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Mathematical Evaluation of Lateral and Torsional Vibrations on Directional Drilling

Odesa David Emudiaga [1], Adewale Dosunmu[2] , Ossia C. Victor[2]

[1] Nigerian Petroleum Development Company Limited

[2]University of Port Harcourt, Nigeria

Abstract- This research work was carried out to study the dynamics of torsional and lateral vibrations and its impact on directional drilling. Harmonic and modal finite element analyses were adopted for the obvious advantage that it is able to approximate the real structure with a finite number of degrees of freedom. The damping effect of drilling mud and friction between drillstring and the wellbore were also considered. The deductions and findings were used to detect, analyze and mitigate against torsional and lateral drillstring vibrations

1.0. Introduction

The oil and gas industry spends about $20billion annually on drilling activities with 15% of that cost lost to Non Productive Time (NPT). Drillstring vibration is a complex and costly problem that often results in failures of Measurement While Drilling (MWD) tools and Bottom Hole Assembly (BHA) components, poor drilling efficiency and high NPT.

Confronted with the challenges of accessing hitherto unaccessible oil and gas reservoirs spurred the technology advancement in directional drilling with well trajectory ranges from simple 'J' or 'S' paths to complex cased junction with pressure isolation and re-entry capabilities (Fig. 1).

As it is with most technological advancement, directional drilling also came with its attendant challenges. With the increase in inclination of the well, when drilling either in sliding or rotary mode, the risk of drillstring vibration increases.

The primary causes of drillstring vibrations are bit/formation and drillstring/borehole interactions. Large vibration levels cause reduced rates of penetration and catastrophic failures while lower levels may lead to a reduced operating life. The benefits of addressing this problem are obvious and include reduced drilling time and costs, reduced maintenance, and lower equipment turnover.

The drill-string vibrations are induced by the characteristics of the bit-rock interaction and by the impacts that might occur between the column and the borehole. If not controlled, vibrations are harmful to the drilling process causing:

1. Premature wear and consequent damage of the drilling equipment and BHA components - bit, motor / RSS, MWD etc. resulting many times in failures, especially due to fatigue (Fig. 2.)
2. Decrease of the rate of penetration (ROP), increasing the well cost
3. Interferences on the measurements performed during the drilling process and damage of the measurement equipment and even failure to acquire evaluation data
4. Significant waste of energy due to increased Tripping Times and inability to run & set casing, torque and drag

5. BHA instability, reducing the directional control.
6. Wellbore instability occasioned by the fracturing effect on the wellbore due to BHA whirl. (Fig. 3).

Factors that affect torsional and lateral drillstring vibrations

1. Material of the drillstring (shear modulus). For the steel with increasing torque, shear stress increases linearly with shear strain until plastic region is reached.
2. Drilling fluid
3. Well geometry (Hole angle)
4. Difference in friction between the static and dynamic friction of the drill bit and bit face
5. The nature of vibration in drillstrings depends on the type of bit, among other factors. PDC bits work by shearing the rock rather than crush the rock. This results in a bit-rock interaction mechanism characterized by cutting forces and frictional forces. The torque on bit and the weight on bit have both the cutting component and the frictional component when resolved in horizontal and vertical direction.

2.0. **TORSIONAL VIBRATION (STICK-SLIP):** This is the alternate slowing down and speeding up of the bit and BHA. This happens when by any reason the bit (or stabilizer) is slowed down or can eventually stall, but the surface Top Drive or Kelly will continue to rotate at constant speed. When the stalled bit can no longer withstand the increasing torque, the bit or stabilizer breaks free with a higher rotational speed. A torsional wave back down the drillstring to the bit, which again stalls. This cycle repeats unless drilling parameters are adjusted to interrupt it. The period of these oscillations is a function of the length of the drillpipe, the mechanical properties of the drilling system, the drilling parameters (WOB - RPM), the nature and location of the down hole friction.

