

Mathematical modeling and optimization of convective drying process of wood

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Abstract. In the paper there is described solving a current scientific problem of mathematical modeling of heat-mass transfer processes and elastic-viscous-plastic deformation taking into account the mechanics and sorption creep in hygroscopic capillary-porous materials with variable anisotropic heat and mechanics characteristics. This finite elements method has been developed for the research of the two-dimensional anisotropic intense-deforming state during the capillary-porous materials drying process in an elastic-viscous-plastic area of deformation taking into account the mechanics and sorption creep. There has been elaborated the applied software which consists of the documented classes and provides the possibility to automate the finite-elemental analysis of timber intense-deformation state during drying process.

Carried out numerical implementation of the mathematical model for the stress-strain state and temperature and moisture fields in the process of drying capillary-porous materials and formulated optimization problem. The analysis function selection for the purpose of building technological modes of drying capillary-porous materials. The method of construction and investigated continuous modes of drying by solving the optimization problem.

Key words: mathematical model, the method of finite elements, heat and mass transfer, elastic-viscous-plastic state, optimization problem, drying process.

Actuality of theme

During drying of hygroscopic capillary-porous bodies irreversible physical processes arise up and cooperate between itself, there are phase transitions, which predetermine changes in physical mechanical properties of materials in general, initial form of body, formation of cracks in it and its possible destruction. These phenomena which take place in the conditions of high changeability of structural and physical properties of hygroscopic materials are basic restrictive factors for development of effective methods of management the capillary-porous materials drying processes. The improvement of existing and creation of new resources saving technologies of drying is closely related to the detailed study deformation-relaxation and heat-and-mass transfer processes in capillary-porous materials. Applying only experimental methods of the deformation run and heat-and-mass transfer research in such systems is connected with considerable financial expenses and technical difficulties. Therefore there is an objective necessity of development of two-dimensional mathematical models and effective programmatic-algorithmic facilities of their realization, which would bind the technological parameters of

drying process to development of the elastic-viscous-plastic state of wood taking into account the anisotropy of heat-mechanic properties and peculiarities of wood deformation mechanisms. Such models will enable to offer more effective, energy saving technologies of capillary-porous materials drying, especially wood.

Analysis of literary sources

The conducted analysis of mathematical models deformation-relaxation processes and methods of their calculation during drying of capillary-porous materials, especially woods, showed that the resilient and viscoelastic area of deformation is mainly investigated in unidimensional and two-dimensional cases for permanent mechanical descriptions, independent of temperature and humidity change [1,2,3,4,5]. Construction of mathematical models of wood deformation during drying taking into account resilient, viscoelastic, plastic deformation and features of their regeneration in a wide turn-down physical mechanical properties of wood is a complicated and not fully decided problem. Various models of idealizing rheological bodies and proper to them differential and integral equalizations were offered and applied for the design of wood standard in the limited intervals of temperature and humidity change indexes.

Taking into account difficult connected with each other processes of deformation and mass transfer during the design of the convective drying of hygroscopic capillary-porous materials substantially complicates mathematical models and requires the improvement of numeral methods for their realization and software development.

The comparative analysis of mathematical models of stress-strain state of wood during drying showed that existent models did not take into account all complex of elastic-viscous-plastic deformation taking into account the anisotropy of heat-mechanic descriptions of material in the conditions of non-isothermal humidity transfer, features of deformation which consider the speed of mass transfer change and the mechanism of deformations regeneration.

Most studies on optimization of processes related to wood drying or optimize various performance cameras drying or drying process automation. And almost no relation to a construction of optimum drying in terms of physical and mechanical processes in the wood, which are crucial to ensure product quality. Most regimes drying constructed empirically, and later improved, optimized for economic or technical criteria. Accordingly known optimization criteria are divided into economic, technical and technological [10, 11].

Synthesis of mathematical model

According to the basic laws of thermodynamics of irreversible processes, mechanics of the inherited environments, contraction of hygroscopic materials, the mathematical model of two-dimensional viscoelastic deformation is formed and heat-and-mass transfer in the process of drying of capillary-porous materials taking into account the anisotropy of heat-mechanical descriptions of material, resilient, viscoelastic, plastic deformations and deformations, caused by a mechanism of mechanical and sorption creep.

