with modified contra-harmonic mean weights.The accuracy of the proposed method is illustrated by a numerical example with a fuzzy initial value problem using trapezoidal fuzzy number.

Keywords— Fuzzy Differential Equations, Third order Runge-kutta method, Modified contra-harmonic mean, Trapezoidal fuzzy number

1.INTRODUCTION

The fuzzy differential equation concept has been most popular and rapidly growing in the last few years. First order linear fuzzy differential equation is one of the simplest fuzzy differential equation,which appear in many applications. The concept of fuzzy derivative was first introduced by S.L.Chang and L.A.Zadeh in[6]. D.Dubois and Prade [7] defined and used the extension principle.Other methods have been discussed by M.L.puri and D.A.Ralescu[23] and R.Goetschel and W.Voxman[10] contributed towards the differential of fuzzy functions.The fuzzy differential equation and initial value problems were extensively studied by O.Kaleva [15,16] and by S.Seikkala[24].Recently many research papers are focused on numerical solution of fuzzy initial value problems (FIVPS).Numerical Solution of fuzzy differential equation has been introduced by M.Ma, M.Friedman, A.Kandel [18] through euler method and by S.Abbasbandy and T.Allahviranloo [1] by taylor method.Runge–Kutta methods have also been studied by authors [2,21].V.Nirmala,N.Saveetha,S.Chenthurpandiyan discussed on numerical solution of fuzzy differential equation by Runge-Kutta method with higher order derivative approximations[20].R.Gethsi sharmila and E.C.Henry Amirtharaj discussed on numerical solutions of first order fuzzy initial value problems by non-linear trapezoidal formulae based on variety of means[13].Runge-kutta third order method with contra-harmonic mean for stiff problems was discussed by Osama Yusuf Ababneh,Rokiah Rozita[17].Following by the introduction this paper is organised as follows :In section 2, some basic results of fuzzy numbers and definitions of fuzzy derivative are given. In section 3, the fuzzy initial value problem is discussed. Section 4 describes the Runge-kutta third order method with modified contra-harmonic mean .In section 5, the Runge-kutta third order with modified contra-harmonic mean method was proposed for solving fuzzy initial value problem and the numerical examples are provided to illustrate the validity and applicability of the new method. Finally the conclusion is given for the proposed method.

2.PRELIMINARIES

Fuzzy number

An arbitrary fuzzy number is represented by an ordered pair of functions $(\underline{u}(r),\overline{u}(r))$ for all $r\in[0,1]$ which satisfy the following conditions.

i) $\underline{u}(r)$ is a bounded left continuous non-decreasing function over $[0,1]$ with respect to any r.

ii) $\overline{u}(r)$ is a bounded right continuous non-decreasing function over $[0,1]$ with respect to any r.

iii) $(\underline{u}(r) \le \overline{u}(r))$ for all $r \in [0,1]$ then the r-level set is $[u]_r = \{x \setminus u(x) \ge r\ \}; 0 \le r \le 1$

Clearly, $[u]_0 = \{x \setminus u(x) \ge 0\}$ *is compact,* which is a closed bounded interval and we denote by $[u]_r = (\underline{u}(r), \overline{u}(r))$

Trapezoidal Fuzzy Number

A trapezoidal fuzzy number *u* is defined by four real numbers $k < l < m < n$, where the base of the trapezoidal is the interval [*k, n*] and its vertices at *x = l, x = m*. Trapezoidal fuzzy number will be written as *u = (k, l, m , n).* The membership function for the trapezoidal fuzzy number *u* = (*k , l , m , n)* is defined as the following :

$$u(x) = \begin{cases} \dfrac{x-k}{l-k} & k \le x \le l \\ 1 & l \le x \le m \\ \dfrac{x-n}{m-n} & m \le x \le n \end{cases}$$

we have :

(1) $u > 0$ *if* $k > 0$

(2) $u > 0$ *if* $l > 0$

(3) $u > 0$ *if* $m > 0$ and

(4) $u > 0$ *if* $n > 0$

Definition: (α - Level Set)

Let I be the real interval. A mapping $y: I \to E$ is called a fuzzy process and its α - *level Set* is denoted by $[y(t)]_\propto = [\underline{y}(t;\alpha), \overline{y}(t;\alpha)\ ],\ t \in I\ , 0 < \propto < 1$

Definition: (Seikkala Derivative)

The Seikkala derivative $y'(t)$ of a fuzzy process is defined by $[y'(t)]_\propto = [\underline{y}'(t;\alpha), \overline{y}'(t;\alpha)\ ]\ \ t \in I\ , 0 < \propto \le 1$ provided that this equation defines a fuzzy number, as in [24].

