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Robust Stability for a Class of Uncertain Singular Time-Delays Systems

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Abstract: This note provides new stability criteria for a class of uncertain singular systems with multiple-state delays. By introducing the state transformation, the study of the robust stability for original systems is changed into this study for the equivalent systems. Based on the Lyapunov-Krasovskii functional combination with LMI techniques, a delay-dependent robust stability criterion for the nominal systems of a class of uncertain singular systems is established, which ensures the nominal systems are asymptotically stable. Furthermore, the delay-dependent robust stability criterion for a class of uncertain singular systems with multiple-state delays is presented, which ensures the class of uncertain singular systems is asymptotically stable. Finally, two numerical examples are given to illustrate the effectiveness of the obtained results.

Keywords: robust stability, time-delay, linear matrix inequality (LMIs), Lyapunov-Krasovskii functional, delay-dependent

1. Introduction

Time delays are frequently encountered in many dynamical systems, such as manufacturing systems, economic systems, biological systems, networked systems, etc. They are generally regarded as a main source of instability and poor performance in such systems. One of the main purposes to analysize linear differential systems is to analyze the stability of the systems, especially the certain and uncertain linear differential control systems with time delay, which is important not only in theory but also in practice and arouses a lot of interests. Some results are obtained [1-5]. A delay-dependent criterion for determining the stability of systems with time-varying delays is obtained by combing a new approach for linear time delay systems based on a descriptor representation with a result on bounding of cross products of vectors in [1]. In [4], the system with single state delay and uncertainty is considered, and non-conservative results are obtained by using new Lyapunov-krasovskii functionals. The stability of regular systems with multiple-state delay and uncertainty is considered in [6], and based on Lyapunov-Krasovskii functionals combined with LMI techniques. Thus delay-dependent robust stability criteria are given.

It should be pointed out that the problem for singular systems is more complicated than that for regular systems, and singular systems better describe physical systems than regular ones, so the singular systems have been extensively studied in the past years [7-10].

Study of singular systems with time delay is of recurring interest [8, 10]. The robust stability and robust stabilization of uncertain singular systems with single state delay are considered in [8]. The delay-dependent robust stability criteria for two classes of singular time-delay systems with norm-bounded uncertainties are proposed in [11], which ensure that the systems are regular, impulse free and asymptotically stable for all admissible uncertainties. A great number of results based on the theory of regular systems have been extended to the area of singular systems [9, 10]. Generally speaking, the existing results of stability and stabilization for singular delay systems can be classified into two types: delay-independent conditions and delay-dependent conditions, and the delay- independent case is more conservative than the delay-dependent case, especially when the time delay is comparatively small. The delay-independent case has been extensively studied [13, 14]. Recently, the problem of delay-dependent robust stability for uncertain discrete singular time-delay systems has been considered. In [15], the delay-dependent robust stabilization result is proposed by transforming the system into a standard state-space system.

This paper considers the robust stability of a class of uncertain singular systems with multiple-state delays. We obtain the transforming uncertain singular systems which are called the equivalent system with multiple-state delays via state transformation matrix, so the research of the original system is changed into the research of equivalent system. First, this brief constructs Lyapunov-Krasovskii functionals, which are applied to the equivalent nominal systems and equivalent systems respectively. Then we present the delay-dependent stability criteria, which ensure that the equivalent systems are asymptotical stable. It ensures that the original systems are asymptotically stable at the same time.

Notations: R denotes the set of real numbers, R^n denotes the n-dimensional Euclidean space over the real and $R^{n\times m}$ denotes the set of all $n\times m$ real matrices. For a real symmetric matrix X, the notation $X\geq 0 (X>0)$ means that the matrix X is positive-semidefinite (positive-definite), and $\lambda_{min}(X)(\lambda_{max}(X))$ denotes the minimum (maximum) eigenvalues of X. $C_{n,\tau} := C([-\tau,0],R^n)$ denotes the Banach space of continuous vector functions mapping the interval $[-\tau,0]$ into R^n, $x_t := x(t+\theta)$, $\theta\in[-\tau,0], t\geq 0$ denotes the function family defined on $[-\tau,0]$ which is generated by n-dimensional real vector valued continuous function $x(t).t\in[-\tau,\infty]$. $\|.\|$ refers to the Euclidean vector norm or spectral matrix norm,

$$\|\phi\| := sup_{-\tau\leq t\leq 0}\|\phi(t)\|$$

stands for the norm of a function $\phi(t)\in C_{n,\tau}$. The symbol * will be used in some matrix expressions to induce a symmetric structure, for example,

$$\begin{bmatrix} X & Y \\ * & Z \end{bmatrix} = \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}$$