System of model equalizations for determination component of deformation vector $\varepsilon = (\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{12})^T$, strains $\sigma = (\sigma_{11}, \sigma_{22}, \sigma_{12})^T$, temperature $T(X, \tau)$ and humidity content $U(X, \tau)$ during drying wooden bar during time $\tau \in [0, \tau_{dry}]$ in the area of cross-cut section $\Omega = \{X = (x_1, x_2); x_1 \in [0, l_1], x_2 \in [0, l_2]\}$ the center of which is combined with beginning of co-ordinates, and the axes of anisotropy coincide with co-ordinate axes, it is built so. The components of tensions vector are satisfied by equalizations of equilibrium with boundary conditions which take into account absence of external efforts:

$$\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} = 0; \frac{\partial \sigma_{12}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} = 0; \sigma_{ij} \Big|_{x_i=l_i, x_j=l_j} = 0. \quad (1)$$

Modeling of connection between the components of tensions and deformations during wood drying is based on Boltzmann-Volterra integral equalizations and laws of contraction of hygroscopic materials [1,6]. For an account mechanical and sorption deformations, predefined speed of change of humidity during drying, the functions of rheological conduct of materials are complemented correlations mechanical and sorption creep. Therefore connection between tensions and deformations taking into account the anisotropy of mechanical properties in material looks like:

$$\begin{aligned} \sigma_{11}(\tau) = & C_{11}(T, U) [\varepsilon_{11}(\tau) - \varepsilon_{v11}] - C_{11}(T, U) \int_0^\tau R_{11}(\tau - s, T, U) \times \\ & \times [\varepsilon_{11}(s) - \varepsilon_{v11}(s)] ds + C_{12}(T, U) [\varepsilon_{22}(\tau) - \varepsilon_{v22}] - \\ & - C_{12}(T, U) \int_0^\tau R_{12}(\tau - s, T, U) [\varepsilon_{22}(s) - \varepsilon_{v22}(s)] ds; \end{aligned}$$

$$\begin{aligned} \sigma_{22}(\tau) = & C_{21}(T, U) [\varepsilon_{11}(\tau) - \varepsilon_{v11}] - C_{21}(T, U) \int_0^\tau R_{21}(\tau - s, T, U) \times \\ & \times [\varepsilon_{11}(s) - \varepsilon_{v11}(s)] ds + C_{22}(T, U) [\varepsilon_{22}(\tau) - \varepsilon_{v22}] - \\ & - C_{22}(T, U) \int_0^\tau R_{22}(\tau - s, T, U) [\varepsilon_{22}(s) - \varepsilon_{v22}(s)] ds; \end{aligned} \quad (2)$$

$$\begin{aligned} \sigma_{12}(\tau) = & 2C_{33}(T, U) \varepsilon_{12}(\tau) - \\ & - 2C_{33}(T, U) \int_0^\tau R_{33}(\tau - s, T, U) \varepsilon_{12}(s) ds, \end{aligned}$$

where $\varepsilon_v = (\varepsilon_{v11}, \varepsilon_{v22}, \varepsilon_{v33})^T$ – a vector component of deformations, predefined the change of wood temperature and humidity content; C_{ij} – components of tensor of resiliency woods, which depend on the temperature and humidity. Values ε_{vi} , C_{ij} are determined with the help of formulas:

$$\begin{bmatrix} \varepsilon_{v11} \\ \varepsilon_{v22} \\ 0 \end{bmatrix} = \begin{bmatrix} \alpha_1 \Delta T + \beta_1 \Delta U \\ \alpha_2 \Delta T + \beta_2 \Delta U \\ 0 \end{bmatrix}; \quad \mathbf{C} = \begin{bmatrix} \frac{E_1}{1 - \nu_1 \nu_2} & \frac{\nu_1 E_2}{1 - \nu_1 \nu_2} & 0 \\ \frac{\nu_1 E_2}{1 - \nu_1 \nu_2} & \frac{E_2}{1 - \nu_1 \nu_2} & 0 \\ 0 & 0 & \mu \end{bmatrix}, \quad (3)$$

where ΔT ΔU – accordingly increase of temperature and humidity content, $E_i(T, U)$ – Young's elastic modulus, $\nu_i(T, U)$ – Poisson's ratio, $\mu(T, U)$ – modulus of shear.

For the modeling of mechanical and sorption deformations, predefined by humidity speed change in wood, such equalizations are used [7]:

$$\frac{\partial \varepsilon_M}{\partial \tau} = m(\sigma - E_m \varepsilon_M(\tau)) \left| \frac{\partial U}{\partial \tau} \right|, \quad (4)$$

where m – is a tensor of mechanical and sorption deformations, which depend on the temperature, in radial and tangential directions of wood anisotropy, the coefficients of which are determined with the help of experimental data.

For the modeling of plastic deformations in wood the Prandtl-Reuss equations of plastic flow is used. The relation between differentials of tensions and deformations for the flat tense state looks like:

$$\begin{aligned} d\sigma_{ij} = & \frac{E}{2(1 + \nu)} \left(d\varepsilon_{ij} + \frac{\nu}{1 - 2\nu} \delta_{ij} d\varepsilon_{kk} - s_{ij} \frac{s_{ke} d\varepsilon_{ke}}{s} \right); \\ s = & \frac{2 - \nu}{3} \sigma^2 \left(1 + \frac{2(1 + \nu)}{3E} \right), \end{aligned} \quad (5)$$

where s_{ij} – deviators of deformation, δ_{ij} – Kronecker's delta.