Lemma:

If the sequence of non-negative number $\{W_n\}_{n=0}^{m}$ satisfy $|Wn+1| \le A|Wn| + B\ , 0 \le n \le N-1$ for the given positive constants *A* and *B*, then $|Wn| \le A^n |W_0| + B\dfrac{A^n - 1}{A-1}\ , 0 \le n \le N$

Lemma:

If the sequence of non-negative numbers $\{W_n\}_{n=0}^{m}, \{V_n\}_{n=0}^{N}$ satisfy $|W_{n+1}| \le |W_n| + A\,max\{|W_n|, |V_n|\} + B,$ $|V_{n+1}| \le |V_n| + A\,max\{|W_n|, |V_n|\} + B$ for the given positive constants *A* and *B*, then $U_n = |W_n| + |V_n|\ ,\ 0 \le n \le N$

we have, $U_n \le \overline{A}^n U_0 + B \frac{\overline{A}^n - 1}{A - 1} \quad 0 \le n \le N$ where $\overline{A} = 1 + 2A$ and $\overline{B} = 2B$.

Lemma

Let $F(t,u,v)$ and $G(t,u,v)$ belong to $C^{'}(R_F)$ and the partial derivatives of *F* and *G* be bounded over R_F .Then for arbitrarily fixed r, $0 \le r \le 1$, $D(y(t_{n+1}), y^0(t_{n+1})) \le h^2 L(1+2C)$ where *L* is a bound of partial derivatives of *F* and *G*, and

$$C = Max\left\{\left|G\left[t_N, \underline{y}(t_N; r), \overline{y}(t_{N-1}; r)\right]\right|, r \in [0,1]\right\} < \infty$$

Theorem

Let $F(t,u,v)$ and $G(t,u,v)$ belong to $C^{'}(R_F)$ and the partial derivatives of *F* and *G* be bounded over R_F .Then for arbitrarily fixed *r*, $0 \le r \le 1$, the numerical solutions of $\underline{y}(t_{n+1}; r)$ and $\overline{y}(t_{n+1}; r)$ converge to the exact solutions $\underline{Y}(t_{n+1}; r)$ and $\overline{Y}(t_{n+1}; r)$ uniformly in *t*.

Theorem

Let $F(t,u,v)$ and $G(t,u,v)$ belong to $C^{'}(R_F)$ and the partial derivatives of *F* and *G* be bounded over R_F and $2Lh < 1$. Then for arbitrarily fixed $0 \le r \le 1$,the iterative numerical solutions of $\underline{y}^{(j)}(t_n; r)$ and $\overline{y}^{(j)}(t_n; r)$ converge to the numerical solutions $\underline{y}(t_n; r)$ and $\overline{y}(t_n; r)$ in $t_0 \le t_n \le t_N$, when $j \to \infty$.

3.FUZZY INITIAL VALUE PROBLEM

Consider a first-order fuzzy initial value problem

$$\begin{cases} y'(t) = f(t, y(t)), t \in [t_0, T] \\ y(t_0) = y_0 \end{cases} \tag{3.1}$$

where *y* is a fuzzy function of t , $f(t, y)$ is a fuzzy function of the crisp variable $'t'$ and the fuzzy variable y, y' is the fuzzy derivative of *y* and $y(t_0) = y_0$ is a trapezoidal or a trapezoidal shaped fuzzy number.

