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The boundary integral equation based method for damages detection in multilayered elastic structures

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ABSTRACT

For two kinds of defects – delamination and transversal failure in the multilayer composite the damages detection method is proposed. The inverse geometrical problems of crack's parameters reconstruction were formulated as a sequence of boundary integral equations (BIE). These boundary integral equations were derived on the basis of the principle of works mutuality applied to the problem of the composite specimen stationary oscillations. To solve the integral equations the finite element method, the boundary element method, and the Tikhonov regularization method were used. As complementary information for inverse problems solving the displacement field measured on a mechanical stress free area of the specimen's surface was served. The numerical examples of the interfacial cracks and transversal failure of the internal layers in the multilayered composite were considered in the framework of two-dimensional elasticity.

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1. Introduction

The model of linear elastic body with cut and negligible interaction between cut's coasts can be successfully used for describing the solid bodies with cracks. Then boundary conditions for stress on the both flaw sides must be set as zero stress condition. In the framework of such linear model a problem of body stationary oscillations can be formulated. Solution of this problem makes possible use of a measured vibration displacement field on the free surface of body for damage parameters identification. The most natural statement of these problems is implemented for the cases when the section, which contains a defect is known; in a common case this section may be non planar. For plane cross-section the problem of defect parameters identification was considered in (Bannour et all., 1997). The problem of plane flaws identification at the harmonic excitation of elastic body was resolved by authors early (Vatulyan and Soloviev, 2003). Analytical methods developed in the mentioned works, require of measuring both stress and displacements on the whole body surface. An identification of interfacial flaws on the internal boundaries of compound elastic body refers to the same kind. Some methods of such inverse problems resolve have been developed. So, in (Vatulyan, 2003; Vatulyan et al., 2000) the method of non-classical BIE was proposed. W. Weikl with coauthors (Weikl et al., 2001) were used the iterative algorithm based on the approach proposed in (Kozlov et al., 1991). Applying to the problem of planar cracks reconstruction a semi-explicit algorithm for Laplase equation has been developed by T. Bannour with coauthors (Bannour et al., 1997).

In order to use of these methods in practical applications it is necessary to provide the measuring of displacement fields not on the whole boundary, but on its free part only. On the base of such a statement we propose three BIE-based approaches to the problem of defects identification. First approach uses statement of BIE relative to displacement jumps on the cracks, and second one use stress jump on internal boundaries containing

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interlayer delaminations and transversal fractures. Defects reconstruction is conducted step-by-step, sequentially from the first external layer to the next one moving into the body's depth from the boundary on which the measurements are made. Next we derive the BIE system by application of mutuality theorems for bodies with defects and without it, which reconstruction is implemented on the previous steps. All constructed equations are the 1st kind Fredholm integral equations with smooth kernel, and therefore their resolving requires the regularization techniques. Here we use a combination of a Finite Element Method (FEM), Boundary Element Method (BEM), the known Tikhonov's regularization techniques (Tikhonov and Arsenin, 1979), and also approaches that has been proposed in (Vatulyan and Soloviev, 2004).

2. Inverse Problem Statement

In the Cartesian coordinate system $Ox_1x_2x_3(\underline{x} = (x_1, x_2, x_3))$ consider a laminated body located in the area $= \bigcup_{k=1}^{K} * V_k$. This body is bounded by a surface S and divided by interfacial surfaces S_{int} into k subdomains V_k. Onto surface S a partition $S = S_U \bigcup S_t \bigcup S_{fr}$ is defined, and $S_U \bigcap S_t = S_U \bigcap S_{fr} = S_t \bigcap S_{fr} = \theta$ (i.e. all subsets not disjointed). An acting stress vector p is defined on the surface S_t, and surface S_{fr} is free from the external stress. On the surface S_0 a subset $S_0 \subset S_{fr}$ is available for measurement of displacement vector. Let considered body, which contains several separated cracks $\Gamma = \bigcup_{q=1}^{M} \Gamma_q$, $(\Gamma_q = \Gamma_q^{(+)} \cup \Gamma_q^{(-)})$ that are located between the nearest layers S_{int} - delaminations case (see. Fig.1).

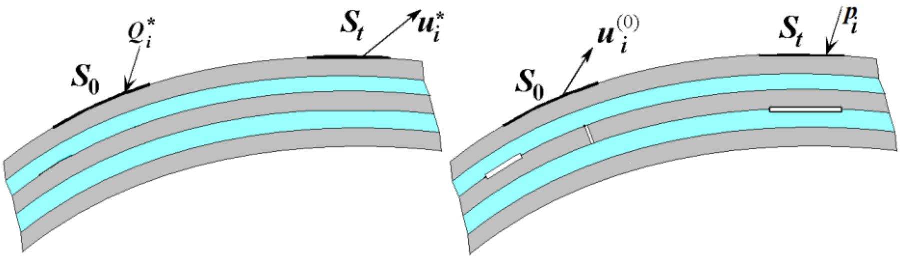


Fig. 1. Defects free (left) and defected (right) bodies.