The functions of rheological conduct of wood during drying taking into account the mechanism of piling up of irreversible deformations choose in a kind:

$$\begin{aligned} R(\tau, U, T) = & \left[a_0 - \sum_{i=1}^M a_i \exp(-b_i \tau) \right] h(\tau) h(\tau_0 - \tau) - \\ & - \left[a_0 - \sum_{i=1}^M \alpha_i \exp(-\beta_i (\tau - \tau_0)) \right] h(\tau - \tau_0) \end{aligned}, \quad (6)$$

where $h(\tau)$ – Heaviside step function, and unknown coefficients a_i , b_i , α_i , β_i are determined by the method of the least squares on the basis of approximation of experimental information of creep of wood samples on-loading and after unloading. They are the functions of temperature and humidity.

A synthesized mathematical model (1)-(6) enables to define stress-strain state of wood during drying taking into account resilient, viscoelastic, plastic deformations and deformations, caused by the mechanism of sorption creep, taking into account savings of irreversible deformations. For its realization needed to determine the temperature and humidity during wood drying, which are in formulas (2), in the tensors of resilient descriptions of \mathbf{C} and mechanical and sorption deformations of \mathbf{m} .

During the convective drying of wood the two-dimensional mathematical model of heat-and-mass transfer taking into account the anisotropy of thermo-physical descriptions is described by the system of

differential equalizations of non-isothermal humidity transfer:

$$c\rho \frac{\partial T}{\partial \tau} = \frac{\partial}{\partial x_1} \left(\lambda_1 \frac{\partial T}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left(\lambda_2 \frac{\partial T}{\partial x_2} \right) + \varepsilon \rho_0 r \frac{\partial U}{\partial \tau};$$

$$\frac{\partial U}{\partial \tau} = \frac{\partial}{\partial x_1} \left(a_1 \frac{\partial U}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left(a_2 \frac{\partial U}{\partial x_2} \right) +$$

$$+ \frac{\partial}{\partial x_1} \left(a_1 \delta \frac{\partial T}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left(a_2 \delta \frac{\partial T}{\partial x_2} \right), \quad (7)$$

Boundary conditions look like:

$$\lambda_i \frac{\partial T}{\partial n} \Big|_{x_i=l_i} + \rho_0 (1 - \varepsilon) \beta_i (U|_{x_i=l_i} - U_p) = \alpha_i (t_c - T|_{x_i=l_i});$$

$$\left(a_i \frac{\partial U}{\partial n} + a_i \delta \frac{\partial T}{\partial n} \right) \Big|_{x_i=l_i} = \beta_i (U_p - U|_{x_i=l_i}); \quad (8)$$

$$\left(\alpha_i \frac{\partial U}{\partial n} + \alpha_i \delta \frac{\partial T}{\partial n} \right) \Big|_{x_i=0} = 0; \quad \frac{\partial T}{\partial n} \Big|_{x_i=0} = 0;$$

$$U|_{\tau=0} = U_0; \quad T|_{\tau=0} = T_0, \quad i = 1, 2.$$

Denotations used here: $T_0(X)$, $U_0(X)$ – primary apportionment of the temperature and humidity content in the material; $U_p(T_c, \varphi)$ – equilibrium humidity; $c(T, U)$ – heat capacity; $\rho(U)$ – density; $\lambda_i(T, U)$ – coefficients of heat-conducting in the directions of anisotropy; ε – coefficient of phase transition; ρ_0 – basic density; r – specific heat of vaporization; $\delta(T, U)$ – thermal-gradient coefficient; $a_i(T, U)$ – coefficients of hydraulic conductivity in the directions of anisotropy; $\alpha_i(T_c, v)$ – coefficient of heat exchange; $\beta_i(T_c, \varphi, v)$ – coefficient of humidity exchange; T_c – environment temperature; $\varphi(\tau)$ and $v(\tau)$ – relative humidity and rate of movement of drying agent; n – vector of external normal of area limit Ω ; τ – current time. During modeling of drying process in the period of the irregular mode the primary apportionment of humidity content in wood is accepted permanent, and in the period of the regular mode initial humidity content changes after a parabolic law.