We denote the fuzzy function '*y*' by $y = [\underline{y}, \overline{y}]$ .It means that the r-level set of $y(t)$ for $t \in [t_0, T]$ is $[y(t)]_r = [\underline{y}(t;r), \overline{y}(t;r)]$, $[y(t_0)]_r = [\underline{y}(t_0;\ r), \overline{y}(t_0;\ r)]$, $r \in (0,1]$,

we write $f(t, y) = [\underline{f}(t, y), \overline{f}(t, y)]$ and

$$\underline{f}(t, y) = F[t, \underline{y}, \overline{y}] \quad , \quad \overline{f}(t, y) = G[t, \underline{y}, \overline{y}],$$

because of $y' = f(t, y)$ we have

$$\underline{f}\left(t, y(t); r\right) = F[t, \underline{y}(t;r), \overline{y}(t;r)], \tag{3.2}$$

$$\overline{f}\left(t, y(t); r\right) = G[t, \underline{y}(t;r), \overline{y}(t;r)] \tag{3.3}$$

by using the extension principle, we have the membership function

$$f(t, y(t))(s) = sup\{y(t)(\tau) \setminus s = f(t,\tau)\}, \ s \in R \tag{3.4}$$

so the fuzzy number $f\left(t, y(t)\right)$ follows that

$$\left[f(t, y(t))\right]_r = [\underline{f}(t, y(t); r), \overline{f}(t, y(t); r)], \ r \in (0,1] \tag{3.5}$$

where $\underline{f}\ (t, y(t); r) = \ min\{f(t,u) \,|\, u\left[y(t)\right]_r\}$ (3.6)

$$\overline{f}\left(t, y(t); r\right) = \ max\{f(t,u) \,|\, u\left[y(t)\right]_r\} \tag{3.7}$$

Definition 3.1: A function $f: R \rightarrow R_F$ is said to be fuzzy continuous function, if for an arbitrary fixed $t_0 \in R$ and $\varepsilon > 0, \delta > 0$ such that $|t - to| < \delta \Rightarrow D\left[f(t), f(t_0)\right] < \varepsilon$ exists.

The fuzzy function considered are continuous in metric D and the continuity of $f(t, y(t); r)$ guarantees the existence of the definition of $f(t, y(t); r)$ for $t \in \left[t_0, T\right]$ and $r \in \left[0,1\right]$ [10]. Therefore, the functions *G* and *F* can be definite too.

4.THIRD ORDER RUNGE-KUTTA METHOD WITH MODIFIED CONTRA-HARMONIC MEAN

The third order Runge-kutta method with modified contra-harmonic mean was proposed for approximating the solution of first order fuzzy initial value problem $y'(t) = f(t, y(t)) \ \ y(t_0) = y_0$.

The basis of all Runge-Kutta methods is to express the difference between the value of 'y' at t_{n+1} and t_n as $y_{n+1} - y_n = \sum_{i=0}^{m} w_i k_i$ (4.1)

where w_i's are constant for all i and $k_i = hf(t_n + a_i h, y_n + \sum_{j=1}^{i-1} c_{ij} k_j)$ (4.2)

Increasing of the order of accuracy of the Runge-Kutta methods have been accomplished by increasing the number of taylor's series terms used and thus the number of functional evaluations required[5].The method proposed by Goeken.D and Johnson.O[9] introduces new terms involving higher order derivatives of 'f' in the Runge-Kutta k_i terms (i > 0) to obtain a higher order of accuracy without a corresponding increase in evaluations of 'f ', but with the addition of evaluations of f'.

Runge-kutta third order method with modified contra-harmonic mean was discussed by Osama Yusuf Ababneh, and Rokiah Rozita [17].