Stick-slip is caused largely by interaction with the formation and frictional forces between the drillstring, BHA and the wellbore, and this is more pronounced in highly deviated and deep wells. If compressive force F is applied directly along the longitudinal axis of the drillstring, buckling will occur.

TORSIONAL VIBRATION

For torsional vibration, monitor

- Torque on bit
- Bit speed

Surface Measurements or Symptoms

- Topdrive /Rotary table stalling
- Large and erratic surface RPM & torque fluctuations, especially noticeable on a top drive
- Whirling sound form the top drive
- Increased delta surface torque
- Torque or RPM cycling
- Loss of tool face
- Reduced ROP

Downhole measurement

a) Increased delta downhole torque
b) Increased torsional (rotational) acceleration
c) Increased stick/slip indicator
d) Increased bit-stuck percentage (percentage of time < 5rpm)
e) Loss of real-time data or measurement

Postrun Evidence

a) Cutters or inserts damaged typically on nose and taper
b) Over torqued connections
c) BHA failure
d) Damage to the bits with heat checking and dull characteristics
e) Fractured or cracked motor drive line components such as bearing mandrels, driveshafts or drive shaft adapters

Causes of torsional vibration

a) Poor bit selection – overly aggressive PDC bit
b) Improper BHA stabilization – undersize stabilizers
c) Excessive bend in mud motors
d) Tortous hole geometries and high dogleg angles
e) High weight on bit with low rate of penetration
f) Inadequate drill collars mass
g) Inadequate drillpipe and BHA stiffness
h) Poor drilling practices
i) Inadequate lubrication in drilling mud
j) Lithology – interbedded formations, formation interfaces hard and/or abrasive stringers, formations with high friction coefficients
k) Reaming/back-reaming, hole opening, drilling out casing, control drilling.

3.0. **LATERAL VIBRATION (WHIRL AND BENDING):** This is the rotation of the drillstring or any part of the BHA around the hole centre line. This type of vibration occurs when by any means, a bit blade or stabilizer digs into the formation after losing its instantaneous center of rotation. As a result, you may say the bit walks around the hole and produces unusual bottomhole patterns. Hole enlargement may result and leading to premature wear of the BHA components. Also, bit whirl causes significant damage to PDC bits due to high and irregular impact loads and off center wear in roller cone bits. Bit whirl can change the tilt or orientation of the bit and replacing the bit requires the removal of what might be a mile-long drillstring, a costly process.

The main factor influencing whirl is the bit geometry. Longer tapers, more aggressive structures or aggressive gauge cutters all significantly increase a bits tendency to whirl. However, higher back rake, flatter (bit) profiles and smooth gauge will tend to reduce bit whirl.

Surface Measurement or Symptom

a) Increased mean surface torque (indicator of backward whirl)
b) Loss of toolface
c) Reduced ROP

Downhole measurement

a) Increased mean downhole torque
b) Large amplitude, high-rate downhole shocks (indicator of chaotic whirl)
c) Increased lateral shocks
d) Loss of real-time data or measurement
e) Increased shock count

Postrun Evidence

a) Cutters or inserts damaged typically on shoulder or gauge
b) Broken PDC cutters
c) Worn hybrid (equivalents) with minimal cutter wear
d) Overguage hole from calipers
e) BHA failure
f) BHA connection failure due accelerated fatigue (backward, chaotic whirl)
g) Washouts
h) One-sided wear on stabilizers and BHA (forward whirl)

Causes

a) Harmonic resonance of drill string
b) Excessive RPM: stick slip
c) High tortuosity of the well
d) Formations with high coefficients of friction and restitution
e) Inadequate BHA stabilization
f) Low weight on bit and high RPM
g) Poor bit selection

3.1. **TORSIONAL VIBRATIONS: ANDRADE, FOSENCA AND WEBER (2013) MATHEMATICAL FRICTION INDUCED STICK-SLIP MODEL**

The model consist of two degrees of freedom where the surface torque is imposed at the top end

Assumptions:

- The dynamic characteristics of the top drive motor are not considered
- The axial and lateral dynamics of the drilling system are neglected.