For numeral realization of mathematical models of connected with each other processes of heat-mass transfer during wood drying (7) – (8) the Finite element method (FEM) is used [8]. Equivalent variation formulation of model is for this purpose got with assumption, that the change of humidity content in time is possible to show as a sum of constituents, related to the stream of mass transfer by the gradient of humidity content and temperature. Eventual system of equalizations for realization of mathematical model (7) – (8) after FEM looks like:

$$[C] \frac{\partial \{U\}}{\partial \tau} + [K] \{U\} + \{F\} = 0;$$

$$[\overline{C}] \frac{\partial \{T\}}{\partial \tau} + [\overline{K}] \{T\} + \{\overline{F}\} = 0, \quad (9)$$

where: $[C] = \int_V \rho_0 [N]^T [N] dV$; $[K] = \int_V [B]^T [D^*] [B] dV + \int_S \rho_0 \beta_w [N]^T [N] dS$; $\{F\} = \int_V [B]^T [H] [B] [T] dV - \int_S \rho_0 \beta U_p [N]^T dS$

– according to the matrix of thermo-physical properties of material, damping and loading, $\{N\}$ – matrix of form functions. Analogical matrixes are those $[\overline{C}]$, $[\overline{K}]$, $\{\overline{F}\}$, related with the coefficient of heat conductivity and heat exchange.

For the values of change of temperature $\{T\}$ and humidity $\{U\}$ in time the Finite difference method is used. Then numeral realization of mathematical model (7) – (8) is taken to the decision of the kind of system equalizations:

$$[A] \{U\}_{next} = \{R\}; \quad [A_T] \{T\}_{next} = \{R_T\}. \quad (10)$$

As thermo-physical descriptions of wood depend on a temperature and humidity, and equalization of model (7) – (8) are related with each other, the iteration process of equalizations realization (10) is carried out on every sentinel step taking into account additional iteration procedure, which specifies influence of humidity on apportionment of temperature in material and vice versa. Completing of iterations for equalizations (10) foresees implementation of conditions: $\{U_n\} - \{U_{n-1}\} \leq 10^{-4}$ i $\{T_n\} - \{T_{n-1}\} \leq 10^{-4}$.

For numeral realization of mathematical model (1) – (6) elastic-viscous-plastic deformation of wood during drying developed FEM for the research of wood deformation taking into account mechanical-sorption and plastic deformations and mechanism of regeneration of deformations. For this purpose on the basis of a minimum of complete potential energy equivalent variation formulation of tasks was received. Lagrange's equation, minimum of which coincides with the decision of mathematical model (1) – (6) is written down like:

$$\Omega = \frac{1}{2} \int_V (\{U\}^T [B]^T [C] [B] \{U\} + 2\{U\}^T [B]^T [C] \times$$

$$\times \int_0^\tau [R(\tau, \tau')] [B] \{U\} d\tau' - \{U\}^T [B]^T [C] \times$$

$$\times (\{\alpha\} \Delta T + \{\beta\} \Delta U + [m] \frac{dU}{d\tau})) + 2\{U\}^T [B]^T [C] \times \quad (11)$$

$$\times \int_0^\tau [R(\tau, \tau')] [B] \{U\} \left(\{\alpha\} \Delta T + \{\beta\} \Delta U + [m] \frac{dU}{d\tau} \right) d\tau'.$$

Finite difference approximation of vectors of moving $\{u(\tau)\}$ and deformation $\{\varepsilon(\tau)\}$ and function of rheological conduct of wood $R(\tau, \tau')$ in time were used to receive the basic correlations of FEM. Especially for $\{\varepsilon(\tau)\}$ and the core of relaxation are got:

$$\{\varepsilon(\tau)\} = \{\varepsilon(\tau_i)\} + \frac{\{\varepsilon(\tau_{i+1})\} - \{\varepsilon(\tau_i)\}}{\tau_{i+1} - \tau_i} (\tau - \tau_i); \quad (12)$$

$$R_i^* = \frac{\Delta \tau}{2} R_i^*(\tau_0) + \Delta \tau \sum_{j=1}^M R_i^*(\tau_j) + \frac{\Delta \tau}{2} R_i^*(\tau_M). \quad (13)$$

From the condition of a minimum of functional $\delta \Omega = 0$ system of equalizations of algebra is received for finding of the unknown moving on every sentinel step $\Delta \tau_i$ ($i = \overline{1, M}$, M – an amount of sentinel intervals):

$$\sum_{n=1}^N [\bar{K}^{(n)}] \{U\} = \sum_{n=1}^N \{\bar{F}^{(n)}\} - \sum_{n=1}^N [\bar{K}^{(n)}] \times \left(\{\alpha\} \Delta T + \{\beta\} \Delta U + [m] \left| \frac{dU}{d\tau_i} \right| \right), \quad (14)$$