Consider $$y(t_{n+1}) = y\left(t_n\right) + \frac{h}{4}\left[\frac{k_1^{\,2} + k_2^{\,2}}{k_1 + k_2} + \frac{3(k_2^{\,2} + k_3^{\,2})}{k_2 + k_3}\right] \tag{4.3}$$

where $k_1 = hf\left(t_n, y(t_n)\right)$ (4.4)

$$k_2 = hf\left(t_n + a_1 h, y(t_n) + a_1 k_1\right) \tag{4.5}$$

$$k_3 = hf\left(t_n + a_2 h, y(t_n) + a_2 k_2\right) \tag{4.6}$$

and the parameters a_1, a_2 are chosen to make y_{n+1} closer to $y(t_{n+1})$ .The value of parameters are $a_1 = \frac{2}{3}$, $a_2 = \frac{4}{21}(3+\sqrt{2})$

5.THIRD ORDER RUNGE-KUTTA METHOD WITH MODIFIED CONTRA-HARMONIC MEAN FOR SOLVING FUZZY DIFFERENTIAL EQUATIONS

Let the exact solution $[Y(t)]_r = [\underline{Y}(t;r), \overline{Y}(t;r)]$,is approximated by some $[y(t)]_r = [\underline{y}(t;r), \overline{y}(t;r)]$ .The grid points at which the solutions is calculated are $h = \frac{T-t_0}{N}$, $t_i = t_0 + ih; 0 \le i \le N$

From 4.3to 4.6 we define

$$\underline{y}(t_{n+1},r) - \underline{y}(t_n,r) = \frac{h}{4}\left[\frac{\underline{k_1}^2(t_n,y(t_n,r)) + \underline{k_2}^2(t_n,y(t_n,r))}{\underline{k_1}(t_n,y(t_n,r)) + \underline{k_2}(t_n,y(t_n,r))} + \frac{3(\underline{k_2}^2(t_n,y(t_n,r)) + \underline{k_3}^2(t_n,y(t_n,r)))}{\underline{k_2}(t_n,y(t_n,r)) + \underline{k_3}(t_n,y(t_n,r))}\right] \tag{5.1}$$

where $k_1 = hF[t_n,\ \underline{y}(t_n,r),\ \overline{y}(t_n,r)]$ (5.2)

$$k_2 = hF[t_n + \frac{2}{3},\ \underline{y}(t_n,r) + \frac{2}{3}\underline{k_1}(t_n,y(t_n,r)),\ \overline{y}(t_n,r) + \frac{2}{3}\overline{k_1}(t_n,y(t_n,r))] \tag{5.3}$$

$$\begin{aligned} k_3 = hF[t_n + \frac{4}{21}(3+\sqrt{2}),\ &\underline{y}(t_n,r) + \frac{4}{21}(3+\sqrt{2})\underline{k_2}(t_n,y(t_n,r)), \\ &\overline{y}(t_n,r) + \frac{4}{21}(3+\sqrt{2})\overline{k_2}(t_n,y(t_n,r))] \end{aligned} \tag{5.4}$$

and

$$\overline{y}(t_{n+1},r) - \overline{y}(t_n,r) = \frac{h}{4}\left[\frac{\overline{k_1}^2(t_n,y(t_n,r)) + \overline{k_2}^2(t_n,y(t_n,r))}{\overline{k_1}(t_n,y(t_n,r)) + \overline{k_2}(t_n,y(t_n,r))} + \frac{3(\overline{k_2}^2(t_n,y(t_n,r)) + \overline{k_3}^2(t_n,y(t_n,r)))}{\overline{k_2}(t_n,y(t_n,r)) + \overline{k_3}(t_n,y(t_n,r))}\right] \tag{5.5}$$

where $k_1 = hG[t_n,\ \underline{y}(t_n,r),\ \overline{y}(t_n,r)]$ (5.6)

$$k_2 = hG[t_n + \frac{2}{3},\ \underline{y}(t_n,r) + \frac{2}{3}\underline{k_1}(t_n,y(t_n,r)),\ \overline{y}(t_n,r) + \frac{2}{3}\overline{k_1}(t_n,y(t_n,r))] \tag{5.7}$$

$$\begin{aligned} k_3 = hG[t_n + \frac{4}{21}(3+\sqrt{2}),\ &\underline{y}(t_n,r) + \frac{4}{21}(3+\sqrt{2})\underline{k_2}(t_n,y(t_n,r)), \\ &\overline{y}(t_n,r) + \frac{4}{21}(3+\sqrt{2})\overline{k_2}(t_n,y(t_n,r))] \end{aligned} \tag{5.8}$$