At the stationary harmonic oscillations the boundary value problem for determination of stress-strain state and geometry of the cracks is formulated using the equations of linear elasticity (Nowacki, 1970).

$$\sigma_{ij,j}^{(k)} = -\rho\omega^2 u_i^{(k)}, k = 1,2, \ldots, K \quad \underline{x} \epsilon V_k, \tag{1}$$

$$\sigma_{ij}^{(k)} = c_{ijml}^{(k)} u_{m,l}^{(k)}, \tag{2}$$

the boundary conditions for the forward problem

$$u_i^{(k)}|S_u = 0, t_i^{(k)}|S_t = \sigma_{ij}^{(k)} n_j|S_t = p_i, t_i^{(k)}|S_{fr} = 0, \tag{3}$$

the definition of continuity constraints on

$$u_i^{(k)}|S_{int} \setminus \Gamma = u_i^{(k+1)}|S_{int} \setminus \Gamma, \ t_i^{(k)}|S_{int} \setminus \Gamma = t_i^{(k+1)}|S_{int} \setminus \Gamma, \tag{4}$$

conditions on the cracks sides

$$t_i^{(k)}|\Gamma_q^{\pm} = 0, \ q = 1,2, \ldots, M, \tag{5}$$

and supplementary conditions, corresponded to measuring of displacement vector on S_0

$$u_i^{(k)}|S_0 = u_i^0. \tag{6}$$

In the expressions (1) - (6) $\sigma_{ij}^{(k)} = c_{ijml}^{(k)}$, are the stress and elastic constant tensors components respectively, $u_i^{(k)}$ are displacement vector components, $\rho^{(k)}$ is material density, ω is an angular frequency, n_j are components of unit normal vector to the corresponding surfaces, and $t_i^{(k)}$ are i-th components of stress vector applied to the surface.

Remark 1. In experimental applications the displacement vector distribution is defined most often not everywhere onto S_0, but on some discrete set of points. These points corresponds to the locations of the sensors, so, Eq. (5) must be replaced by

$$u_i^{(k)}(\underline{x}_m) = u_{im}^0, x_m \in S_0, \ m = 1,2, \ldots, M. \tag{7}$$

3. The Auxiliary Problems

3.1. *Problem I*

Let us consider the problem I for defects free body V^0. Then boundary conditions on the inner surface S_{int} correspond to the continuity of displacements and stress vectors. The problem statement consists of differential Eq. (1), (2) for $u_i^{(k)*}$, $\underline{x} \in V_k$, the boundary conditions

$$u_i^{(k)*}|S_u = 0, \ t_i^{(k)*}|S_t = \sigma_{ij}^{(k)*} n_j|S_t = 0, \tag{8}$$

$$t_i^{(k)*}|S_0 = \sigma_{ij}^{(k)*} n_j|S_0 = Q_i^*\left(\underline{x}, \underline{\xi}\right), \ \underline{x}, \underline{\xi} \in S_0, \tag{9}$$

and continuity constraints on S_{int}

$$u_i^{(k)}|S_{int} = u_i^{(k+1)*}|S_{int}, \ t_i^{(k)*}|S_{int} = t_i^{(k+1)*}|S_{int}. \tag{10}$$

3.2. *Problem II*

Next consider the boundary value problem II for body $V^2 = \bigcup_{k=1}^{K_1} * V_k$ at $K_1 < K$ the same load acting on S_0; and surface $S_{int}^{K_1}$ of last layer is stress free. At first we study this problem assuming the defects free body V^2.

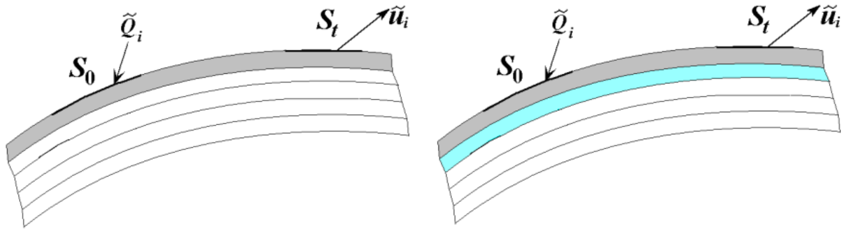


Fig. 2. The sequence of auxiliary problems for $K_1 = 1$ (left), and for $K_1 = 2$ (right).