where, integrals $[\bar{K}^{(n)}]$ define the matrix of key inflexibility of material, which is defined by resilient or plastic characteristics of wood and geometrical sizes of elements of laying out. In case of resilient deformation it is taken that $[\bar{K}^{(n)}] = [K^{(n)}]$. For viscous-plastic deformation $(\sqrt{\sigma_{11}^2 + \sigma_{22}^2} - \sigma_{11} \sigma_{22} + 3\sigma_{12}^2 > \sigma_{pl})$ matrix of inflexibility consists of two matrices $[K^{(n)}]$ i $[K^{(n)pl}]$, and $[C^{(n)pl}]$ is calculated with the help of (5). The matrix of loading $\{\bar{F}^{(n)}\}$ is determined by the rheological conduct of wood, and also temperature and humidity descriptions of material. Vector of component to be found $\{U\}$ on the i -step after laying out on time is unknown in relation to calculations $\{U\}$ on previous $i-1$ steps depending on apportionment of temperature and humidity, which are determined on those steps using the algorithm from a previous paragraph.

In this way the mathematical modeling of heat-mass transfer is carried out and deformation-relaxation processes during drying of capillary-porous materials taking into account the anisotropy of heat mechanic descriptions, mechanical and sorption creep, plastic deformations and piling up of irreversible deformations.

Programmatic realization of mathematical models

We described the application software for numeral realization of mathematical models of heat-mass transfer (7) – (8) and elastic-viscous-plastic deformation (1) – (6) of wood during drying developed within the limits of the object-oriented approach [9]. Programmatic complex, developed with the help of programming language *Java*, contains an informative model and interface of the programmatic system, which looks like packages of classes and relations between them with the use of graphic diagrams of UML, components of programmatic code, calculative charts of FEM realization. Developed classes represent the essence of the object-oriented realization of Finite element method. It creates possibility of integration of the developed programs to the existent systems of the automated modeling with the purpose of expansion of their functional possibilities.

There are separate packages for the classes, which realize: geometrical and physical and mechanical descriptions of researches object; laying out of area on finite elements with the help of mesh of knots and elements; determination of basic functions within the limits of eventual elements; calculable classes (squaring for numeral integration); interpolation functions; decision of the system of linear equations; classes, which are oriented on the concrete calculations of matrix and vectorial algebra; classes of saving loaded and received data; user interface. All classes can be repeatedly used for the computer modeling of other mathematical models with the use of Finite element method.

The order of convergence of close decision is analyzed using FEM and errors assessment estimate is shown. A mathematical model of heat-mass transfer verified by the comparisons of calculating values of temperature and humidity with the known experimental data.

For calculations permanent temperature conditions are chosen: (1st process – $t_s = 70$ °C, $\varphi = 50$ %, $v = 2$ m/s; 2nd process – $t_s = 80$ °C, $\varphi = 60$ %, $v = 1$ m/s). Geometrical sizes of the wood sample of pine are: $90 \times 50 \times 25$ mm.

Verification of adequacy of mathematical model of elastic-viscous-plastic deformation of wood taking into account the mechanism of sorption creep during drying took place by comparing of results of numeral modeling with the known researches without taking into account the sorption creep deformations ($m = 0, E_{ms} = 0$) and viscous-plastic deformation ($R(\tau-s) = 0$) with the known resilient models. Also the results of two-dimensional modeling were compared with the results of modeling the elastic-viscous-plastic state of unidimensional case. The analysis of the conducted test calculations of the deformation fields and tensions testifies the satisfactory convergence of numeral and experimental definitions of the probed values.

Analysis of modeling results and their discussion

Application of the developed mathematical models and applied programmatic facilities is shown for the research of processes of heat-mass transfer of the elastic-viscous-plastic state of wood during drying. For numeral experiments some thermo-physical descriptions of wood are specified. Especially, on the basis of working of experimental data dependence of coefficient of wood hydraulic conductivity as functions from a temperature and humidity is used: $a_{m1}(T, U) = a_{m1}(T) a_{mu}(U)$, $a_{m1} / a_{m2} = 1,25$. For determination of coefficient of humidity exchange we use the dependence: $\alpha = 0,95(T/\varphi \varepsilon \exp(-2\sigma V_p / rTR)) 10^{-9}$, where V_p , σ – molar volume and superficial strain of liquid, φ – relative humidity of environment. Value $r = r(U)$ have been received on the basis of wood structure modeling by the system of inconstant capillaries of radius r , that depends on humidity.

