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DESIGN AND IMPLEMENTATION OF OBJECT-ORIENTED COMPUTER SOFTWARE TO SOLVE SECOND ORDER ORDINARY DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS

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Articles

Abstract. With the advent of computers, a lot has been achievable including the use of computers to solve Mathematics and Mathematically related problems. In this work an appraisal of the solution to second order linear ordinary differential equations with constant coefficients is done. Methods of solution of homogeneous second order ordinary differential equations, Initial Value Problem inclusive are described, considering three cases: distinct roots, repeated roots and complex roots. Non-homogeneous differential equations, IVP inclusive are also treated using the method of undetermined coefficients.

Object oriented software is developed to solve these second order linear ordinary differential equations with constant coefficients. The core essence of this work is to develop a system which would be user-friendly, reliable (accurate in solution), fast and timely to meet the demands which are not achievable by the traditional method.

Keywords: *Differential Equation, Homogeneous, Initial Value Problem, Nonhomogeneous, Object Oriented Software.*

1. INTRODUCTION

Wikipedia Encyclopedia defines a differential equation as a mathematical equation for an unknown function of one or several variables that relates the values of the function itself and its derivatives of various orders.

Differential equations arise in many areas of science and technology as it plays a prominent role in engineering, physics, economics, and other disciplines, specifically whenever a deterministic relation involving some continuously varying quantities (modeled by functions) and their rates of change in space and/or time (expressed as derivatives) is known or postulated. This is illustrated in classical mechanics, where the motion of a body is described by its position and velocity as the time value varies. Newton's laws allow one (given the position, velocity, acceleration and various forces acting on the body) to express these variables dynamically as a differential equation for the unknown position of the body as a function of time.

An example of modeling a real world problem using differential equations is the determination of the velocity of a ball falling through the air, considering only gravity and air resistance. The ball's acceleration towards the ground is the acceleration due to gravity minus the deceleration due to air resistance. Gravity is considered constant, and air resistance may be modeled as proportional to the ball's velocity. This means that the ball's acceleration, which is a derivative of its velocity, depends on the velocity (and the velocity depends on time). Finding the velocity as a function of time involves solving a differential equation.

The problem we seek to solve in this work is that of developing an automated (computer application) solution which solves second order linear ordinary differential equations with constant coefficients.

Linear differential equation is one of the

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fundamental equations used in solving differential equations, many of which have real life applications. Hence studying it will be of prime importance. Additionally, the system developed would help in reducing the time spent manually in solving differential equation problems.

1.1. Terminologies

Equation: This is the relationship between the dependent and independent variables. An equal sign "=" is required in every equation.

Differential Equation: Equations that involve dependent variables and their derivatives with respect to the independent variables.

Ordinary Differential Equation: Differential equations that involve only one independent variable.

Order: The order of a differential equation is the highest derivative that appears in the differential equation.

Degree: The degree of a differential equation is the power of the highest derivative term.

Linear: A differential equation is called linear if there are no multiplications among dependent variables and their derivatives. In other words, all coefficients are functions of independent variables.

Non-linear: Differential equations that do not satisfy the definition of linear.

Homogeneous: A differential equation is homogeneous if every single term contains the dependent variables or their derivatives.

Non-Homogeneous: Differential equations which do not satisfy the definition of homogeneous.

IVP: Initial Value Problem

1.2. Theoretical Framework

According to Zever (2012), if given the general form of a second order differential equation as

$$y'' + ay' + by = g(x) \quad (1.1)$$

Equation (1.1) can be classified as either homogenous or non-homogeneous differential equation.

Homogenous equations are equations of the form

$$y'' + ay' + by = 0$$

Comparing with (1.1), we have that $g(x) = 0$,$g(x) = 0$, and a, b are real constants.

Giving examples of homogenous equations with constant coefficients according to Zever (2012) as:

$$y'' + 4y' + y = 0y'' + 4y' + y = 0.$$

$$y'' + 8y' + 2y = 0y'' + 8y' + 2y = 0.$$

Nonhomogeneous equations are equations of the form (1.1) with $g(x) \neq 0g(x) \neq 0$ written as

$$y'' + ay' + by = g(x)$$

where a, b are constants and g is a continuous function on the open interval I.