The problem statement consists of differential Eq. (1), (2) for $\tilde{u}_i$, , $\underline{x} \in V^2$ the boundary conditions

$$\tilde{u}_i|S_u = 0,\ \tilde{t}_i|S_t = \tilde{\sigma}_{ij}n_j|S_t = 0\ , \tag{11}$$

$$\tilde{t}_i|S_u = 0,\ t_i|S_t = \tilde{\sigma}_{ij}n_j\, \Big|S_0 = \tilde{Q}_i\left(\underline{x},\underline{\xi}\right),\ \ \underline{x},\underline{\xi} \in S_0\ , \tag{12}$$

and $S_{int}^{K_1}$ condition when is stress free

$$t_i|S_{int}^{K_1} = 0\ . \tag{13}$$

3.3. *Problem III*

This boundary value problem is considered for a body $V^3 = \bigcup_{k=1}^{K_1-1} V_k \cup \hat{V}_{K_1}$, $K_1 < K$ which undergoes the same loads on the surface S_0. The surface $S_{int}^{K_1}$ (see Fig. 3, right) is wholly belongs to the K_1-th layer. Three types of conditions can be set on this surface: 1st – fixed surface, 2nd – sliding contact (restricted normal displacements and zero tangential stress), and 3rd – restricted tangential displacements and zero normal stress. At first we study this problem assuming the free defects body V^3.

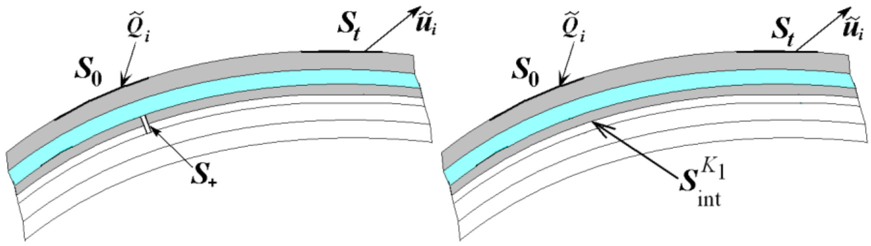


Fig. 3. The 3rd auxiliary problem (right K_1=3).

The problem statement consists of differential equations (1), (2) for $\hat{u}_i$, $\underline{x} \in V^3$, the boundary conditions

$$\hat{u}_i|S_u = 0,\ \hat{t}_i|S_t = \hat{\sigma}_{ij}n_j|S_t = 0\ , \tag{14}$$

$$\hat{t}_i|S_0 = \hat{\sigma}_{ij}n_j|S_0 = \hat{Q}_i\left(\underline{x},\underline{\xi}\right),\ \ \underline{x},\underline{\xi} \in S_0\ , \tag{15}$$

and also one from three boundary conditions types $S_{int}^{K_1}$, e.g. for 1st type of boundary conditions

$$\hat{u}_i|S_{int}^{K_1} = 0\,. \tag{16}$$

Remark 2. Here we assume that ξ dependence of functions $Q_i^*\left(\underline{x},\underline{\xi}\right)$, $\tilde{Q}_i\left(\underline{x},\underline{\xi}\right)$ can be considered as one parameter set

$$Q_i^*\left(\underline{x},\underline{\xi}\right) = P_i^*\delta\left(\underline{x}-\underline{\xi}\right) \quad \tilde{Q}_i\left(\underline{x},\underline{\xi}\right) = \tilde{P}_i\left(\underline{x}-\underline{\xi}\right). \tag{17}$$

Remark 3. The technique of BIE design with the aid of the problem II or problem III does not changed if the bodies V^2 and V^3 contain known imperfections (possibly identified on the previous step).

4. Derivation of BIE on the Basis of Auxiliary Problems Solution

4.1. Derivation of BIE on the basis of problem I solution

Let's consider the multilayered material with the presence of some delaminations and absence of any transversal failures. Assuming that the problem I is resolved denote

$u^{(k)*}|S_t = \phi^*\left(\underline{x},\underline{\xi}\right),\ \underline{x} \in S_t\,,\ \ \underline{\xi} \in S_0\,.$

Regarding an operator

$$G\left(\underline{u}^0,\underline{p},\underline{\phi}^*,\underline{Q}^*\right) = \int_{S_0} u_i^0(\underline{x}) Q_i^*\left(\underline{x},\underline{\xi}\right) dS_x - \int_{S_t} \phi_i^*\left(\underline{x},\underline{\xi}\right) p_i(\underline{x}) dS_x = G_1\left(\underline{\xi}\right), \tag{18}$$

we apply the theorem of works mutuality (Nowacki, 1970) to the bodies V and V^1. Taking into account the continuity conditions (4) on $S_{int}\backslash\Gamma$ and (10) on the S_{int}, such equation can be obtained

$$\int_\Gamma t_i^{(k)*}\left(\underline{x},\underline{\xi}\right) X_i(\underline{x}) dS_x = G_1\left(\underline{\xi}\right), \tag{19}$$

where $X_i(\underline{x})$ - the jumps of displacement vector on the cracks. It is important to mark that on $S_{int}\backslash\Gamma$ these jumps have zero value. Entering a designation

$$X_i(\underline{x}) = \begin{cases} X_i & at\ \underline{x} \in \Gamma \\ 0 & at\ \underline{x} \in S\backslash\Gamma \end{cases},$$

into relationship (19) transform it to the system of integral equations for functions $X_i(\underline{x})$, which are defined on $\underline{x} \in S_{int}$.