we define $$F(t_n,y(t_n,r)) = \frac{h}{4}\left[\frac{\underline{k_1}^2(t_n,y(t_n,r)) + \underline{k_2}^2(t_n,y(t_n,r))}{\underline{k_1}(t_n,y(t_n,r)) + \underline{k_2}(t_n,y(t_n,r))} + \frac{3(\underline{k_2}^2(t_n,y(t_n,r)) + \underline{k_3}^2(t_n,y(t_n,r)))}{\underline{k_2}(t_n,y(t_n,r)) + \underline{k_3}(t_n,y(t_n,r))}\right] \tag{5.9}$$

$$G(t_n, y(t_n,r)) = \frac{h}{4}\left[\frac{\overline{k_1}^2(t_n, y(t_n,r)) + \overline{k_2}^2(t_n, y(t_n,r))}{\overline{k_1}(t_n, y(t_n,r)) + \overline{k_2}(t_n, y(t_n,r))} + \frac{3(\overline{k_2}^2(t_n, y(t_n,r)) + \overline{k_3}^2(t_n, y(t_n,r)))}{\overline{k_2}(t_n, y(t_n,r)) + \overline{k_3}(t_n, y(t_n,r))}\right] \quad (5.10)$$

Therefore we have

$$\underline{Y}(t_{n+1}, r) = \underline{Y}(t_n, r) + F[t_n, Y(t_n, r)]$$

$$\overline{Y}(t_{n+1}, r) = \overline{Y}(t_n, r) + G[t_n, Y(t_n, r)] \quad (5.11)$$

and

$$\underline{y}(t_{n+1}, r) = \underline{y}(t_n, r) + F[t_n, y(t_n, r)]] \quad (5.12)$$

$$\overline{y}(t_{n+1}, r) = \overline{y}(t_n, r) + G[t_n, y(t_n, r)]$$

Clearly $\underline{y}(t;r)$ and $\overline{y}(t;r)$ converge to $\underline{Y}(t;r)$ and $\overline{Y}(t;r)$ whenever $h \to 0$

6.NUMERICAL EXAMPLE

Consider fuzzy initial value problem

$$\begin{cases} y'(t) = y(t), & t \geq 0 \\ y(0) = (0.8 + 0.125r, 1.1 - 0.1r) \end{cases} \quad (6.1)$$

The exact solution is given by

$$Y(t,r) = [(0.8 + 0.125r)e^t, (1.1 - 0.1r)e^t]$$

At t=1we get

$$Y(1,r) = [(0.8 + 0.125r)e, (1.1 - 0.1r)e], \; 0 \leq r \leq 1$$

The values of exact and approximate solution with h= 0.1 is given in Table : 1. The exact and approximate solutions obtained by the proposed method is plotted in Fig:1.The estimation of Error 1 and Error 2 is plotted in Fig:2.

Table:1

r	Exact Solution t=1		Approximate Solution (h=0.1)		Error 1	Error 2
	$\underline{Y}(t;r)$	$\overline{Y}(t;r)$	$\underline{y}(t;r)$	$\overline{y}(t;r)$		
0.0	2.174625 ,	2.990110	2.205551 ,	3.032633	3.092588e-002	4.252308e-002
0.1	2.208604 ,	2.962927	2.240013 ,	3.005064	3.140910e-002	4.213651e-002
0.2	2.242583 ,	2.935744	2.274475 ,	2.977494	3.189231e-002	4.174994e-002
0.3	2.276561 ,	2.908562	2.308937 ,	2.949925	3.237553e-002	4.136336e-002
0.4	2.310540 ,	2.881379	2.343398 ,	2.922356	3.285875e-002	4.097679e-002
0.5	2.344518 ,	2.854196	2.377860 ,	2.894786	3.334196e-002	4.059022e-002

0.6	2.378497 , 2.827013	2.412322 , 2.867217	3.382518e-002	4.020364e-002
0.7	2.412475 , 2.799830	2.446784 , 2.839647	3.430840e-002	3.981707e-002
0.8	2.446454 , 2.772647	2.481245 , 2.812078	3.479161e-002	3.943050e-002
0.9	2.480432 , 2.745465	2.515707 , 2.784509	3.527483e-002	3.904392e-002
1.0	2.514411 , 2.718282	2.550169 , 2.756939	3.575805e-002	3.865735e-002