2. System Description

Consider the following uncertain time-delay singular systems described by

$$\begin{cases} E\dot{X}(t) = (A_0+\triangle A_0(x,t))x(t)+\sum_k^{i=1}(A_i+\triangle A_i(x,t))x(t-h_i) \\ x(t)=\phi(t)\forall t\in\text{[-h,0]} \end{cases} \quad (1)$$

where $x(t)\in R^n$ is the state vector, $A_j, j=0,1,...,k$, are known constant matrices with appropriate dimension, $\triangle A_j(x,t), j=0,1,...k$ are matrix functions representing the uncertainties in the matrices $A_j, j=0,1,...k$. E is a singular matrix. Without the lose of generality, we can assume that

$$\begin{aligned} &E=\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \\ &A_i=\begin{bmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{bmatrix}, A_{i11}\in R^{r\times r}, i=0,1,...k, \\ &x(t)=\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \phi(t)=\begin{bmatrix} \phi_1(t) \\ \phi_2(t) \end{bmatrix}, \\ &x_1(t),\phi_1(t)\in R^{r\times r}, \\ &\triangle A_j(x,t)=D_jF_j(x,t)E_j, j=0,1,...,k \end{aligned} \quad (2)$$

where $F_j(x,t)\in R^{k_j\times g_j}$ are unknown real time-varying matrices bounded by

$$F_j^T(x,t)F_j(x,t)\leq I, \forall t, j=0,1,...k \quad (3)$$

D_j and E_j are known real constant matrices, $h_i, i=0,1,...,k$, are the unknown constant delay terms, but bounded $0\leq h_i\leq h$. $\phi(t)$ is a smooth vector-valued initial function in $-h\leq t\leq 0$.

In the following, for simplicity, we denote $\triangle A_j(x,t)=\triangle A_j(t)$.

The aim of this paper is to develop delay-dependent conditions for robust stability of the uncertain time-delay system (1).

The following lemma is needed in the proof of this paper.

Lemma 1[12] Let D,E and F be real matrices of appropriate dimensions with $\|F\|\leq 1$, then for any scalar $\varepsilon>0$, we have the following inequality:

$$DFE+E^TF^TD^T\leq\varepsilon^{-1}DD^T+\varepsilon E^TE \quad (4)$$

Lemma 2[16] (Schur complement) Given constant symmetric matrices S_1,S_2,S_3, where $S_1=S_1^T, 0<S_2=S_2^T$, then $S_1+S_3S_2^{-1}S_3<0$ if and only if

$$\begin{bmatrix} S_1 & S_3^T \\ S_3 & -S_2 \end{bmatrix}<0, or \begin{bmatrix} -S_2 & S_3 \\ S_3^T & S_1 \end{bmatrix}<0,$$

3. Main Results

Let

$$z(t) = e^{\alpha t}x(t), t > 0 \tag{5}$$

where $\alpha > 0$ is stability degree. Differentiating $z(t)$ with respect to t, we have

$$\dot{z}(t) = \alpha e^{\alpha t}x(t) + e^{\alpha t}\dot{x}(t) = \alpha z(t) + e^{\alpha t}\dot{x}(t) \tag{6}$$

Then, from (1), (5) and (6), we have

$$\begin{aligned} E\dot{z}(t) &= \alpha Ez(t) + e^{\alpha t}E\dot{x}(t) \\ &= (\alpha E + A_0 + \triangle A_0)z(t) \\ &\quad + \sum_{i=1}^{k}(A_i + \triangle A_i)e^{\alpha h_i}z(t-h_i) \\ Z(t) &= e^{\alpha t}\phi(t), \forall t \in [-h, 0] \end{aligned} \tag{7}$$

So the research for the system (1) is changed into the research for the system (7).