$$\int_{S_{int}} t_i^{(k)*}\left(\underline{x},\underline{\xi}\right) X_i\left(\underline{x}\right) dS_x = G_1\left(\underline{\xi}\right), \xi \in S_0\,, \tag{20}$$

where integration spread over the known surface . It is clear that last equation can be used for the cracks shape reconstruction at variation of excitation frequency and load distribution.

4.2. Derivation of BIE on the basis of problem II solution end examine the integral operator

As well as in the previous section we consider the multilayered material with the presence of some delaminations and absence of any transversal failures. Let auxiliary problem II is resolved. Then denote

$\tilde{u}^{(k)}|S_t = \tilde{\phi}\left(\underline{x},\underline{\xi}\right), \underline{x} \in S_t\,,\ \ \underline{\xi} \in S_0\,,$

and examine the integral operator

$$F\left(\underline{u}^0,\underline{p},\underline{\phi}^*,\underline{Q}^*\right) = \int_{S_0} u_i^0(\underline{x}) Q_i^*\left(\underline{x},\underline{\xi}\right) dS_x - \int_{S_t} \phi_i^*\left(\underline{x},\underline{\xi}\right) p_i(\underline{x}) dS_x = F_1\left(\underline{\xi}\right). \tag{21}$$

Applying the theorem of works mutuality (Nowacki, 1970) to the body, which occupy the subdomain V^2 we can obtain

$$\int_{S_{int}^{K_1}\backslash\Gamma} \left(\underline{x},\underline{\xi}\right) t_i^{(K_1)}\left(\underline{x}\right) dS_x = F_1\left(\underline{\xi}\right). \tag{22}$$

Taking into account that multipliers under integral (22) are defined everywhere on $S_{int}^{K_1}$, and because of (5)

$$t_i^{(K_1)}|\Gamma = 0\,, \tag{23}$$

the relationship (22) can be treated as BIE system on the known boundary S

$$\int_{S_{int}^{K_1}} \left(\underline{x},\underline{\xi}\right) t_i^{(K_1)}\left(\underline{x}\right) dS_x = F_1\left(\underline{\xi}\right);\ \xi \in S_0\,, \tag{24}$$

It is necessary to mark herein an important difference between the system (20) and (24). Identification based on the Eq. (20) use that searched variables are not zero inside the cracks area. Unlike the previous case the crack reconstruction using Eq. (24) assume zero value of stress vector components on the crack's coasts, and also their singular behavior at come to the crack coast out of intact area. This singularity appears numerically as a rapid growth of these components.

4.3. Derivation of BIE on the basis of problem III solution

Now consider the multilayered material with the presence of some transversal fracture and absence of any delaminations. Let auxiliary problem III is resolved. Then denote

$F\left(\underline{u}^0,\underline{p},\underline{\phi}^*,\underline{Q}^*\right)\hat{u}^{(k)}|S_t = \hat{\phi}\left(\underline{x},\underline{\xi}\right),\ \underline{x} \in S_t,\ \underline{\xi} \in S_0$

Considering operator

$$T\left(\underline{u}^0,\underline{p},\underline{\phi}^*,\underline{Q}^*\right) = \int_{S_0} u_i^0(\underline{x}) Q_i^*\left(\underline{x},\underline{\xi}\right) dS_x - \int_{S_t} \phi_i^*\left(\underline{x},\underline{\xi}\right) p_i(\underline{x}) dS_x = T_1\left(\underline{\xi}\right), \tag{25}$$

and applying the mutuality theorem [9] to the body, which locates in a subdomain V^3, we shall obtain

$$\int_{S_{int}^{K_1}} \hat{t}_i^{(k)}\left(\underline{x},\underline{\xi}\right) u_i\left(\underline{x}\right) dS_x + \int_{S_+}^{\hat{t}_i^{(k)}} \left(\underline{x},\underline{\xi}\right) X_i\left(\underline{x}\right) dS_x = T_1\left(\underline{\xi}\right), \tag{26}$$

where $X_i(\underline{x})$ - jumps of displacement vectors on the transversal cracks (ply fracture). However, in practice, using of Eq. (26) is impossible because of unknown boundary S_+. Therefore we use a surface $S_{int}^{K_1}$, which is located in adjacency to the boundary separating the layers $K_1 - 1$ and K_1. As S_+ is a small quantity we can neglect second summand in Eq. (26) in the first time. Then Eq. (26) is substituted by

$$\int_{S_{int}^{K_1}} \hat{t}_i\left(\underline{x},\underline{\xi}\right) u_i^{(1)}\left(\underline{x}\right) dS_x = T_1\left(\underline{\xi}\right). \tag{27}$$