Giving examples of non-homogenous equations with constant coefficients according to Zever (2012) as:

$$y'' - y' - 4y = 5e^{2x}y'' - y' - 4y = 5e^{2x}.$$

$$y'' + 2y' - 5y = 2x^2 + 3x.$$

$$y'' + 5y' + 3.5y = cos\ x\ .$$

Zever (2012) illustrated that an initial value problem for second order differential equations of

$$y'' + ay' + by = 0$$

Consists of finding a solution y of the differential equation that also satisfies initial conditions of the form

$$y(x_0) = y_0, \quad y'(x_0) = y_1$$

Richard and Gabriel (2006) stated that a second order linear homogeneous differential equation with constant coefficient has the form

$$y'' + a_1 y' + a_0 y = 0 \quad (2.1)$$

Corresponding to the differential equation in (2.1) in which a_1 and a_0 are constants, is the algebraic equation

$$m^2 + a_1 m + a_0 = 0 \quad (2.2)$$

Which is obtained from equation (2.1) by replacing y'', y' and y by m^2, m^1 and $m^0 = 1$, respectively. Equation (2.2) is called the characteristic equation of (2.1). The characteristic equation can be factored into

$$(m - m_1)(m - m_2) = 0 \quad (2.3)$$

Richard and Gabriel (2006) also stated that the General Solution of (2.1) is obtained from the roots of (2.3). Richard & Gabriel (2006) considered the three cases below:

CASE I: m_1 and m_2 both real and distinct:

Two linearly independent solutions are $e^{m_1 x}$ and $e^{m_2 x}$ and the general solution is

$$y = C_1 e^{m_1 x} + C_2 e^{m_2 x} \quad (2.4)$$

In the special case $m_2 = -m_1$, the solution (2.4) can be written as

$$y = k_1 cosh\, m_1 x + k_2 sinh\ \ m_1 x$$

CASE II: $m_1 = a + ib$, a complex number:

Since a_1 and a_0 in (2.1) and (2.2) are assumed real, the roots of (2.2) must appear in conjugate pairs; thus the other root is

$$m_1 = a + ib$$

Two linearly independent solutions are $e^{(a+ib)x}$ and $e^{(a-ib)x}$, and the general complex solution is

$$y = d_1 e^{(a+ib)x} + d_2 e^{(a-ib)x}$$
(2.5)

which is algebraically equivalent to

$$y = C_1 e^{ax} \cos bx + C_2 e^{ax} \sin bx \quad (2.6)$$

CASE III: $m_1 = m_2$

Two linearly independent solutions are $e^{m_1 x}$ and $e^{m_1 x}$, and the general solution is

$$y = C_1 e^{m_1 x} + C_2 e^{m_1 x} \quad (2.7)$$

According to Richard and Gabriel (2006), this is applicable only if $\emptyset(x)$ of the general solution to the linear differential equation of the form

$$L(y) = \emptyset(x)$$

and all of its derivatives can be written in terms of the same finite set of linearly independent functions, which we denote by

$$\{y_1(x), y_2(x), \dots, y_n(x),\}.$$

The method is initiated by assuming a particular solution of the form

$$y_p(x) = A_1 y_1(x) + A_2 y_2(x) + \dots + A_n y_n(x)$$
(3.0)

where $A_1, A_2, \dots, A_n$ denote arbitrary multiplicative constants. These arbitrary constants are then evaluated by substituting the proposed solution into the given differential equation and equating the coefficients of like terms. The general solution of the method of undetermined coefficients of non-homogeneous differential equation is given as

$$y = y_h + y_p$$

Richard and Gabriel (2006) considered the three cases stated below:

CASE I: $\emptyset(x) = P_n(x)\emptyset(x) = P_n(x)$, an nth degree polynomial in xx

Assume a solution of the form

$$y_p = A_n x^n + A_{n-1} x^{n-1} + \dots + A_1 x + A_0$$
(3.1)

where $A_j (j = 0, 1, 2, \dots, n)$ is a constant to be determined (Richard and Gabriel, 2006)

CASE II: $\emptyset(x) = ke^{ax}$ where k and a are known constants

Assume a solution of the form

$$y_p = Ae^{ax} \quad (3.2)$$

where A is a constant to be determined (Richard and Gabriel, 2006).

