

# The Space-Time Fractal Feature of Deformation at Convex Corner of Deep Foundation Pit

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**Abstract.** The study on the space-time feature of foundation pit deformation has important significance to ensure the stability of foundation pit engineering. In the present study, the relationship between the space-time fractal feature of foundation pit deformation and the stability of the foundation pit is expressed with simple indexes, such as the time and position of the maximum value of deformation. By combining the concrete engineering example, the fractal theory is introduced, and the correlation dimension is calculated with the measured deformation data for a period of time. By combining the concrete engineering example, the fractal theory was introduced, and used the correlation dimension calculated with the measured deformation data to analyze the space-time fractal feature of deformation at convex corner. Further researched on the relationship between the correlation dimension of the foundation pit deformation and the stability of foundation pit. The research showed that the correlation dimension could reveal the complex space-time feature of the foundation pit deformation. From the aspects of time, the correlation dimension is related to the foundation pit condition, construction disturbance, the change of supporting structure and so on, and

has a certain degree of decline with time. From the aspects of space, the difference of correlation dimension between stable and unstable regions is relatively large while there is little difference in the stability region. With the correlation dimension, it is more easily to identify the stable and the unstable regions of the foundation pit, compared with the accumulated deformation.

## **Introduction**

With the development of urbanization, a large number of foundation pits are emerging in China, and the foundation pits are becoming more and more complex. At present the safety and stability of foundation pit is a hot research topic. For the stability of foundation pit, the researchers mainly focus on the deformation of foundation pit. The researches on the deformation of foundation pit include the time effect and space effect. Time effect on foundation pit mainly includes the prediction of foundation pit deformation based on the time series of displacement <sup>[1-8]</sup>. In the prediction researches on the deformation of foundation pit, the intelligent algorithm was mainly used to predict the accumulated deformation of foundation pit. In most of the prediction research the influence of hydro geological conditions, construction state, surrounding load and so on was ignored which greatly limited the forecast precision. Although the prediction accuracy for accumulated deformation was acceptable, for the daily deformation quantity the error is unacceptable. In some prediction researches of foundation pit, the time series of deformation was divided into trend term and random term <sup>[3,6]</sup>. but the analysis of the time series data is not enough.

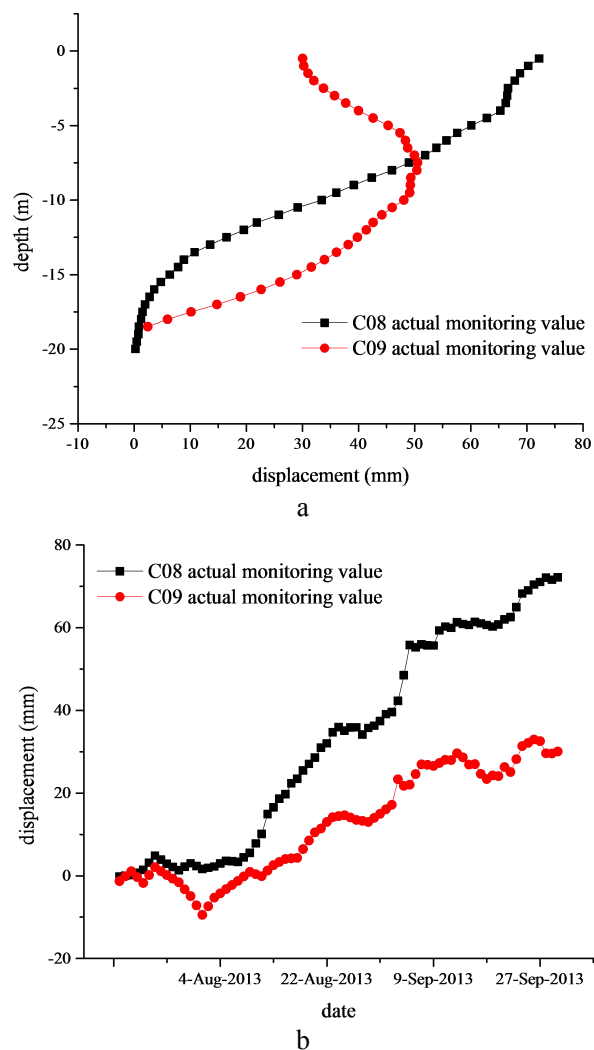
In order to ensure the safety and stability of foundation pit, the establishment of early warning value is the key point of the research. Because of the complexity of the soil and the construction conditions of the foundation pit, the early warning value is difficult to establish. Specifications of some regions only provide a deformation allowable value, and don't combine the deformation with the stability of foundation pit to decide the early warning value of its monitoring indicators <sup>[9]</sup>. In the existing researches, the relationship between the time effect and space effect on foundation pit deformation with the stability of the foundation was taken a simple index, for example the maximum value of the