To research the elastic-viscous-plastic state taking into account the mechanism of sorption creep resilient and mechanical-sorption descriptions were chosen according to the temperature and humidity change on the basis of approximation of known experimental data that look like:

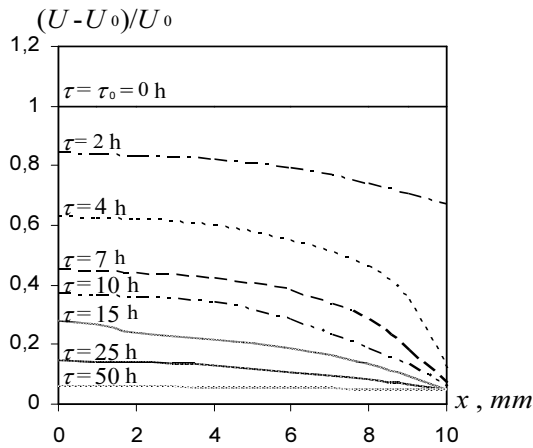
$$\begin{aligned} E_i &= E_{i_0} (1 + E_{iT} (T_0 - T)) + E_{iU} (U_p - U); \\ m_i &= m_{i_0} (1 + m_i (T_0 - T)), \end{aligned} \quad (15)$$

where $E_{i_0}, E_{iT}, E_{iU}, m_{i_0}, m_i$ – coefficient, determined on the basis of approximation of known experimental data.

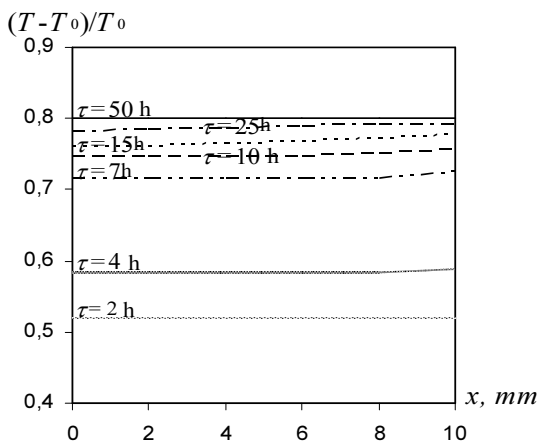
For determination of the module of wood plasticity used experimental data, according to which the determination of the modules of plasticity have been received.

The analysis of apportionment of temperature and humidity dynamics affirms that mathematical models enable to take into account interconnectivity of processes, their physical non-linearity, predefined by dependence of

thermo-physical properties of material temperature and humidity. Curves of the temperature $(T - T_0) / T_0$ and humidity content $(U - U_0) / U_0$ shown on the scheme 1 and scheme 2, testify that the presence of internal resistance of heat and humidity transference predetermines the unevenness of apportionment of the temperature-moisture fields and their interconnectivity. A process of humidity diffusion in wood is less intensive during the humidity change of material surface with an environment. Therefore the value of humidity on-the-spot diminishes and goes to balanced, and the maximal gradients of humidity content appears inside the material.



Scheme 1. Dependence $(U - U_0) / U_0$ of different determinations of time



Scheme 2. Dependence $(T - T_0) / T_0$ of different determinations of time

In a high-quality plan received results coordinate with the results of temperature and humidity fields modeling of drying wood in partial cases. The analysis of graphic dependences testifies the sharp intensity of heat and humidity transfer growth, which is related not only with the fact described higher about less influence of diffusion in comparison to external humidity content, but with the effect of thermo-hydraulic conductivity. The process of thermo-diffusion influences on thermo humidity transfer. Especially, in case of only diffusive transference the curves of humidity transfer have monotonous character and evenly diminish from a maximal value in a central layer to the minimum value on-the-spot of wood sample. The presence of humidity transfer taking

into account a thermo-diffusion, a value of humidity content in wood can be greater than an initial. It predetermines appearance of maximal values of humidity content not only "in a middle" layer but also in other "near a middle" layers of wood sample. Diminishing of humidity exchange coefficient and increase of diffusion coefficient predetermines the increase of thermal-diffusion stream influence. Diminishing of gradient of temperature predetermines the decline of thermal-diffusion stream, and diminishing of humidity on the surface U_s slows humidity exchange with an environment. If a diffusive stream is greater than a total stream, namely thermal-diffusion from the surface and humidity exchange with the agent of drying, humidity content of superficial layers increase. This growth is observed to the moment of smoothing of the described streams.

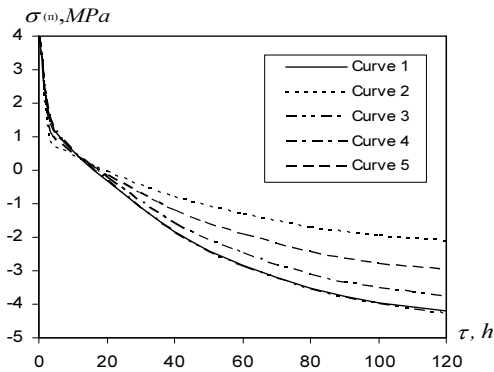
To research the influence of anisotropy of structural properties of wood the numeral experiment of determination the temperature-humidity fields in radial and tangential samples have been made. For the exposure of influence exactly of anisotropy of thermo-physical properties conditions of external environment were set such, for which withering coefficients would be the smallest. The analysis of researches testifies that humidity of superficial area of standard substantially does not influence on apportionment of humidity under the thickness of tangential board. During the long period of time humidity in central part of board doesn't change. In initial moments of time the maximal values of humidity are observed in "near surface" layers. Diminishing of humidity in central layers occurs in a regular mode period. For a radial board humidity diminishes in all points of wood sample. To graphic dependences the humidity change is not peculiar inflection points for initial moments of wood drying, as in the case of tangential boards. The speed of humidity change in a radial board is more intensive, than in tangential. The increase of duration of drainage slows the change of humidity of radial board in comparison to the change of humidity of tangential board.