CASE III:

$\emptyset(x) = k_1 sin\beta x + k_2 cos\beta x$ where k_1, k_2 and β are known constants

Assume a solution of the form

$$y_p = A\sin\beta x + B\cos\beta x \quad (3.3)$$

where AA and BB are constants to be determined. (Richard and Gabriel 2006).

Kreyszig (2006) stated that the development of engineering phenomenon in solving fluid mechanics, heat transmission, wave motion or electromagnetic can only be successfully carried out using second order ordinary differential equations. Linear ordinary differential equations of second order are the most important ones because of their application in mechanical and electrical engineering.

Boyce and Diprima (2001) have talked about the use of series methods in solving differential equations with particular emphasis on second order linear homogenous differential equation. One of such is Bessel's equation of the form

$$x^2y'' + xy' + (x^2 - v^2)y = 0$$

where v is a constant.

1.3. Materials and method

The minimum hardware and software needed for the implementation of the system which solves second order linear ordinary differential equations with constant coefficients are shown in Table 1.

Table 1. Minimum hardware and software for the proposed system

S/N	Hardware/Software	Minimum Specification
1.	Operating system	Windows XP
2.	Processor	Pentium IV
3.	Hard disk space	1GB
4.	RAM	16MB
5.	Software development environment	Visual Basic 6.0

The Structured System Analysis and Design methodology was used. The method has the following stages:

Problem Definition: The problem identified was that some scientist still solve second order linear ordinary differential equations with constant coefficients using the traditional method and not all are have been able to develop user-friendly applications (for those who have software to do this) hence, the need to develop high quality software for solving these differential equations.

Feasibility study: This is a preliminary study to determine how the solution to second order linear ordinary differential equations with constant coefficients will be feasible with respect to economic, technical and operational considerations.

Economically computerizing the method of solution to second order linear ordinary differential equations with constant coefficients will save time which is normally used in solving manual method and as is popularly said time is money. It will lead to increased interaction with the computer by users thereby increasing computer awareness. Technically, the predominance of microcomputers and with the growing cheap computing powers makes it easy for the game to be easily available. Operationally, the proposed system if computerized will be readily accepted by its numerous users who currently use the traditional method.

2. ANALYSIS OF THE EXISTING SYSTEM

The existing system is a manually based system where, second order linear ordinary differential equations with constant coefficients are solved manually, i.e making use of paper and pen to find the solutions to these systems. The analysis of an existing system of second order linear ordinary differential equation with constant coefficients can be described in the four steps below:

Identification of the differential equation under the following conditions:

If it is a second order linear differential equation with constant coefficient of the form

$$y'' + ay' + by = g(x)$$

If it is a homogenous differential equation of the form

$$y'' + ay' + by = 0$$

If it is a non-homogenous differential equation of the form

$$y'' + ay' + by = g(x)$$

where $g(x) \neq 0$

If it is an initial value problem (IVP) of the form

$$y'' + ay' + by = g(x)$$

satisfying the initial conditions of the form

$$y(x_0) = y_0, \quad y'(x_0) = y_1$$

If conditions (Ia) and (Ib) holds then, the differential equation is solved using the general equation which is gotten from the roots of the characteristic equation considering the three different types of roots with their distinct general solutions: Both real and distinct roots, complex roots and equal roots.

If conditions (Ia) and (Ic) holds then, the differential equation is solved using the method of undetermined coefficients solution of non-homogenous linear differential equation with constant coefficients considering the different cases for solving the equation which are:

CASE I: $\emptyset(x) = P_n(x)$, an nth degree polynomial in x,

CASE II: $\emptyset(x) = ke^{ax}$ where kk and a a are known constants,

CASE III: $\emptyset(x) = k_1 sin\beta x + k_2 cos\beta x$ where k_1, k_2 and β are known constants.