- The Bottom Hole Assembly (K_{BHA}) is assumed to be a rigid body since its stiffness is much greater than the stiffness of the drill pipe (K_{DP})
- The two degrees of freedom modeling

The equivalent mass moment of inertia

$J_1 = \rho_{BHA} I_{BHA} L_{BHA}$

The area moments of inertia was given as

$$I_{BHA} = \frac{\pi}{32}\left(OD_{BHA}^4 - ID_{BHA}^4\right) \qquad 1$$

$$I_{DP} = \frac{\pi}{32}\left(OD_{DP}^4 - ID_{DP}^4\right) \qquad 2$$

The stiffness is given as

$K = \frac{GI_{DP}}{L_{DP}}$

Where

The shear modulus,

$G = \frac{E}{2(1+\nu)}$

The mud damping is written in terms of a damping factor of the mud Dr,

$C_1 = D_r L_{DP}$

The structural damping is given by

$Cs = 2\xi \sqrt{KJ_1}$

The underlined matrices, A contains the properties of the system

$$q^1 = Aq + T \qquad 3$$

They based their work on Pavonne and Desplans (1994) friction model developed from field data using the Televigile measurement device

In their experiment, they observed that the vibration range of surface RPM increases when the WOB value is increased, the amplitude of vibration increasing with WOB also.

The vibration amplitudes when the WOB increases while the surface RPM stays constant were evaluated and also the vibration amplitudes increase when the WOB is increased and the effect of WOB were demonstrated in the bifurcation.

3.2. LATERAL VIBRATION: CRISTIANO AND RODRIGO (2014) MATHEMATICAL MODEL ON LATERAL VIBRATION

The work was based on Jansen, J.D. (1993) model which takes into account the pumping fluid, contact of the stabilizers with the borehole walls, contact of the drillstring with the borehole walls, and excitation due to unbalance. The model was complemented with additional degrees-of-freedom of torsional and longitudinal movement.

Assumptions

- Pumping Fluid: drag in the annular gap between the drillstring and the borehole is proportional to the square of the rotating speed.
- Stabilizers: the hydrodynamic effects in the gap between the stabilizers and the borehole walls are neglected
- Borehole: the cross section of the borehole is circular, and contact between the drillstring and the borehole obey the Coulomb law
- Drillstring Vibration: the adopted rotating speeds are close to the first natural frequency of the drillstring associated to the first bending mode. Hence, lateral vibration of the drillstring will be limited to that of the bending mode of a simply supported beam, which means that both stabilizers will be in contact at the same time;
- Longitudinal Movement: Coulomb friction is considered in the longitudinal direction of movement.
- The weight of the drillstring, and its effects, will not be considered in the model.

Vibration is analyzed in the mid-section of the beam (section A-A Figs. 4). The model has two degrees of freedom to represent the lateral movements of the BHA in directions x_1 and x_2, one degree-of-freedom to represent the torsion of the drillstring, and one degree-of-freedom to represent the longitudinal motion in x. The unbalance of the BHA is represented by an eccentricity, given by the difference between the centre of mass of the BHA and it's the geometric Centre.

The inertia forces due to acceleration of the BHA and of the fluid can be described by

$$F_{mx_1} = -(m_b + m_f)x_1 + m\Omega^2 e_o \cos(\emptyset - \Omega t) \qquad 4$$

$$F_{mx_2} = -(m_b + m_f)x_2 + m\Omega^2 e_o \sin(\emptyset - \Omega t) \qquad 5$$

4.1. HARMONIC FINITE ELEMENT ANALYSIS OF DRILLSTRING LATERAL VIBRATION

Lateral vibration will cause fatigue and failure of drillstring, broaden hole and change bending angle of bit. The major difficulty however encountered in controlling lateral vibration amplitude and impact intensity is the fact that lateral vibration and its consequent drillstring impact cannot be observed at the surface without the expensive MWD apparatus.