deformation. However, for the deformation with more complex characteristics the analysis is not enough. The fractal theory was introduced to study the complex characteristics of foundation pit deformation. The most of the present researches applying fractal theory in foundation pit was focused on the microstructure of soil<sup>[10-11]</sup>. In this paper, with the specific engineering example, analyzed the space-time fractal feature of deformation at convex corner of deep foundation pit and explored the relationship between fractal dimension and stability of foundation pit.

### **Engineering examples**

The foundation pit built by Wuhan Wangjiadun CBD Construction& Investment Co., Ltd. located in Wuhan wangjiadun CBD. The plane shape of foundation pit is irregular. Combining the geological data and the surrounding environment of the engineering, the importance level of the foundation pit is the first level and the effective running time is 12 months.

In the actual construction process, the measured displacement of the convex corner region peaked at 214.54mm, which caused the fracture of top beam and the cracks in the plant outside of the foundation pit. The construction company took the measures such as backfilling soil and applying diagonal brace in time, which limited the further development of the deformation at the convex corner region. And in the follow-up process of construction applied support to prevent the occurrence of a greater accident. The monitoring points close to the convex corner region were C08, C09. The actual monitoring date was from July 17, 2013 to June 16, 2014. The end date of foundation pit excavation was September 30, 2013. The actual monitoring values were as shown in Fig.1. From Fig.1, it is found that the actual monitoring values show the greater volatility and more obvious time effect.



**Fig.1.** The contrast between the actual monitoring values with the simulated values.

### The correlation dimension and foundation pit stability

**The correlation dimension.** For the deformation of foundation pit, the deformation rate is more effective than the accumulated deformation to reflect the stability in a period of time. Meanwhile, the deformation of foundation pit is complex, so the deformation of one day is not enough to prove the stability of foundation pit. As a supplement to the deformation of foundation pit, the deformation data in a period of time was used to calculate the correlation dimension, and judge the stability. The correlation dimension calculation principle of deformation time series data is as follows:

In theory, Takens proved that from the time series of single variable  $X = [x_1, x_2, x_3 \dots x_n]$ , a series of  $m$  dimension vector could be constructed, and the phase space reconstruction of time series was achieved by selecting the dimension  $m$  of embedded space. When the dimension of embedded space

is high enough, in phase space the dynamic characteristics can be recovered in the sense of topological equivalence<sup>[12]</sup>. The principle of G-P algorithm for calculating the correlation dimension from one dimension time series data is as follows:

First, construct a series of m dimension vector with Eq.(1).

$$y_i = (x_i, x_{i+\tau}, x_{i+2\tau}, \dots, x_{i+(m-1)\tau}) \quad (1)$$

The  $\tau$  is the time lag. With Eq.(1) in the reconstruction space, there are N vector, and in order to calculate the correlation dimension of time series, the distance  $d$  between different vectors need to be defined. In the paper, the distance  $d$  was defined with 1- norm which was as follows:

$$d_{ij} = \|y_i - y_j\|_1 \quad (2)$$

Given a positive integer  $r$ , when the distance between two vectors is less than  $r$ , it is called an association vector. The frequency of the association vector in the embedded space is called the correlation integral. The calculation formula is as follows:

$$C(r, N, m, \tau) = \frac{2}{(N-1)N} \sum_{1 \leq i < j \leq N} H(d_{ij}) \quad (3)$$

In the Eq.3, the  $H$  is Heaviside unit step function that is  $H(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$ .