If conditions (Ia) and (Id) holds then, the differential equation is solved by finding a solution of the differential equation that satisfies the initial condition.

3. THE PROPOSED SYSTEM

After going through the feasibility studies and analysis of the system, the following algorithm which will adequately and efficiently replace the existing traditional method was arrived at.

3.1. Algorithm for the Proposed System

The algorithm to solve second order linear ordinary differential equation with constant coefficients is given below:

```
Algorithm solve_second_order_ODE_with_constant_coefficients
Begin
  Identify equation
     If type of Eqns. → Nonhomo Eqns. then
          If type of Nonhomo Eqns. → Nonhomo then
               If form of Nonhomo → Constant then
                 Read values for constant equation
                    If discriminant → 0 then
                         Solve equation for equal roots solution
                      Else if Discriminant > 0 then
                         Solve equation for real and distinct roots solution
                      Else
                         Solve equation for complex roots solutions
                    End if
                 Else if form of Nonhomo → Exponential then
                   Read values for exponential equation
                    If Discriminant → 0 then
                         Solve equation for equal roots solution
```

```
            Else if Discriminant > 0 then
                Solve equation for real and distinct roots solution
            Else
                Solve equation for complex roots solution
          End if
      Else if form of Nonhomo → Trigonometric then
        Read values for trigonometric equation
          If Discriminant → 0 then
                Solve equation for equal roots solution
            Else if Discriminant > 0 then
                Solve equation for real and distinct roots solution
            Else
                Solve equation for complex roots solution
          End if
      Else if form of Nonhomo → Polynomial then
        Read values for polynomial equation
          If Discriminant → 0 then
                Solve equation for equal roots solution
            Else if Discriminant > 0 then
              Solve equation for real and distinct roots solution
          Else
              Solve equation for complex roots solution
        End if
    End if
Else if type of Nonhomo Eqns. → Nonhomo IVP then
    If form of Nonhomo IVP → Constant then
      Read values for constant equation
        If discriminant → 0 then
              Solve equation for equal roots solution
          Else if Discriminant > 0 then
              Solve equation for real and distinct roots solution
          Else
```

```
                    Solve equation for complex roots solutions
                End if
            Else if form of Nonhomo IVP → Exponential then
              Read values for exponential equation
                If Discriminant → 0 then
                    Solve equation for equal roots solution
                  Else if Discriminant > 0 then
                    Solve equation for real and distinct roots solution
                  Else
                    Solve equation for complex roots solution
                End if
            Else if form of Nonhomo IVP → Trigonometric then
              Read values for trigonometric equation
                If Discriminant → 0 then
                    Solve equation for equal roots solution
                  Else if Discriminant > 0 then
                    Solve equation for real and distinct roots solution
                  Else
                    Solve equation for complex roots solution
                End if
            Else if form of Nonhomo IVP → Polynomial then
              Read values for polynomial equation
                If Discriminant → 0 then
                    Solve equation for equal roots solution
                  Else if Discriminant > 0 then
                    Solve equation for real and distinct roots solution
                  Else
                    Solve equation for complex roots solution
                End if
          End if
      End if
ELSE IF type of Eqns. → Homo then
```

```
                If type of Homo Eqns. → Homo then
                   Read values for homogeneous equation
                      If Discriminant → 0 then
                            Solve equation for equal roots solution
                        Else if Discriminant > 0 then
                            Solve equation for real and distinct roots solution
                        Else
                            Solve equation for complex roots solution
                      End if
                  Else if type of Homo Eqns. → Homo IVP then
                    Read values for homogeneous IVP equation
                      If Discriminant → 0 then
                            Solve equation for equal roots solution
                        Else if Discriminant > 0 then
                            Solve equation for real and distinct roots solution
                        Else
                            Solve equation for complex roots solution
                      End if
                End if
          End if
      Print solution to equation
  End
```

The above algorithm is illustrated in the flow chart below:

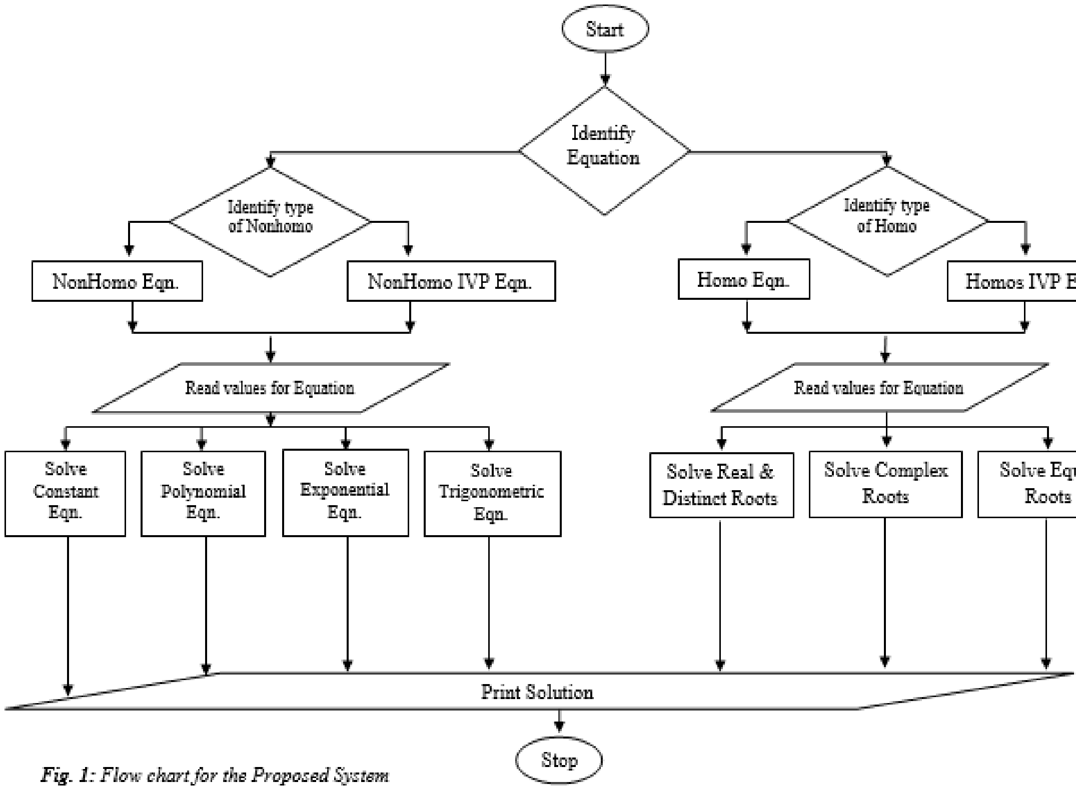

Fig. 1: Flow chart for the Proposed System

System Design

Use Case Diagram for the proposed system

Below is the use case for the proposed system.