Bit whirl is predominant with PDC bits because tricone bits penetrate the bottom of the borehole more and do not allow sideways movement of the bit. Whirl generation is caused by two factors: a centrifugal force generated as a result of the high rotary speed. The added force creates more friction which further reinforces whirl. The second is the center of rotation which is no longer the centre of the bit. This fact however contradicts the basic bit design assumption that the geometric center of the bit is the center of rotation. The impact loads associated with this motion cause PDC cutters to chip, which in turn, accelerates wear.

In this analytical model, we would be able to control lateral vibration by controlling some factors such as under-gauge stabilizer, initial phase angle, initial deformation, WOB and ROP.

The Euler equation for beams will be used to analyze the drillstring vibration. The length of the drillstring AB of length l whose upper end is fixed B and lower end (bit) free to move (Fig. 5).

where P = crippling load that causes the drillstring just to deflect, expressed in N.

The rotation of the cross section as measured by $\theta \approx \frac{dy}{dx}$ is less than 1.0, one radian

At any section XX distant X from the fixed point B, for the curved beam, bending moment is expressed as given as

$$\frac{M}{EI_a} = \frac{d2y}{dx2} \qquad \text{(Leonhard Euler equation)}$$

Recall, M = Force x distance

$$EI_a \frac{d2y}{dx2} = P(\delta - y)$$

$$EI_a \frac{d2y}{dx2} + Py = P\delta$$

$$\therefore \frac{d2y}{dx2} + \frac{P}{EI_a} y = \frac{P\delta}{EI_a}$$

Solving the differential equation,

$$y = C_1 \cos(x \sqrt{\frac{P}{EI_a}}) + C_2 Sin\left(x \sqrt{\frac{P}{EI_a}}\right) + \delta \qquad 6$$

Where

E = Modulus of elasticity for steel expressed in pa (from 200,000 to 220,000MPa)

I_a = Moment of inertia of the straight section of the drillstring expressed in m^4

$l =$ Free buckling length which depends on the actual length of the pipe and the way the end s are fixed, expressed in m

C_1C_2 = Arbitrary constants of integration

At the fixed point, B, the deflection is zero

Boundary conditions:

$y = 0 \,@\, x = 0$

$y = \delta \,@\, x = l$

So

$$0 = C_1 + \delta \; or \;\; C_1 = -\delta$$

The slope at any section is given by

$$\frac{dy}{dx} = -C_1\sqrt{\frac{P}{EI_a}}\sin(x\sqrt{\frac{P}{EI_a}}) + -C_2\sqrt{\frac{P}{EI_a}}\cos(x\sqrt{\frac{P}{EI_a}})$$

At the fixed point, B, slope is zero

$$\therefore x = 0, \frac{dy}{dx} = 0$$

So

$$0 = C_2\sqrt{\frac{P}{EI_a}}$$

$$\therefore C_2 = 0$$

At A, the deflection is δ

$$\therefore x = l, y = \delta$$

$$y = C_1\cos(x\sqrt{\frac{P}{EI_a}}) + C_2 Sin\left(x\sqrt{\frac{P}{EI_a}}\right) + \delta$$

$$\therefore \delta = -\delta\cos(l\sqrt{\frac{P}{EI_a}}) + \delta$$

$$\cos(l\sqrt{\frac{P}{EI_a}}) = O$$

$$l\sqrt{\frac{P}{EI_a}} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \dots$$

Where K = 1, 3, 5, …

Considering the first critical value,

$$l\sqrt{\frac{P}{EI_a}} = \frac{\pi}{2}$$

$p = \frac{\pi^2 EI_a}{4l^2}$ (Euler formula)

$$Ia = \frac{\pi}{4}(R_e^4 - R_i^4)$$

Where

R_e: outside radius of the pipe expressed in m

R_i: inside radius of the pipe expressed in m

$$Ia = \frac{\pi}{64}(D_e^4 - D_i^4)$$

D_e: outside diameter of the pipe expressed in m

D_i: inside diameter of the pipe expressed in m

P, the crippling load is the maximum limiting load at which the column tends to have lateral displacement. Buckling occurs about the axis having least moment of inertia.