It has proved that when the  $r$  goes to 0, the relationship between the correlation integral and  $r$  is shown in Eq.(4).

$$\lim_{r \rightarrow 0} C(r, N, m, \tau) \propto r^D \quad (4)$$

The  $D$  is the correlation dimension of time series and can be got by transformation Eq.(4).

$$D = \lim_{r \rightarrow 0} \lim_{N \rightarrow \infty} \frac{d[\ln C(r, N, m, \tau)]/dr}{d[\ln(r)]/dr} \quad (5)$$

In the practical solution, with a series of  $r$ , the approximate estimation of the correlation dimension is obtained by least square method. The principle is shown in Eq.(6).

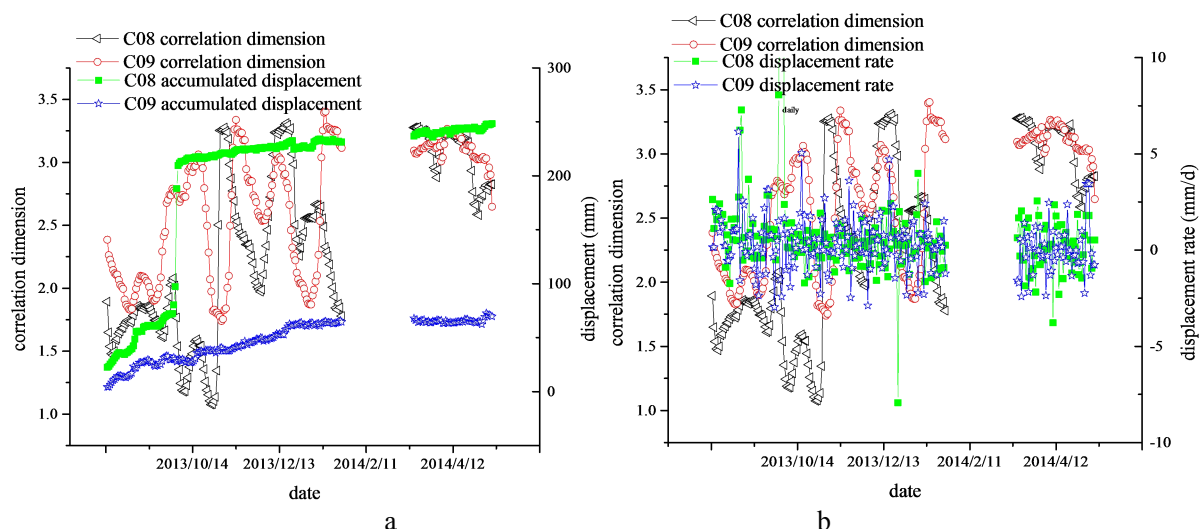
$$D = \frac{l \sum_{i=1}^l \ln(r_i) \ln C(r_i) - \sum_{i=1}^l \ln(r_i) \sum_{i=1}^l \ln C(r_i)}{\sum_{i=1}^l (\ln(r_i))^2 - (\sum_{i=1}^l \ln(r_i))^2} \quad (6)$$

From the calculation principle of correlation dimension, when the difference between the time series data is relatively small, that is, the correlation degree is high, the value of correlation dimension is also large. In the time series data of the deformation of the foundation pit, when the correlation dimension is large, the deformation of foundation pit is smaller and the foundation pit is stable. Therefore, can judge the unstable region of foundation pit with the comparing the correlation dimension at different region of foundation pit.