Fig-1 (Approximate & Exact)

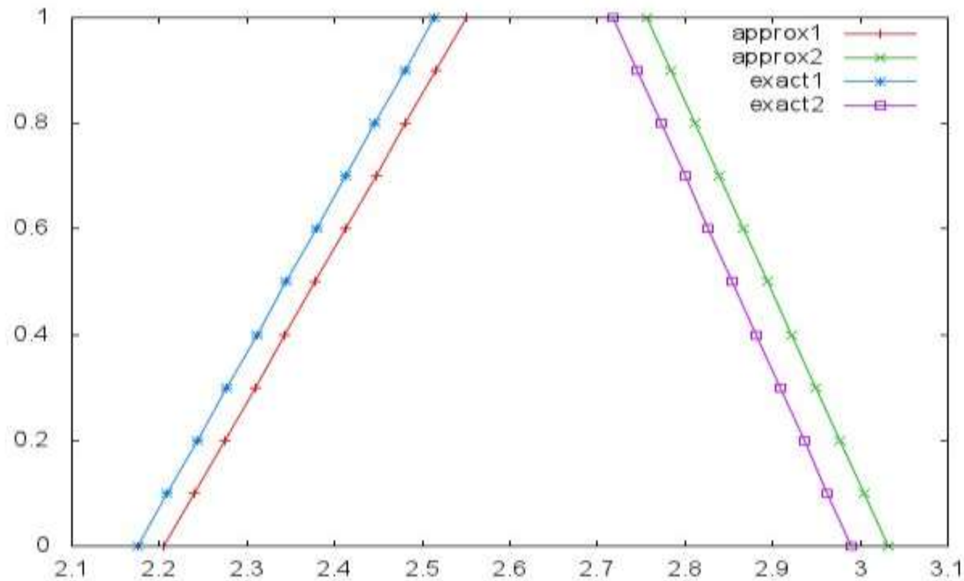
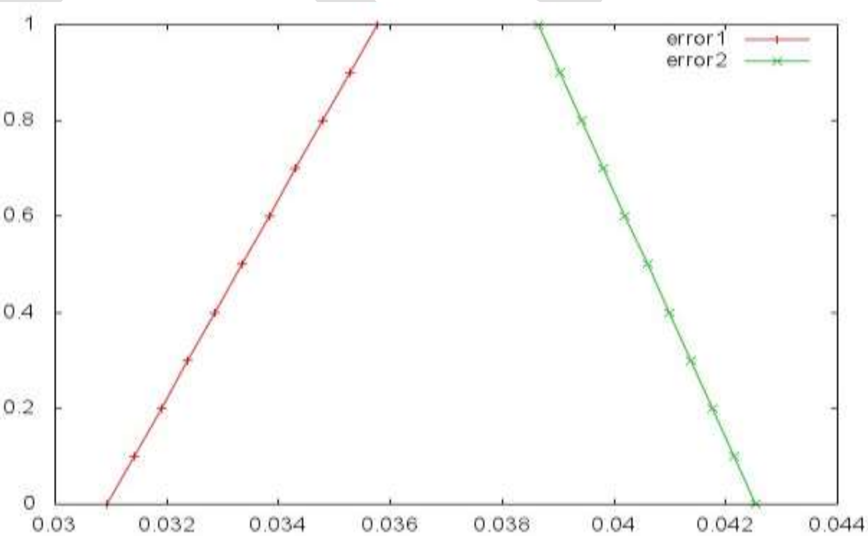


Fig-2 (Error-1 & Error-2)



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CONCLUSION

In this paper the runge-kutta third order method with modified contra-harmonic mean has been applied for finding the numerical solution of first order fuzzy differential equations using trapezoidal fuzzy number.The efficiency and the accuracy of the proposed method have been illustrated by a suitable example. From the numerical example it has been observed that the discreet solutions by the proposed method almost coincide with the exact solutions.

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