On the basis of the developed mathematical models we investigated influence of anisotropy of visco-plastic descriptions of wood, descriptions mechanical and sorption creep, withering coefficients on apportionment of two-dimensional stress-strain state of wood during drying taking into account an isothermal process. For this purpose during numeral modeling the probed index changed to the half of its value and the change of normal and tangential tensions on-the-spot and on the "near surface" layers of wood was probed.

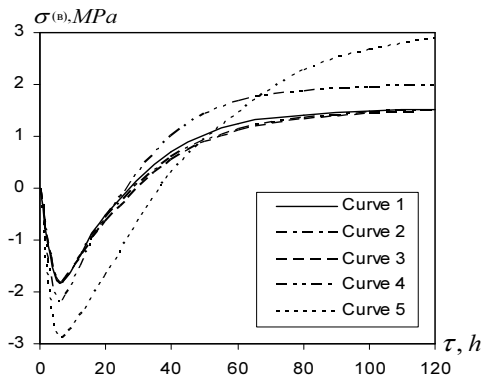
Especially, on the scheme 3 graphic dependences of normal tension σ_x on-the-spot tangential saw-timbers for different values component of tensor of resiliency are shown. Continuous curve 1 answers a numeral calculation without the change of resilient descriptions. The biggest influence on the tension apportionment σ_x has the change of Young's elastic modulus in tangential direction, thus a curve 2 (for which $E_2^{(2)} = 0,5 E_2$ and other coefficient didn't change during numeral modeling). Especially, the increase of deviation σ_x , for the case σ_x of $E_2^{(2)} = 0,5 E_2$ without change E_2 is more considerable with the drying time increase. A change of the module of resiliency in radial direction $E_1^{(3)} = 0,5 E_1$ doesn't influence the

apportionment a lot σ_x on the surface (a curve σ_x in this case almost coincides with σ_x without change of characteristics). The diminution of the Poisson's ratio $\nu_{12}^{(4)} = 0,5 \nu_{12}$ to the change of component σ_x is characterized by a curve 4, and the diminution of the shear module $G_{12}^{(5)} = 0,5 G_{12}$ on σ_x describes the curve 5. The analysis of graphic dependences testifies that growth of change of wood anisotropic mechanical descriptions, dependent upon humidity on apportionment of normal tensions σ_x on the surface during the time of wood drying.

On the scheme 4 influence of anisotropy coefficients of mechanical and sorption creep on tension in the "near surface" layer of wood sample is shown. Curve 1 meets a set value for calculation. The analysis of graphic dependences testifies that diminishing of coefficients m_r ($m_r^{(2)} = 0,5 m_r$, curve 2), μ_{rt} ($\mu_{rt}^{(3)} = 0,5 \mu_{rt}$, curve 3) doesn't substantially influence on the value of normal tensions σ_x . The change of m_{rt} ($m_{rt}^{(4)} = 0,5 m_{rt}$, curve 4) increases the value of normal tensions from 0,5 to 0,72 MPa after the initial stage of drying process. The most important influence has the change of m_t ($m_t^{(5)} = 0,5 m_t$, curve 5) on incremental tensions in a "near surface" area. On the initial stage of drying process maximal value increases from 0,75 MPa to 1,24 MPa. The increase of tension σ_x is observed with the duration of drying process. Diminishing m_t moves the change of sign of normal tensions on time axis.



Scheme 3. The influence of anisotropy of tensor resiliency component on the apportionment of normal tension σ_x on the surface.



Scheme 4. Influence of mechanical and sorption creep on apportionment of normal tension in a "near surface" layer.

Thus, received mathematical models and developed programmed complex give the opportunity to determine the changes of technological process

parameters in wood drying to the apportionment of heat-and-mass transfer and elastic-viscous-plastic state of wood taking into account the mechanical-sorption creep and anisotropy of heat mechanic characteristics of wood.