**The time evolution of correlation dimension.** To guarantee a certain amount of data, the correlation dimension was calculated with the deformation data in the past 30 days, that was, the correlation dimension represented the deformation state of foundation pit in the last 30 days. The embedded space dimension was 10, and the time lag is 1. According to the results of the calculation, drew the curve about the correlation dimension with the change of the excavation time of the foundation, as shown in Fig.2. From the Fig.2, the accumulated deformation isn't sensitive to the change of the state of the foundation pit, while the deformation rate is too sensitive. According to the relationship between the correlation dimension and time of the foundation pit, in the initial stage of the foundation pit excavation, the change of C08 correlation dimension was small which was related with the incomplete excavation of foundation pit. When the C08 displacement mutation of foundation pit occurred, the correlation dimension reduced sharply, which mean the larger deformation of foundation pit over the past period of time and the poorer stability of foundation pit. In the actual construction process, the top beam was broken. The construction company took the measures such as backfilling soil and applying diagonal brace in time, which limited the further development of the deformation at the convex corner region. And in the follow-up process of construction applied support to prevent the occurrence of a greater accident. But in this stage, the correlation dimension of the foundation pit had a large fluctuation, and the minimum value of the correlation dimension was

similar to that in the initial stage of excavation. With stopping the construction during the Spring Festival the correlation dimension of foundation pit increased, and with restarting the construction the correlation dimension of foundation pit reduced. The trend of C09 correlation dimension was similar to the C08 correlation dimension. However the correlation dimension of C09 monitoring points was bigger than the C08 and in the actual monitoring process the region near the C08 was more stable than the C09. At the same time, the change of C09 correlation dimension lagged behind the C08 correlation dimension. With stopping the construction, the incremental of C08 correlation dimension was larger than C09 which indicated that the stability of convex corner region was more sensitive to the construction disturbance.

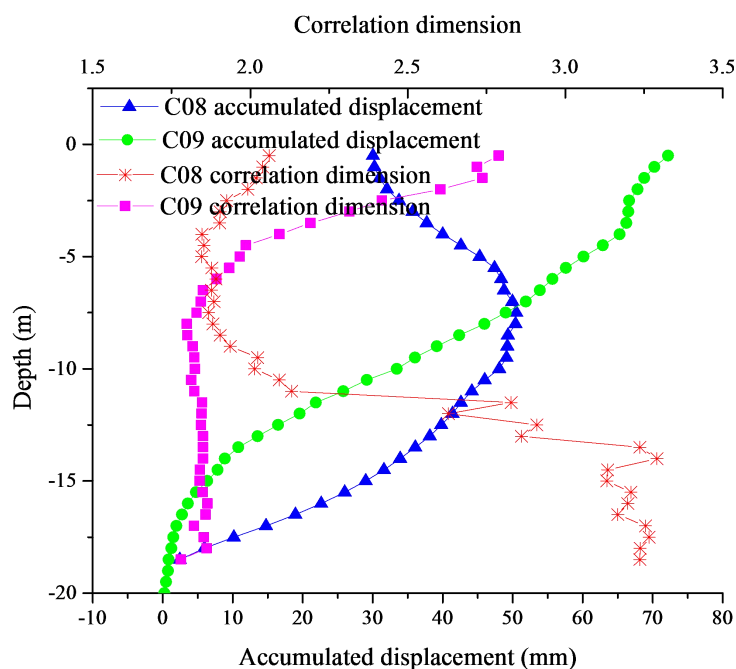
With combining the Fig.2 with the actual engineering, it could be found that the correlation dimension is related to the foundation pit condition, construction disturbance, the change of supporting structure and so on, and has a certain degree of decline with time. The traditional prediction methods about foundation pit deformation was based on dynamic characteristics of time series and due to the lack of the consideration of the foundation pit condition, construction disturbance, the change of supporting structure and so on, the results were poor. Therefore, for the foundation pit whose construction period is short and the construction states is complex, the effective way to ensure the stability of foundation pit is based on the analysis of the actual monitoring date.



**Fig.2.** The relationship between deformation of foundation pit and correlation dimension. (a) The relationship between accumulated displacement and correlation dimension, (b) The relationship between displacement rate and

correlation dimension, in which the larger deformation of three day was deleted to fully show the deformation characteristics.