From above, the factors affecting the vibration/deflection are critical load, length of drillstring, young modulus and moment of inertia.

4.2. HARMONIC ANALYSIS OF NATURAL FREQUENCY OF DRILLSTRING LATERAL VIBRATIONS

To determine the natural frequency of free lateral vibrations, consider a drillstring whose end is fixed and the other end carries a body of weight, W as seen in the figure 6 below

Where

k = stiffness of the drillstring (N/m)

m = mass (Kg)

W = weight of drillstring = mg

δ = static deflection due to weight of the body

x= displacement of the body from the equilibrium position

w_n = circular natural frequency (rad/s)

f_n = natural frequency, Hz

f= frequency of mass body

A = Cross-sectional area

In equilibrium position, the gravitational pull, W = mg and is balanced by force of spring, such that W = k. δ

Since the mass is displaced from its equilibrium position by a distance of x, as shown in Fig 6. Above and is then released, therefore after time t,

Restoring force = $W - k(\delta + x)$

$$= W - k\delta - kx$$

Recall $W = K\delta$

$$\therefore K\delta - k\delta - kx = -kx \;\; (taking\ upward\ force\ as\ negative)\ 7$$

Accelerating force = mass x acceleration $m\frac{d2y}{dt2}(taking\ downward\ force\ as\ positive)$8Equating equation 7 and 8, the equation of motion becomes

$$m\frac{d2y}{dt2} = -kx$$

$$m\frac{d2y}{dt2} + kx = 0$$

$$\frac{d2y}{dt2} + \frac{k}{m}x = 0 \qquad 9$$

Recall that the fundamental equation of simple harmonic equation is

$$\frac{d2y}{dt2} + \text{ɯ}x = 0 \qquad 10$$

Comparing equation 9 and 10, we have,

$$\text{ɯ}^2 = \frac{k}{m}$$

$$\therefore \text{ɯ} = \sqrt{\frac{k}{m}}$$

The periodic time of the vibration is

$$t_p = \frac{2\pi}{\text{ɯ}_n}$$

And the natural frequency = $\frac{1}{t_p} = \frac{\text{ɯ}_n}{2\pi}$

$= \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$

Since the static deflection due to gravity,

$$\delta = \frac{mg}{k}$$

Taking the value of g as 9.81m/s^2 and δ in metres

Therefore, the Natural Frequency, f_n

$$f_n = \frac{1}{2\pi}\sqrt{\frac{9.81}{\delta}} = \frac{0.4985}{\sqrt{\delta}}Hz$$

The value of static deflection can be obtained from the relation

$$E = \frac{Stress}{strain} = \frac{W\ x\ L}{A\ X\ \delta}$$

Implication

The external forces causing vibration to the drillstring should not operate at this natural frequency of the drillstring to avoid resonance.

5.0. EFFECT OF THE MUD ON LATERAL VIBRATION

Chen et al (1974) explained that when a structural component vibrates in a viscious fluid, the presence of the fluid gives rise to a fluid reaction force which can be interpreted as an added mass and a damping contribution to the dynamic response of the component.

Added mass and damping are known to be dependent on fluid properties (density and viscosity) as well as the hole geometry and adjacent boundaries.

In general, added mass will decrease the component natural frequencies and can also have a significant effect on the response and the potential for large amplitude motion caused by a resonance. ϴ (Fig. 7)

Let r = radius of drillstring oscillating along a diameter in a viscious fluid annulus

The wellbore is stationary

For small aplitude, the equations of state and motion can be linearized.