The nominal singular delay system of (1) can be written as

$$\begin{cases} E\dot{X}(t) = A_0x(t) + \sum_{k}^{i=1}A_ix(t-h_i) \\ x(t) = \phi(t)\forall t \in [-h, 0] \end{cases} \tag{8}$$

Throughout this paper we shall use the following concept of robust stability for uncertain systems (7).

Definition 1[7] The singular system (7) is said to be robust stable if for any $\varepsilon > 0$, there exist a $\delta(\varepsilon) > 0$ such that for all continuous $e^{\alpha t}\phi(t)$, with $e^{\alpha t}\phi(t)$ satisfying the solution to (7) satisfying $\sup_{-\tau \le r \le 0} \|e^{\alpha t}\phi(t)\| \le \delta(\varepsilon)$, the solution $z(t)$ to (7) satisfying $\|z(t)\| \le \varepsilon$ for all $t \ge 0$ The solution of (7) is said to be robust asymptotically stable if it is stable and furthermore $z(t) \to 0$,when $t \to \infty$.

Remark 1 Because $x(t) = e^{-\alpha t}z(t)$, where $\alpha > 0$, the asymptotical stability of the system (7) ensures the asymptotically stability of the system (1), and we transform the research for the system (1) into the research for the system (7).

First of all, we will investigate the nominal system stability of system (7).

The nominal singular delay system of (7) can be written as

$$\begin{aligned} E\dot{X}(t) &= \alpha Ez(r) + e^{\alpha t}(A_0x(t) + \sum_{i=1}^{k}A_ox(t-h_i)) \\ &= (A_0 + \alpha E)z(t) + \sum_{i=1}^{k}A_ie^{\alpha h_i}z(t-h_i) \\ z(t) &= e^{\alpha t}\phi(t), \forall t \in [-h, 0] \end{aligned} \tag{9}$$

Remark 2 If A_{022} is non-singular, then the system (9) is regular and impulse free [17]. In this case, the compatible initial condition is

$$\begin{aligned} 0 &= A_{021}\phi_1(0) + A_{022}\phi_2(0) + \sum_{i=1}^{k}e^{\alpha h_i}A_{i21}z_1(t-h_i) \\ &\quad + \sum_{i=1}^{k}e^{\alpha h_i}A_{i22}z_2(t-h_i) \end{aligned}$$

Define the difference operator $D : C_{n-r,r} \to R^{n-r}$

$$D(z_{2t}) = z_2(t) + \sum_{i=1}^{k}e^{\alpha h_i}A_{022}^{-1}A_{i22}z_2(t-h_i)$$

That the operator is stable means that the equation $D_{z_{2t}} = 0$ is asymptotically stable.

We need the following lemma.

Lemma 3 [10] If the operator D is stable and there exist positive number α, β, γ and a continuous functional $V : C_{n,\tau} \to R$ such that

$$\begin{aligned} \alpha\|Z_1(t)\| \le V(z_t) &\le \beta\|z_t\|_c^2 \\ \dot{V}(z_t) &\le -\gamma\|z(t)\|^2 \end{aligned} \tag{10}$$

and the function $\bar{V}(t) = V(z_t)$ is absolutely continuous for z_t satisfying (9), then (9) is asymptotically stable.

In the following, we give the asymptotically stable condition of the system (8).