**The space evolution of correlation dimension.** In order to further study the relationship between the correlation dimension of foundation pit and the stability of foundation pit, the change of correlation dimension of the different space regions was studied. The relationship between correlation dimension with accumulated displacement in September 30, 2013 was shown in Fig.3. In depth, the distribution of the accumulated displacement of the C08 monitoring points is triangular and the deformation of the upper part is large, which show that the unstable region is located in the upper part. The region where the correlation dimension of C08 monitoring points is small has a certain consistency to the unstable region. The distribution of the accumulated displacement of the C09 monitoring points is arched. Therefore the upper stability is higher, which is same as the stable region judged by the bigger correlation dimension. In Fig.3, the difference of correlation dimension between stable and unstable regions is relatively large, while the difference of correlation dimension in the stable regions is small. Comparing with the accumulated displacement, taking the correlation dimension to judge the stable and unstable regions is easier.

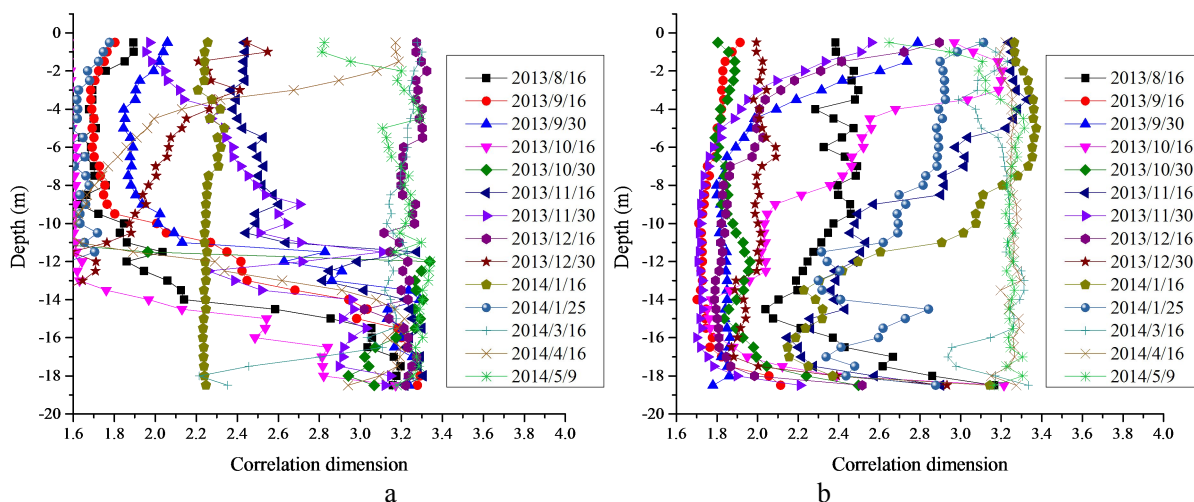


**Fig.3.** The relationship between correlation dimension with accumulated displacement in September 30, 2013. In order to further show the space-time evolution of the correlation dimension of the foundation pit,

the correlation dimension-depth curves with time of the C09 and C08 monitoring points were plotted



in Fig.4. In the initial stage of the excavation, the correlation dimension of the foundation pit had a decreasing trend, and the stability was weakened. Along with the backfill of the foundation pit and the decrease of the construction disturbance, the correlation dimension and the stability increased. The boundary points of the space distribution of C08 correlation dimension appeared in the foot of the foundation pit, which showed that the stability of the foot of the foundation pit was poor. In contrast, the boundary points of the space distribution of C09 correlation dimension appeared in the lower part of the foot of the foundation pit. The root cause of this difference lied in the large deformation at convex corner region and the unstable boundary points' rise, which led to that the underground continuous wall had not played a full role in resisting deformation. Therefore, from the point of view of the correlation dimension's space distribution, when the boundary points of the correlation dimension shift up to the foot of the foundation pit, the stability of the foundation pit became poor and need to focus on the stability of the foot of the foundation pit in the construction process. Through the analysis of Fig.4, the difference of correlation dimension in different regions of foundation pit became small when the foundation pit stability was enhanced at the end of the excavation.



**Fig.4.** The space-time evolution of the correlation dimension of the monitoring points. (a) The space-time evolution of the C08 correlation dimension, (b) The space-time evolution of the C09 correlation dimension.