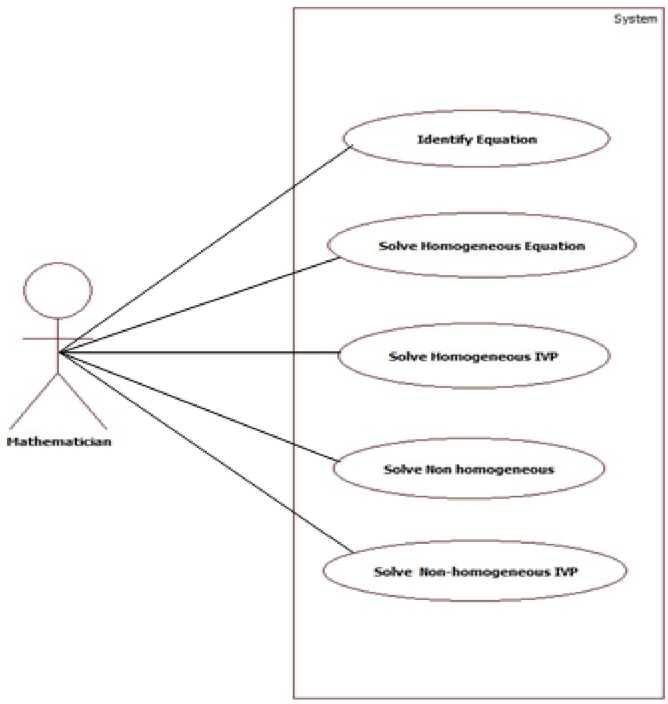

Fig. 2.0: Use Case Diagram for the proposed System

4. CODING, TESTING AND IMPLEMENTATION OF THE PROPOSED SYSTEM

The program was developed using Visual Basic 6.0 version. Visual basic offers a graphical user interface (GUI) for all programs. It makes creation of widow-based application easy. The software was tested using unit testing as well as system testing. The software provides solutions to second order linear ordinary differential equations with constant coefficients.

5. CONCLUSION AND RECOMMENDATIONS

It is pertinent to note that the transformation of the traditional method of solving second order linear ordinary differential equation with constant coefficients into an automated system is important and necessary due to the reliability, accessibility, accuracy, speed and efficiency provided by the automated system. This can only be manifest if the automated system meets its requirement. It is also apparent that despite the advantages that will be gained by the use of the automated system; there would still be few persons who would prefer the traditional system. Helping them to understand the advantages of this new system would go a long way.

The researcher of this paper recommends that the developed system be implemented in tertiary institutions for the purpose of teaching and learning, especially for students offering Mathematics and Computer Science as well as lectures in these fields. This research work has some limitations and it is recommended that further work be done in order to meet these limitations.

Conflict of interests

Author declare no conflict of interest.

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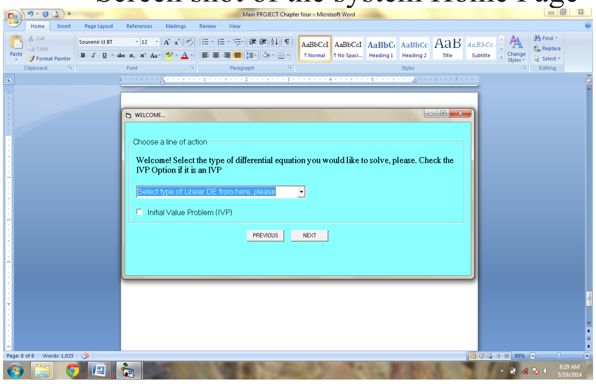
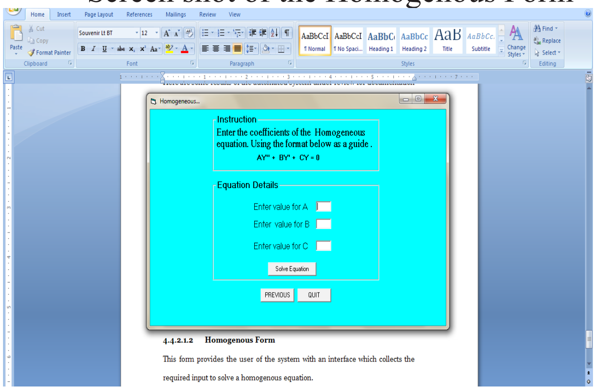
Appendix

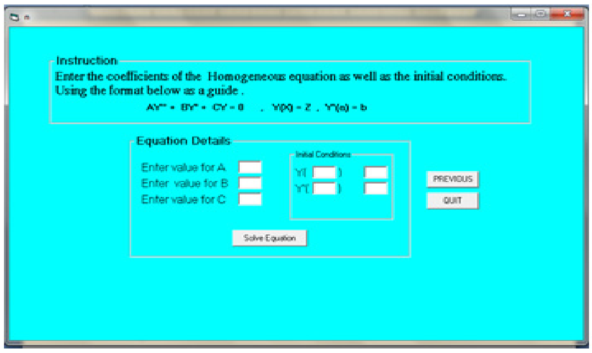
Screen shot of the system Home Page



Screen shot of the Homogenous Form



Screen shot of the Homogenous Initial Value Problem (IVP) Form



Screen shot of the Non-Homogenous Form

Screen shot of the Non-Homogenous Initial Value Problem (IVP) Form