Damping factor

$$\vartheta n = \frac{Viscious\ damping\ coefficient}{critical\ damping\ coefficient}$$

$$= \frac{C_v}{C_{CR}}$$

But

$$C_{CR} = 2(MC_m + m)w_n$$

When oscillating at the drillstring frequency

$$w_n = w$$

Damping can be measured and calculated by different method including log decrement from auto correlation of response to white noise input of from a “pluck test”, magnification factor (Q) at resonance, bandwith of frequency response function, and measurement of input power at resonance. The advantages of the bandwith method over the log decrement method and pluck test is that higher modes are not involved and response levels can readily be controlled, if amplitude dependency is suspected. The advantage over the magnification factor method is that only the shape of the curve is involved, calibration factors and the fact that energy may go into different modes is not of concern.

Damping is readily obtained from the transfer function or frequency response curve as

$$\vartheta = \frac{1}{2\sqrt{(N^2 - 1)}}\frac{\Delta f_N}{f_N} \qquad 11$$

Where

$$\Delta f_N = f_N{}^1 - f_N{}^2$$

f_N *is the natural (resonant) frequency*

$f_N{}^1$ & $f_N{}^2$ *are the frequencies at which at*

which the response is a factor

The greater the value of Δf_N, the greater the accuracy of the measurement.

6.0. **STICK-SLIP**

Stick-slip problem starts to become significant with drill strings exceeding a length of about 2000 m (Morten, 2010). In stick-slip situations, from a standstill position, the top drive rotates many times before enough torque is applied to the bit for it to overcome the static friction between the bit and rock. When the bit slips and starts to rotate, it will get a high acceleration. Often the angular velocity of the bit can reach up to three times the velocity of the top drive before the bit speed reduces again and reaches the stick-situation. Then the top drive will 'spin up' the string again until the bit slips, and so it goes on.

To achieve realistic RPM_{bit} and $RPM_{surface}$, the WOB must be reduced to reduce slick-slip effect. The chances of stick-slip are higher in deviated hole sections.

- Stick-slip likely when operating in region of "falling friction" that is as the torque decreases, stick-slip is more likely to occur
- Increase RPM to reduce likelihood of stick-slip

Stick-slip likely

Stick-slip unlikely

For the stick-slip model, the entire model comprises 3 degrees of freedom: the tw tw

To measure stick-slip downhole, stick-slip index (SSI) is used (SSI is usually encoded in the bits and transmitted to the surface) (Table 1):

$$SSI = \frac{RPM_{\max - RPM_{min}}}{2\, XRPM_{avg}}\, x\, 100 \qquad 12$$

Downhole torsional oscillations and stickslip behavior are visible as periodic torque fluctuations on surface.

CONCLUSION

The results show that

1. Using the harmonic finite element analysis, critical limits for different components of the drillstring and BHA should be determined, beyond which the drillstring will be subjected to lateral and torsional vibrations.
2. Using the Euler equation, the crippling load that causes each drillstring to begin to buckle and the effect of the mud weight and length of drillstring were analyzed
3. It was confirmed that Stick-slip problem starts to become significant with drill strings exceeding a length of about 2000 m (Morten, 2010). As a matter of fact, from this work, Lateral and torsional vibration becomes visible from 1500m.
4. The diameter of the components of the drillstring is inversely proportional to the critical speed of the drillstring.
5. The greater the moment of inertia of the drillstring and BHA when considered in finite terms, the higher the tolerance and therefore the higher the allowable critical speed of the drillstring

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APPENDIX

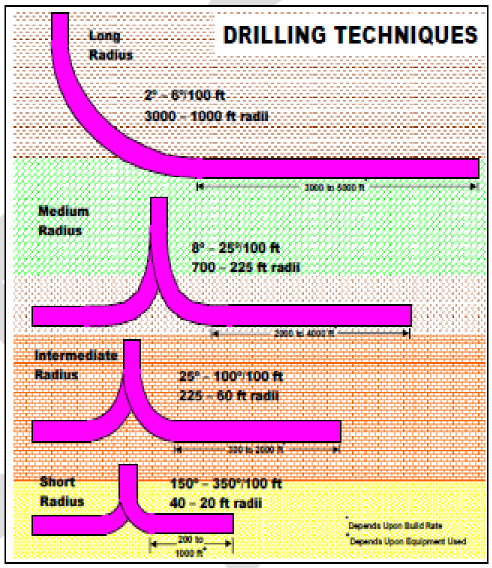


Fig.1. Build curve for short, medium and long range horizontal wells. *Courtesy: Petrolskills, 2013.*