Theorem 1 Consider the singular delay system (8) with all constant delays $h_i \in [0, h], i = 0, 1, ..., k$. Then the system (8) is asymptotically stable if there exist $0 < R \in R^{n\times n}, i = 1, ..., k, P = \begin{bmatrix} P_1 & 0 \\ P_2 & p_3 \end{bmatrix} \in R^{n\times n}$, with $0 < P_1 \in R^{r\times r}$, and a constant $\alpha > 0$, such that the following LMI holds:

$$\begin{bmatrix} X_1 & M_1 \\ M_1^T & -N_1 \end{bmatrix} < 0 \tag{11}$$

where

$$\begin{aligned} X_1 &= P^TA_0 + A_0^TP + \alpha P^TE + \alpha EP + \sum_{i=1}^{k}R_i \\ M_1 &= [e^{\alpha h_1}P^TA_1 \ldots e^{\alpha h_k}P^TA_1] \\ N_1 &= diagR_1, ..., R_k \end{aligned}$$

Proof Let $z(t) = e^{\alpha t}x(t)$ then the system (8) is transformed into the system (9). From (11), we get . (12)

$$X_1 < 0 \tag{12}$$

From (2) and (12), we have

$$\begin{bmatrix} \Gamma_{11} & P_1^T A_{012} + P_2^T A_{022} + A_{021}^T P_3 + \sum_{i=1}^k R_{i12} \\ \Gamma_{21} & P_3^T A_{022} + A_{022}^T P_3 + \sum_{i=1}^k R_{i22} \end{bmatrix} < 0$$

where

$$\Gamma_{11} = P_1^T A_{011} + P_2^T A_{021} + A_{011}^T P_2 + A_{021}^T P_2 + \alpha P_1 + \alpha P_1^T + \sum_{i=1}^k R_{i11}$$

$$\Gamma_{21} = A_{012}^T P_1 + A_{022}^T P_2 + P_3^T A_{021} + \sum_{i=1}^k R_{i12}^T$$

Hence, it is yielded that

$$P_3^T A_{022} + A_{022}^T P_3 + \sum_{i=1}^k R_{i22} < 0$$

Noticing $\sum_{i=1}^k R_{i22} > 0$ we get $P_3^T A_{022} + A_{022}^T P_3 < 0$, which implies that A_{022} is non-singular. Therefore, the system (9) is regular and impulse free [17]. According to Lemma 2 in [7], we know that the operator D is stable.

Constructing the Lyapunov-Krasovskii functionals for the system (9) as follows:

$$V(z(t), z(t-h_1), ..., z(t-h_k)) = z^T(t)EPz(t) + \sum_{i=1}^k \int_{t-h_i}^t z^T(\theta) R_i z(\theta) d\theta$$

we get

$$t \geq h_i (i = 1, 2, ...k), \|z_t\|_c = \sup_{\theta \in [-\tau, 0]} \|z(t+\theta)\| \quad (13)$$

Because

$$\frac{d}{dt}(z^T(t)EPz(t)) = 2z_1^T(t)P_1\dot{z}_1(t) = 2z_1^T(t)P^T \begin{bmatrix} \dot{z}_1(t) \\ 0 \end{bmatrix}$$

the time derivative of along the trajectory of (9) is given by

$$\dot{V}(z(t), z(t-h_1), ..., z(t-h_k)) = 2z^T(t)P^T \begin{bmatrix} \dot{z}_1(t) \\ 0 \end{bmatrix} + \sum_{i=1}^k (z^T(t)R_i z(t) - z^T(t-h_i)R_i z(t-h_i))$$

$$= 2z^T(t)P^T[(A_0 + \alpha E)z(t) + \sum_{i=1}^k A_i e^{\alpha h_o} z(t-h_i)] + \sum_{i=1}^k (z^T(t)R_i z(t) - z^T(t-h_i)R_i z(t-h_i))$$

$$= z^T(t)(2P^T A_0 + 2\alpha P^T E + \sum_{i=1}^k R_i)z(t) + z^T(t)(2P^T \sum_{i=1}^k A_i e^{\alpha h_i})z(t-h_i) - \sum_{i=1}^k z^T(t-h_i)R_i z(t-h_i)$$

We get

$$\dot{V}(z(t), z(t-h_1), ..., z(t-h_k)) = \xi^T S \xi$$

where

$$\xi = [\dot{V}(z(t), z(t-h_1), ..., z(t-h_k))]^T$$
$$S = \begin{bmatrix} X_1 & M_1 \\ M_1^T & -N_1 \end{bmatrix} \quad (14)$$

M_1, X_1, N_1 are defined in (11)

From condition (11), we have . Thus, from the stability of operator , Lemma 3 and (13), it follows that system (9) is asymptotically stable. So the system (8) is asymptotically stable. This completes the proof.