It is worth to notice that at presence of transversal fractures and at successfully selected oscillation frequency the functions $u_i^{(1)}$ are piecewise continuous, and the points of discontinuity coincide with surfaces intersection. So, it is possible to use of Eq. (26) at the second stage with variation of S_+ position relative to its initial position founded on the first stage.

5. Frequency Scanning and Cracks Identification

If a used experimental setup and tested structure do not restrict of the scanning frequency range, it is desirable to expand this range so that it includes as much as possible of structure's eigenfrequencies. Then functions in eq. (20) and (24) will be defined in the following frequency domain

$$\omega \in \Omega = \cup_{n=1}^{N} \Omega_n , \; \Omega_n = \left[\omega_n^{(b)}, \omega_n^{(e)}\right] ,$$

where intervals Ω_n can be chosen after the preliminary modal analysis of defects free structure. The selected frequency intervals should include those vibration modes, at which there is an intensive opening of cracks. Usually, it is the tensile-compression or shear deformations modes in a neighborhood of a surface S_{int}, while the flexural modes are less sensitive to existence of the crack-like imperfections.

Now the system (20) can be rewritten in the form

$$\int_{S_{int}} t_i^{(k)*} \left(\underline{x}, \underline{\xi}, \omega\right) X_i \left(\underline{x}, \omega\right) dS_x = G_1 \left(\underline{\xi}, \omega\right), \; \xi \in S_0,$$
$$\omega \in \Omega . \qquad (28)$$

As the carrier (crack's geometry) of functions $X_i\left(\underline{x}, \omega\right)$ does not depend on ω, the solution of the equation (28) in a frequency set Ω allows achieving a high accuracy of the cracks numerical reconstruction. Let's mark here that function $G_1(\xi, \omega)$ equal to zero everywhere in Ω if the body has no cracks. This circumstance gives a simple means of cracks detection even if the surface S_{int} is a priory unknown. For this purpose it is enough to choose any scheme of loading in a problem I, for instance, scheme (9) at $\underline{\xi} = \underline{\xi_k}$ and to construct a frequency response

$$G_2(\omega) = G_1 \left(\underline{\xi_k}, \omega\right), \quad \omega \in \Omega .$$

At known precision of the displacement field u_i^0 (see Eq. 6) measurements, and known exactitude of integrals (15) calculation one can conclude a crack's occurrence by form of this frequency response.

At the frequency scanning the system (28) is replaced by the following system

$$\int_{S_i^{K_1}} \tilde{u}_i^{(k)} \left(\underline{x}, \underline{\xi}, \omega\right) t_i^{(K_1)} \left(\underline{x}, \omega\right) dS_x = F_1 \left(\underline{\xi}, \omega\right), \; \xi \in S_0,$$
$$\omega \in \Omega , \qquad (29)$$

where the principle of geometry Γ reconstruction does not depend on frequency ω , as well as for (28).

Remark 4. As the functions $\underline{Q}^*$, $\underline{\tilde{Q}}$ are defined on a discrete set of measuring points x_m, the integrals in (18) and (21) can be calculated explicitly.

6. Numerical Examples of Cracks Parameters Reconstruction

6.1. Example 1

At first consider the problem of identification of two longitudinal cracks Γ_1 =KL and Γ_2 =MN located between two layers of semi-passive bimorph that has a rectangular shape AOBCED [A(0;-0.03), O(0;0), B(0;0.03), C(0,1;0.03), E(0.1;0), D(0.1;-0.03), K(0.03;0), L(0.05,0), M(0.065;0), N(0.08;0)]– all dimensions in meters. The upper layer OBCE is made of Cu, and lower layer AOED – made of piezoelectric ceramic PZT-4. The present resolving scheme leads to Eq. (21), which is resolved in the framework of plane strain. The forced oscillations at frequency $f = \left(\omega/(2\pi)\right) = 20$ kHz are excited by alternating voltage $V_0 = 1000$ V applied to electrodes located on the sides OE and AD. The boundary AB is fixed; the remaining exterior boundaries are stress free.