Fig. 2. Drill collars failure in Okono-9 offshore Niger Delta due to PDC Bit Vibration. *Courtesy: NPDC, 2013.*

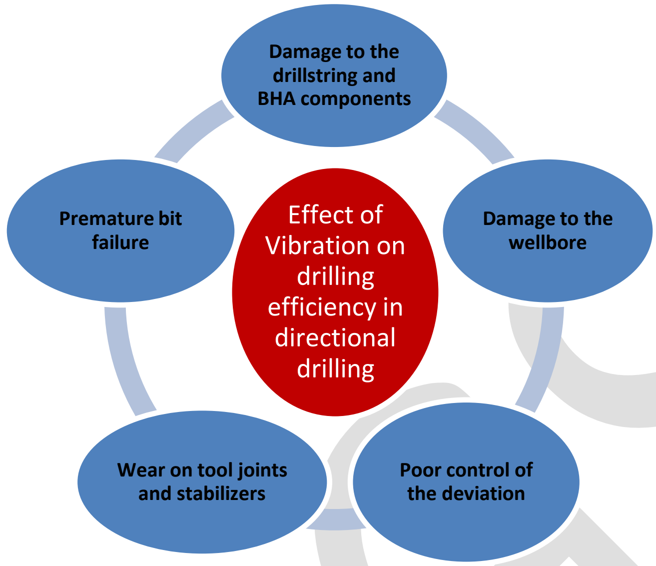


Fig. 3. Effect of vibration on drilling efficiency in directional drilling

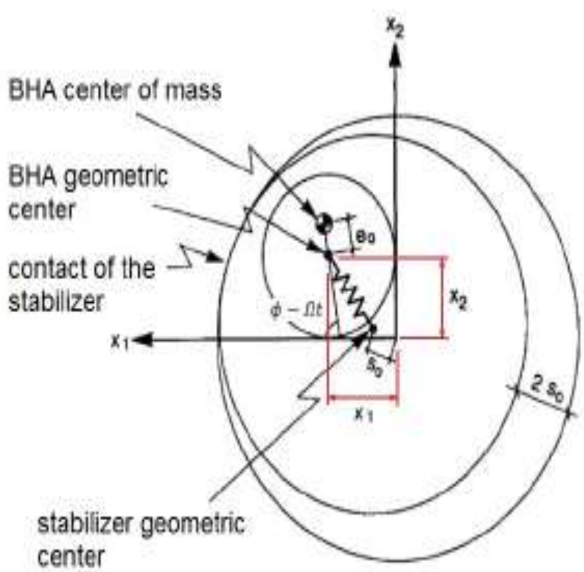


Figure 4. Section A-A with bending of the BHA and contact of the stabilizers with the borehole wall