Theorem 2 Consider the singular delay system (1) with $\triangle A_j(t) = D_j F_j(t) E_j, j = 0, 1, ...k, \|F\| \leq 1$ For all delays given $h_i \in [0, h], i = 1, 2, ...k$, the system (1) is robust asymptotically stable if there exist $0 < R \in R^{n \times n}, i = 1, ..., k, P = \begin{bmatrix} P_1 & 0 \\ P_2 & p_3 \end{bmatrix} \in R^{n \times n}$, with $0 < P_1 \in R^{r \times r}$ and scalars $\varepsilon_j > 0, j = 1, ..., k$, such that the following LMI holds:

$$\begin{bmatrix} X_2 & M_1 & K & M_3 \\ M_1^T & -N_2 & 0 & 0 \\ K^T & 0 & -T & 0 \\ M_3^T & 0 & 0 & -N_3 \end{bmatrix} < 0 \quad (15)$$

where

$$\begin{array}{l} X_2 = P^T A_0 + A_0^T P + \alpha P^T E + \alpha E P + \\ \qquad + \sum_{i=1}^k R_i + \varepsilon_0 E_0^T E \\ K = PTD_1 \quad ... \quad P^T D_k \\ T = diag e^{-2\alpha h_1} \varepsilon_1 I_{k_1} \quad ... \quad e^{-2\alpha h_k} \varepsilon_k I_{k_k} \\ M_3 = P^T D_0, N_3 = \varepsilon_0 I_{k_0} \\ M_1 = [e^{-2\alpha h_1} P^T A_1 \quad ... \quad e^{-2\alpha h_k} P^T A_k] \\ N_2 = diag R_1 - \varepsilon_1 E_1^T E_1, ..., R_k - \varepsilon_k E_k^T E_k \end{array} \quad (16)$$

Proof Using we transform (1) into the system (7).

Similar to the proof in theorem 1, we get that the operator D is stable.

Constructing the Lyapunov-Krasovskii functional for system (7) as follows:

$$V(z(t),z(t-h_1),...,z(t-h_k)) = z^T(t)EPz(t) + \sum_{i=1}^{k}\int_{t-h_i}^{t} z^T(\theta)R_i z(\theta)d\theta$$

$$\dot{V}(z(t),z(t-h_1),...,z(t-h_k)) = 2z^T(t)P^T\begin{bmatrix}\dot{z}_1(t)\\0\end{bmatrix} + \sum_{i=1}^{k}(z^T(t)R_iz(t) - z^T(t-h_i)R_iz(t-h_i))$$
$$= 2z^T(t)P^T[(\alpha E + A_0 + \triangle A_0)z(t) + \sum_{i=1}^{k}(A_i + \triangle A_i)e^{\alpha h_i}z(t-h_i)] + \sum_{i=1}^{k}(z^T(t)R_iz(t) - z^T(t-h_i)R_iz(t-h_i))$$

Hence we have

$$\dot{V}(z(t),z(t-h_1),...,z(t-h_k)) = \xi^T(S+\bar{S})\xi \quad (17)$$

where

$$\bar{S} = \begin{bmatrix}\Gamma & M_2\\ M_2^t & 0\end{bmatrix}$$
$$\Gamma = P^T\triangle A_0 + \triangle A_0^T P \quad (18)$$
$$M_2 = [e^{\alpha h_1}P^T\triangle A_1, ..., e^{\alpha h_k}P^T\triangle A_k]$$

and S is given by (14).