## Discussion

Based on the actual monitoring data of the foundation pit, the deformation of foundation pit is evaluated by the correlation dimension, and further judge the stability of the foundation pit. However,

it should be noted that there is a certain time lag with the correlation dimension to judge the stability. In this paper, the correlation dimension was calculated based on the data of the past month, and it was not enough to reflect the current state of the foundation pit. Therefore, the method proposed in this paper is just a good supplement to the existing evaluation system. Through a large number of foundation pits statistics can provide a reference for setting up the risk warning value of correlation dimension. By the Eq.(4) used the deformation date in past month to calculate the correlation dimension, the enlightenment can be obtained that there also is the power function relationship between the deformation rate  $r$  and the number of days whose deformation rate is greater than  $r$ . If do further promotion, the maximum deformation rate of different foundation pit should have some power function relationship. Liao Shaoming <sup>[13]</sup> through a large number of statistics obtained the distribution of the maximum deformation rate. The fitting degree is good by fitting the power function which is shown in Fig.5. So it is reasonable and effective to describe the deformation feature of the foundation pit with the fractal theory. The method used in this paper can be further applied to landslide monitoring, dam deformation monitoring, surrounding rock displacement and so on.

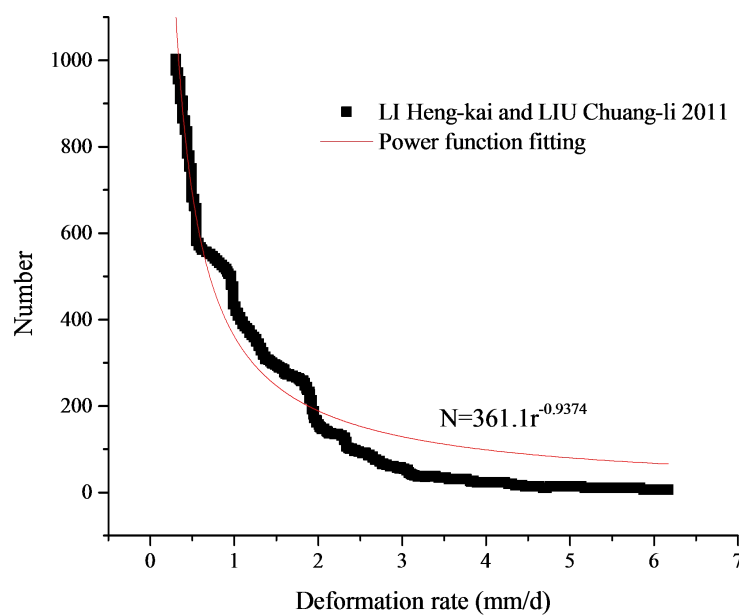


Fig.5. Comparison of the maximum velocity distribution and power function fitting

## Conclusions

By calculating the correlation dimension of the deformation data of the foundation pit, the deformation feature of C08 monitoring point in the unstable convex corner region of foundation pit was studied, and the deformation feature of the C08 monitoring point are compared with the C09 monitoring point in the stable region. The space-time fractal feature of the deformation of the foundation pit was studied systematically. Through the research, mainly conclusions are as follows:

1. The correlation dimension is related to the foundation pit condition, construction disturbance, the change of supporting structure and so on, and has a certain degree of decline with time. For the foundation pit whose construction period is short and the construction states is complex, the effective way to ensure the stability of foundation pit is based on the analysis of the actual monitoring date. The prediction based on time series has little effect in ensuring the stability of foundation pit.

2. Comparing with the accumulated displacement, the difference of correlation dimension between stable and unstable regions is relatively large, so it is easier that use the correlation dimension to judge the stable and unstable regions. From the point of view of the correlation dimension's space distribution, when the boundary points of the correlation dimension shift up to the foot of the foundation pit, the stability of the foundation pit became poor and need to focus on the stability of the foot of the foundation pit in the construction process. The difference of correlation dimension in different regions of foundation pit became small when the foundation pit stability was enhanced at the end of the excavation.

3. There is a certain time lag with the correlation dimension to judge the stability. The method is not enough to reflect the current state of the foundation pit. Therefore, the method proposed in this paper is just a good supplement to the existing evaluation system.

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