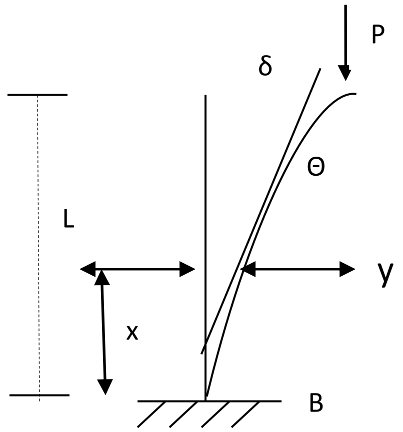
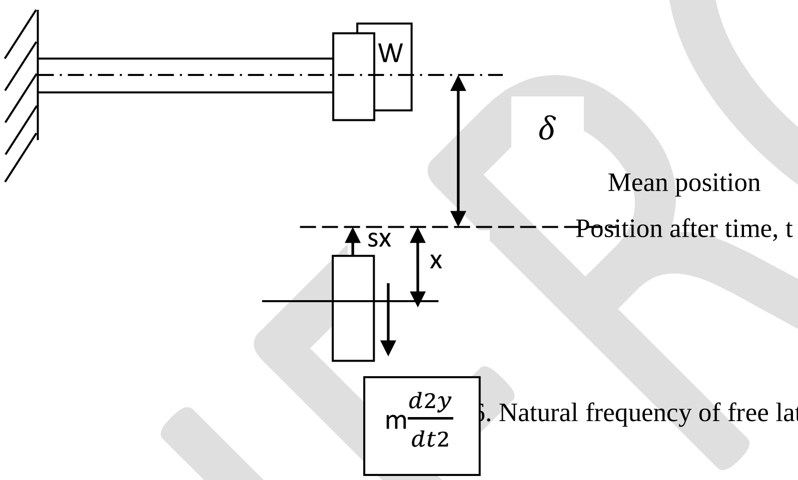


Fig 5. Drillstring deflection



. Natural frequency of free lateral vibration

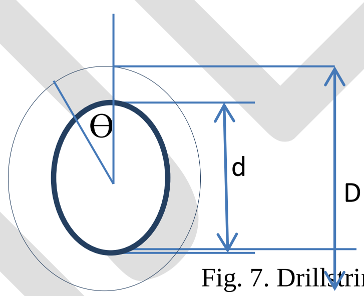


Fig. 7. Drillstring vibration in a wellbore

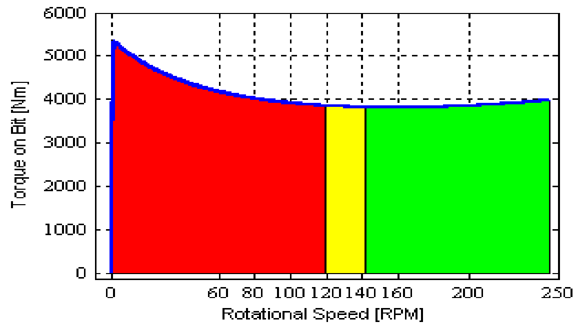


Fig 8. Stick-slip analysis. Source: Reckmann, J. H., 2007.

Table 1. Stick-Slip Index

S/N	SSI	Mode	Risk	Time limit
1.	0-40	None	Low	None
2.	40-60	Torsional oscillations	Medium	None
3	60-80		Medium	Recommended to mitigate
4.	80-100	Stick-slip	High	Onset of full-stall stick slip, mandatory to mitigate
5.	>100	Stick-slip	Severe	30 minutes (formal notification)

SYMBOLS

T_1 = torque on bit

T_2 = surface torque

J_1J_2 = equivalent moment of inertia at bottom end and top end respectively

C_1C_2 = mud damping

C_S = structural damping

Ω_1 = bit speed

Ω_2 = top drive speed (surface RPM)

φ_1 φ_2=rotational displacement (angle) of the bit starting and top drive respectively with zero at time t = 0

k = equivalent stiffness of the drillpipe

The equivalent mass moment of inertia

$J_1 = \rho_{BHA} I_{BHA} L_{BHA}$

ξ = the damping factor

T' =vector with efforts

q = state-space coordinate

m = Mass

k = stiffness

x = displacement of m from the equilibrium position

E = Modulus of elasticity for steel expressed in pa (from 200,000 to 220,000MPa)

I_a = Moment of inertia of the straight section of the drillstring expressed in m^4

l = Free buckling length which depends on the actual length of the pipe and the way the end s are fixed, expressed in m

C_1C_2 = constants of integration

L = length of drillstring in inches