Since

$$S+\bar{S} = S + \gamma_0 F_0\Xi_0 + (\gamma_0 F_0 \Xi_0)^T + \sum_{i=1}^{k}\gamma_i F_i \Xi_i + \sum_{i=1}^{k}(\gamma_i F_i \Xi_i)^T$$

where

$$\gamma_0 = [D_0^T P0\ldots 0]^T$$
$$\Xi_0 = [E_0 0\ldots 0]$$
$$\gamma_i = [e^{\alpha h_i}D_i^T 0\ldots 0]$$
$$\Xi_i = [0\ldots 0E_i0\ldots 0]$$

E_i is the i-th column of Ξ_i. Using Lemma 1, we have

$$S+\bar{S} \le S + \sum_{i=0}^{k}\varepsilon_i \Xi_i^T\Xi_i + \sum_{i=0}^{k}\varepsilon_i^{-1}\gamma_i\gamma_i^T$$

Using Lemma 2, we know that

$$S + \sum_{i=0}^{k}\varepsilon_i\Xi_i^T\Xi_i + \sum_{i=0}^{k}\varepsilon_i^{-1}\gamma_i\gamma_i^T < 0 \quad (19)$$

is equivalent to

$$\begin{bmatrix} X_2 & M_1 & K & M_3\\ M_1^T & -N_2 & 0 & 0\\ K^T & 0 & -\bar{N}_1 & 0\\ M_3^T & 0 & 0 & -N_3\end{bmatrix} < 0 \quad (20)$$

where

$$\bar{N}_1 = diag\{e^{-2\alpha h_1}\varepsilon_1 I_{k_1}, \ldots, e^{-2\alpha h_k}\varepsilon_k I_{k_k}\}$$

From (15) and (16), we know that (20) is true, so $S+\bar{S} < 0$. Thus it is yielded that $\dot{V} < 0$ whenever ξ is not zero. According to the stability of the operator D, Lemma 3 and (13), it follows that the system (7) is robust asymptotically stable. So the system (1) is robust asymptotically stable. This completes the proof.

Remark 3 Let $k = 1$, then the system (7) only has one single state delay, and (15) can be transformed into (21) as LMI problem on a single state delay.

$$\begin{bmatrix} \Pi & \Pi_1 & \Pi_2 & M_3\\ \Pi_1^T & -B_1 & 0 & 0\\ \Pi_2^T & 0 & -B_2 & 0\\ M_3^T & 0 & 0 & -N_3\end{bmatrix} < 0 \quad (21)$$

where

$$\Pi = P^TA_0 + A_0^TP + \alpha P^TE + \alpha EP + R_1 + \varepsilon_0 E_0^T E$$
$$\Pi_1 = e^{\alpha h_1}P^TA_1, \quad \Pi_2 = P^TD_1, \quad B_1 = R_1 - \varepsilon_1 E_1^T E_1$$
$$B_2 = e^{-2\alpha h}\varepsilon_1 I_{k_1}, \quad M_3 = P_T D_0, \quad N_3 = \varepsilon_0 I_{k_0}$$

4. Numerical Examples

Example 1 Consider the following uncertain linear time-delay singular systems

$$\begin{bmatrix}1&0\\0&0\end{bmatrix}\begin{bmatrix}\dot{z}_1(t)\\ \dot{z}_2(t)\end{bmatrix} = \begin{bmatrix}-1+0.1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}z_1(t)\\ z_2(t)\end{bmatrix} + \sum_{i=1}^{2}\begin{bmatrix}-0.01 & 0\\ 0 & -0.01\end{bmatrix}e^{0.1h_i}z(t-h_i) \quad (22)$$

It is obvious that

$$\alpha = 0.1, \quad E = \begin{bmatrix}1&0\\0&0\end{bmatrix}, \quad A_0 = \begin{bmatrix}-1&0\\0&-1\end{bmatrix}$$
$$A_i = E = \begin{bmatrix}-0.01&0\\0&-0.01\end{bmatrix}, \quad h_i \le h, i = 1,2$$