All numerical simulations were implemented using FEM package ACELAN (Belokon, 2000). The cracks were modeled by holes which transversal size is sufficiently short comparing to their length (~ 10-7). It was supposed that the crack walls do not interact. Near the crack tip a FEM mesh was essentially condensed. The side BC was accessible to measurements of the displacement vector, and on this side 39 interior equidistant nodes were selected, in which displacements were calculated. The excitation frequency was chosen from reasons of intensive crack opening, but it should differ from the natural frequencies of a boundary value problem II. These natural frequencies are shown in Table 1.

Table 1. The first eigenfrequencies of the studied semi-passive bimorph (kHz).

Eigenfrequency No	Free of defects	Boundary problem II
1	2.851	1.721
2	9.194	7.946
3	9.858	9.575
4	19.01	17.51
5	22.32	27.18
6	24.16	27.88

BIE (24) was resolved on the basis of the Boundary Element method. For discretization of Eq. (24), a piecewise constant and piecewise linear continuous approximation of unknown depending variables was used. However, our numerical experiments have shown that the first scheme is more preferable. The discrete analog of Eq. (24) was resolved by the Tikhonov regularization method (Tikhonov and Arsenin, 1979) and regularization parameter for *n*=40 was accepted 2·10-14 (*n*- number of points, in which displacements were "measured").

Some obtained results are present in Figs. 4 and 5. Fig. 4 demonstrates the stress amplitude distributions along a sample with one crack KL. Two extremes of stress there correspond to coordinates of the crack's tips. The diagrams of a stress distribution in a specimen with two cracks are present on a Fig. 4, where for comparison the graphs created by the solution of the problem (24) and FEM solution results are displayed. The tips of both cracks also easily identified.

In the performed numerical experiment the influence of the number *n* "measured" points, which provide the input information, on the precision of crack reconstruction was also studied. Relative errors of the crack tips determination at different number of the "measured" points on a side BC at distance 0.25 mm between adjacent points are present in Table 2.

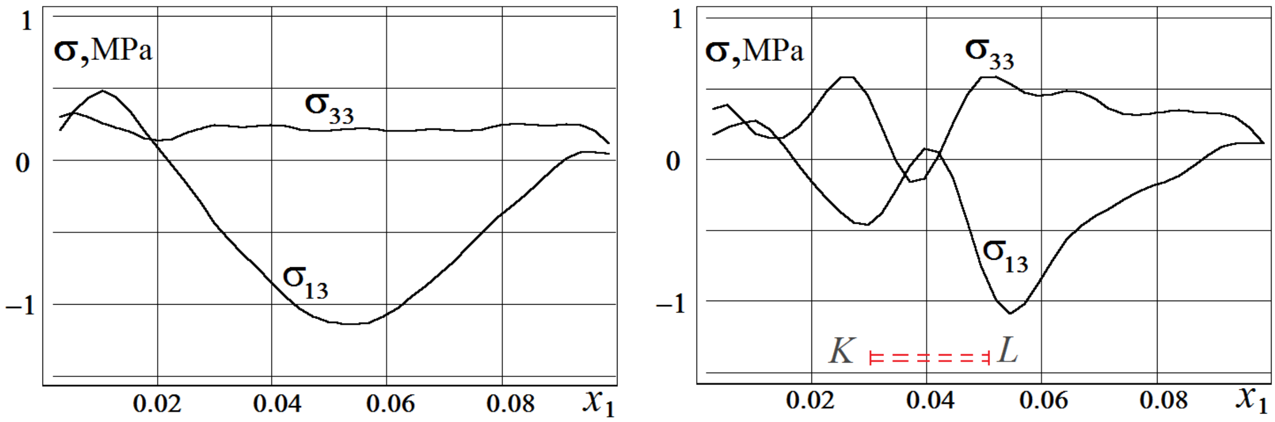


Fig. 4. The stress distributions in the intact specimen (left diagram) and in the specimen with interfacial crack KL (results of the inverse problem (24) solution).

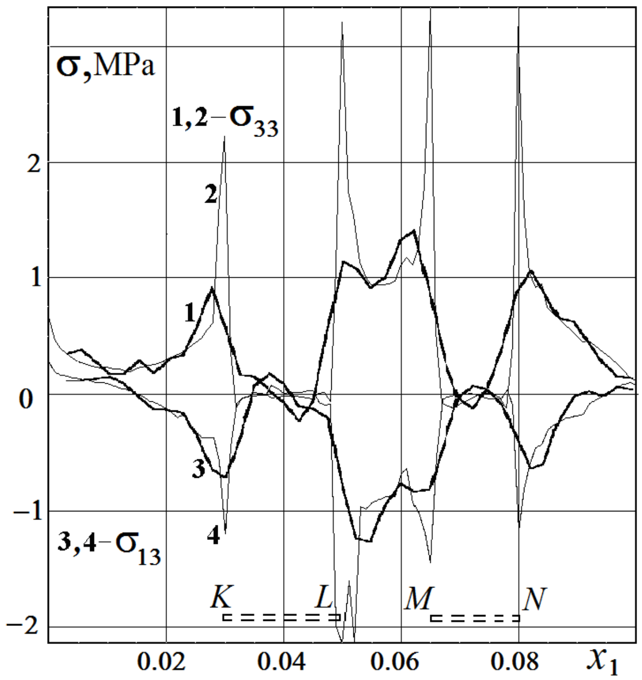


Fig. 5. The stress distributions in a specimen with two interfacial cracks KL and MN (bold lines – FEM solution results; subtle line – BIE solution result).