Set

$$R_i = \begin{bmatrix}1&0\\0&1\end{bmatrix}, i = 1,2$$

then

$$N_1 = diag R_1, R_2$$

For simple, set

$$P = \begin{bmatrix}c&0\\0&d\end{bmatrix}$$

where c and d are positive scalars and will be determined later. Noticing (11), we have

$$X_1 = \begin{bmatrix} -2c+2\times 0.1c+2 & 0 \\ 0 & -2d+2 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} e^{0.1h_1}\begin{bmatrix} -0.01c & 0 \\ 0 & -0.01d \end{bmatrix}^T \\ e^{0.1h_2}\begin{bmatrix} -0.01c & 0 \\ 0 & -0.01d \end{bmatrix}^T \end{bmatrix}^T$$

Using Lemma 2, (11) is equivalent to

$$X_1 + M_1 N_1^{-1} M_1^T < 0$$

It is obtained that

$$X_1 + M_1 N_1^{-1} M_1^T < \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix}$$

where

$$\begin{aligned} \Sigma_1 &= -2c+0.2c+2+2\times 0.01^2 e^{0.2h} c^2 \\ \Sigma_2 &= -2d+2+2\times 0.01^2 e^0.2hd^2 \end{aligned}$$

So if

$$\begin{cases} -2c+0.2c+2+2\times 0.01^2 e^{0.2h} c^2 < 0 \\ -2d+2+2\times 0.01^2 e^0.2hd^2 < 0 \end{cases} \quad (23)$$

then

$$X_1 + M_1 N_1^{-1} M_1^T < 0$$

Solving (23), we get

$$\begin{aligned} \frac{1.8-\sqrt{\triangle_1}}{2\times 2\times 0.01^2 e^{0.2h}} < c < \frac{1.8+\sqrt{\triangle_1}}{2\times 2\times 0.01^2 e^{0.2h}} \\ \frac{2-\sqrt{\triangle_2}}{2\times 2\times 0.01^2 e^{0.2h}} < c < \frac{2+\sqrt{\triangle_2}}{2\times 2\times 0.01^2 e^{0.2h}} \end{aligned} \quad (24)$$

where

$$\begin{aligned} \triangle_1 = 1.8^2 - 4\times 2\times 0.01^2 e^{0.2h}\times 2 \\ \triangle_2 = 2^2 - 4\times 2\times 0.01^2 e^{0.2h}\times 2 \end{aligned}$$

Solving the following inequalities:

$$\begin{cases} \triangle_1 > 0 \\ \triangle_2 > 0 \end{cases}$$

we get $h < 10ln45$. Hence when $h < 10ln45$ we have $\triangle_1 > 0$, $\triangle_2 > 0$. Take c and d satisfying (24), then the following inequality holds:

$$X_1 + M_1 N_1^{-1} M_1^T < 0$$

According to Theorem 1, the system (22) is asymptotically stable.

Example 2 Consider the uncertain time-delay systems

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \end{bmatrix} = \left\{\begin{bmatrix} -1+0.1 & 0 \\ 0 & -1 \end{bmatrix} + \triangle A_0\right\}\begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} + \sum_{i=1}^{2}\left\{\begin{bmatrix} -1+0.1 & 0 \\ 0 & -1 \end{bmatrix} + \triangle A_i\right\} e^{0.1h_i} z(t-h_i) \quad (25)$$

and the uncertainties can be described by

$$\triangle A_j(x,t) = D_j F_j(x,t) E_j, \quad j = 0,1,2$$

with

$$D_j = E_j \begin{bmatrix} \sqrt{0.5} & 0 \\ 0 & \sqrt{0.5} \end{bmatrix}, F_j = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$$

where σ_1 and σ_2 are unknown real parameters with $\sigma_1 < 10^{-4}$, $\sigma_2 < 10^{-4}$ Set

$$R_j = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad i = 1,2$$

and

$$P = \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix}$$

where c and d are positive scalars and will be determined later.