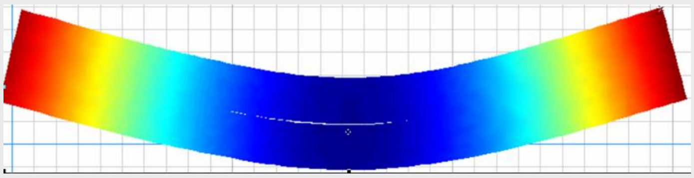
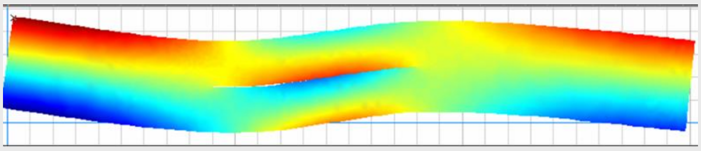
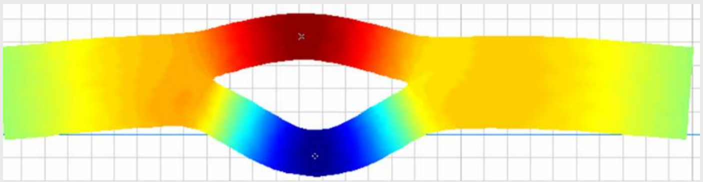
Table 2. Relative tolerance of crack's tip coordinate identification (in percents).

Number of points n	x_K	x_L	x_M	x_N
10	5.0	6.4	3.8	5.5
20	4.2	3.6	2.0	3.3
30	4.2	2.5	1.9	3.1
40	4.2	2.5	1.9	3.1

6.2. Example 2

In this example the interface delamination in free bimorph (Fe, Cu) plate (0.3x0.04 м) is identified. From the Table 3, where the first three natural frequencies and vibration modes are shown, one can see that the frequency near the first flexural mode is not suitable for the efficient crack reconstruction. On the second eigenfrequency the principal role a horizontal displacement of crack's side is played, and on the third mode – crack opening along a vertical direction. For this crack identification the Eq. (20) was used. The discretization of the searched functions - discontinuities of the crack's coasts displacements by 10 piecewise constant elements was implemented. Fig. 6 demonstrates the found jumps of horizontal displacements (left graph) near the second eigenfrequency (16000 rad/s), and vertical displacement (right graph) near the third eigenfrequency.

Table 3. Eigenfrequencies and natural oscillation modes, which are used for the longitudinal crack (local delamination) identification in the bimorph plate.

Eigenfrequency, kHz		Natural oscillation mode shape
1.82	Vertical displacements	
2.57	Horizontal displacement	
5.97	Vertical displacements	

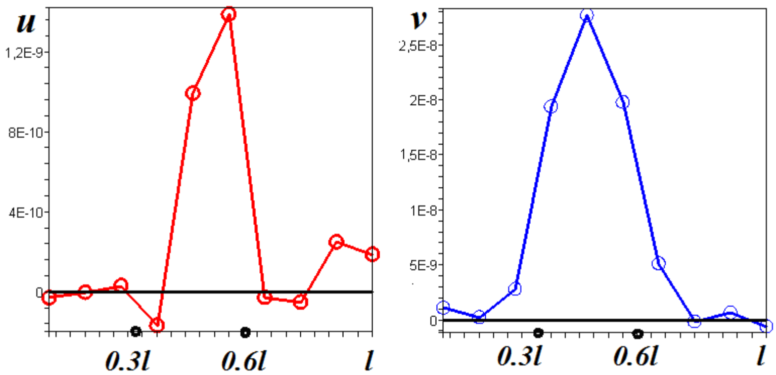
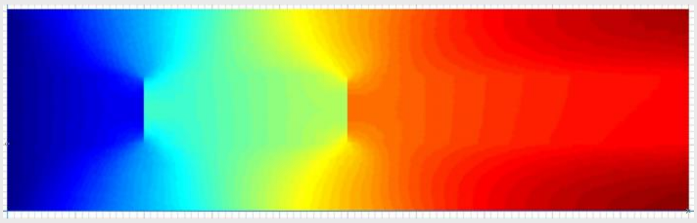
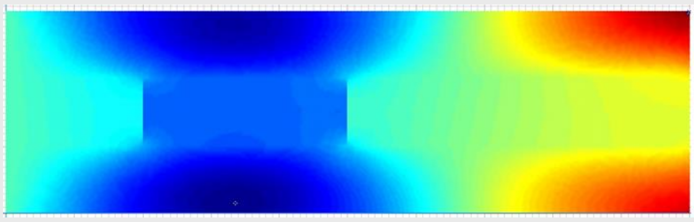