Noticing (11), we have

$$X_1 = \begin{bmatrix} \Sigma_3 & 0 \\ 0 & \Sigma_4 \end{bmatrix},$$

$$M_0 = \begin{bmatrix} e^{\alpha h_1}\begin{bmatrix} -0.01c+0.5\sigma_1 c & 0 \\ 0 & -0.01d+0.6\sigma_2 d \end{bmatrix}^T \\ e^{\alpha h_2}\begin{bmatrix} -0.01c+0.5\sigma_1 c & 0 \\ 0 & -0.01d+0.6\sigma_2 d \end{bmatrix}^T \end{bmatrix}^T$$

$$\Sigma_3 = 2c(-1+0.5\sigma_1) + 2\times 0.1c + 2$$

$$\Sigma_4 = 2d(-1+0.6\sigma_2) + 2$$

Using Lemma 2, (11) is equivalent to

$$X_0 + M_0 N_1^{-1} M_0^T < 0$$

It is obtained that

$$X_0 + M_0 N_1^{-1} M_0^T < \begin{bmatrix} \Sigma_5 & 0 \\ 0 & \Sigma_6 \end{bmatrix}$$

where

$$\begin{aligned} \Sigma_5 &= 2c(-1+0.5\sigma_1) + 0.2c + 2 + 2(0.01c - 5\sigma_1 c)^2 e^{0.2h} \\ \Sigma_6 &= 2d(-1+0.6\sigma_2) + 2 + 2(0.01d - 0.6\sigma_2 d)^2 e^{0.2h} \end{aligned}$$

Noticing $\sigma_1^2 < 10^{-4}, \sigma_2^2 < 10^{-4}$ we get

$$\begin{aligned}\Sigma_5 &= 2(-0.01+0.5\sigma_1)^2 e^{0.2h} c^2 + \\ &+(-1.5+\sigma_1)c+2 \\ &< 2\times(-0.01-0.5\times 10^{-2})^2 e^{0.2h} c^2 \\ &+(-1.8+10^{-2})c+2 \\ \Sigma_6 &= 2(-0.01+0.6\sigma_2)^2 e^{0.2h} d^2 + \\ &+(-2+1.2\sigma_2)d+2 \\ &< 2\times(-0.01-0.6\times 10^{-2})^2 e^{0.2h} d^2+2 \\ &+(-2+1.2\times 10^{-2})d\end{aligned} \tag{26}$$

Solving the following inequalities

$$\begin{aligned}&2\times(-0.01-0.5\times 10^{-2})^2 e^{0.2h} c^2 \\ &\quad +(-1.8+10^{-2})c+2<0 \\ &2\times(-0.01-0.6\times 10^{-2})^2 e^{0.2h} d^2+2 \\ &\quad +(-2+1.2\times 10^{-2})d<0\end{aligned} \tag{27}$$

we get

$$\begin{aligned}&\frac{1.79-\sqrt{\triangle_3}}{9\times 10^{-4} e^{0.2h}} < c < \frac{1.79+\sqrt{\triangle_3}}{9\times 10^{-4} e^{0.2h}} \\ &\frac{1.988-\sqrt{\triangle_4}}{1.024\times 10^{-3} e^{0.2h}} < d < \frac{1.024+\sqrt{\triangle_4}}{1.024\times 10^{-3} e^{0.2h}}\end{aligned} \tag{28}$$

where

$$\begin{aligned}\triangle_3 &= 1.79^2 - 16\times 1.5^2\times 10^{-4} e^{0.2h} \\ \triangle_4 &= 1.988^2 - 16\times 1.6^2\times 10^{-4} e^{0.2h}\end{aligned}$$

we get $h < 10ln\frac{179}{6}$. So when $h < 10ln\frac{179}{6}$, we have $\triangle_3 > 0, \triangle_4 > 0$. Take c and d satisfy (28), then the following inequality holds:

$$X_0 + M_0 N_1^{-1} M_0^T < 0$$

According to Theorem 1, system (25) is asymptotically stable for any

$$\sigma_1^2 \leq 10^{-4}, \quad \sigma_2^2 \leq 10^{-4}$$

5. Conclusion

This paper deals with the problem of robust stability criteria for a class of uncertain linear time-delay singular systems. Firstly, we consider the equivalent systems of the original systems by introducing the state transformation. The Lyapunov-Krasovskii functional is constructed, and is applied to the equivalent systems and the nominal system of the equivalent systems. The delay-dependent robust stability criterion for the equivalent systems is obtained, which also ensures the original system is asymptotically stable.

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