Fig. 6. The jumps of horizontal (left) and vertical (right) displacements on the crack's coasts (explanation in text).

6.3. Example 3

Next example illustrates an application of BIE derived in sub-section 4.3 for reconstruction of two separated transversal fractures in intermediate layer of three-ply laminate. Coordinates of fractures are x=0.02 and x=0.05 (see Table 4). Preliminary FEM analysis has shown that on the natural frequency 20.59 kHz, which provides the major amplitude of horizontal displacements, there an intensive opening of cracks is. Fig. 8 demonstrates the charts of horizontal displacement on a center line of interior layer at harmonic oscillation near to eigenfrequency. Observed displacements discontinuities on the charts are caused by the presence of two cracks. On both charts the continuous bold lines present the solution obtained using finite element method, whereas the light lines with small circles – solutions obtained by Boundary Element method implemented using GMRES algorithm (Rjasanow and Steinbach, 2007). The left plot in Fig. 7 demonstrates solution of the Eq. (27), and right plot - specified solution of the Eq. (26). BEM based solutions are implemented by using of piecewise continuous approximation, which is the most preferable for discontinuous solutions. One can see that location of both cracks is reconstructed with the satisfied precision.

Table 4. Eigenfrequencies and natural oscillation modes, which are used for two transversal crack identification in the in the three-layered plate.

Eigenfrequency, kHz		Natural oscillation mode shape
7.11	Horizontal displacement	
20.59		

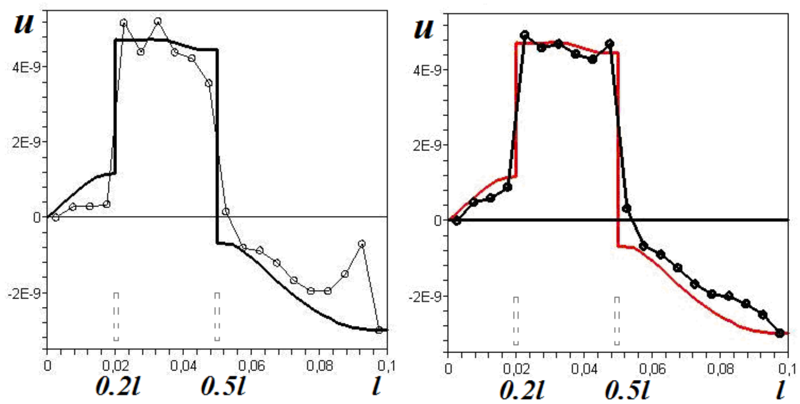


Fig. 7. The results of two transversal fracture reconstructions in the intermediate layer of three-ply laminate (explanation in text).

7. Conclusions

The boundary integral equation method is proposed and effectively used to solve the problem of longitudinal and transversal crack identification in multilayered elastic structures. These boundary integral equations were derived using the principle of works mutuality applied to the problem of stationary oscillations of the multilayered elastic structures. In order to excite these oscillations into a broad frequency range, which contains some eigenfrequencies of the studied structure, the local sources of harmonical excitation are provided. These sources can be disposed on some part of tested structure's surface. As input information for the defects identification problem the displacements, which are measured on the discrete set of points belonging to the stress free area of the tested specimen's surface is used.

- Precision of the crack's tips coordinates determination essentially depends on location of the displacement field measurements area. Therefore at the use of discrete set of points for measuring of displacement field (the positional probe) their location should be extended on the overall accessible area. The obtained overdetermined equations system can be resolved by application of some regularization scheme.
- If the choice of oscillation frequencies is not restricted by a measuring apparatus and by specificity of a tested structure, it is necessary to perform the preliminary numerical modal analysis with the purpose of operational frequencies selection. It is necessary to select those frequencies, which cause intensive motions of a material close to guessed imperfections. It is possible also to expand the resolving linear equation system by scanning in a broad frequencies band (frequency scanning).
- To increase the reliability of the identification results in the areas with high gradients of the solution, the iterative reconstruction scheme with sequentially finer finite element mesh is desirable.
- Use of the step by step reconstruction scheme allows identifying multiple imperfections located in the different layers of a multilayered composite material.

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