Optimization problem formulation and analysis of simulation results

One way to intensify and improve drying capillary-porous materials, including wood, is the development of continuous control heat and moisture exchange that would realized using modern tools. The basis of this approach is the justification and optimization of continuous technological modes of drying wood. Structure continuous (stepless) regimes characterized performance conditions $(t_c(U(\tau)) - U_p(U(\tau)))^2$, where t_c - ambient temperature, U , U_p - humidity and equilibrium moisture content, τ - drying time.

Based on mathematical models of viscoelastic deformation and processes heat and mass transfer in capillary-porous materials, taking into account thermo-mechanical characteristics formulate the optimization problem for constructing continuous techno-logical modes of drying wood that way.

Need to find such features ambient temperature $t_c(x, \tau)$ and relative humidity $\varphi(x, \tau)$ with regard to their limits

$$t_{c1} \leq t_c(x, \tau) \leq t_{c2}; \quad \varphi_1 \leq \varphi(x, \tau) \leq \varphi_2 \quad (16)$$

and the stress-strain state

$$|\sigma_{11}(x, \tau)| \leq \sigma_{11r}; \quad |\sigma_{12}(x, \tau)| \leq \sigma_{12r}; \quad |\sigma_{22}(x, \tau)| \leq \sigma_{22r}, \quad (17)$$

that for wooden beams with an initial moisture content of $U(x, 0) = U_0(x)$ over a given time drying to minimize the condition

$$J(U) = (t_c(U(x, \tau)) - U_p(U(x, \tau)))^2 \rightarrow \min, \quad (18)$$

where $t_{c1}, t_{c2}, \sigma_{11r}, \sigma_{12r}, \sigma_{22r}$ - set the value.

To solve the formulated optimization problem (16) - (17) the method of genetic algorithms. In accordance with the basic definitions and provisions of the theory of genetic algorithms [10, 12] for solving the need to specify the form chromosomes, which represent a solution and ask fitness function is determined by the most appropriate chromosome is the best solution.

To simulate the temperature dependence of the medium used

$$t_c(U(x, \tau)) = t_0(x) + (t_k(x) - t_0(x)) \frac{U_0(x) - \bar{U}(\tau)}{U_0(x) - U_k(x)}, \quad (19)$$

where t_k - final value of ambient temperature, U_k - end of the moisture content.

An expression that binds moisture equilibrium with the function of moisture content according to [10] can be written as

$$U_p = U_{pk} + (U_{pn} - U_{pk}) \exp(-\exp(b_1 + b_2 U(x, \tau))), \quad (20)$$

where U_{pk}, U_{pn} - equilibrium moisture content at the beginning and at the end of the drying process, b_1, b_2 - some coefficients that are determined by the empirical relationship [12].

Thus, the temperature and humidity environment drying varies from linear (19) and exponential (20) dependencies. The corresponding chromosome genetic algorithm has the form

$$\chi = (\tau_1, \dots, \tau_{k-1}, t_{c1}, t_{c2}, \dots, t_{ck}, U_{p1}, U_{p2}, \dots, U_{pk}). \quad (21)$$

Fitness function written as

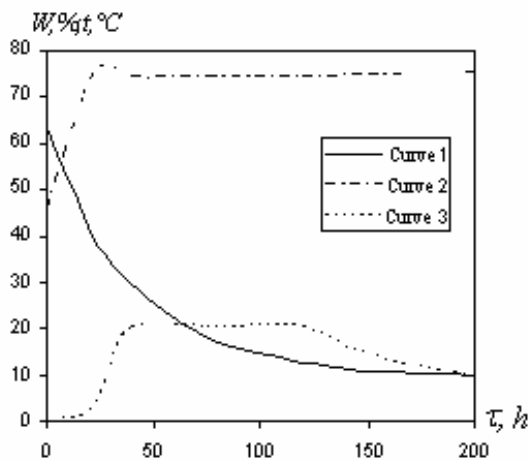
$$P(\tau_1, \dots, \tau_{k-1}, t_{c1}, \dots, t_{ck}, U_{p1}, \dots, U_{pk}) = \begin{cases} 0, & \max \sigma_{ij} \geq 75\% \cdot \sigma_{TM}; \\ J(U), & \text{other wise,} \end{cases} \quad (22)$$

where σ_{TM} – experimentally determined boundary strength of wood [1].

Numerical solution of an optimization problem by using Matlab 7.01 Genetik Algorithm and Direct Search Toolbox, which is used for the expansion Optimization Toolbox genetic algorithms [13].

Results of the numerical solution of the optimization problem is shown in Scheme 5. to compare the influence of the regime and the regime with constant parameters of the environment on the drying process of wood moisture shows the distribution of stresses in the surface and central layers of wooden model.

The simulation results shown in Scheme 5. The initial value for t_{c1} and t_{c2} according 45 °C and 75 °C. The temperature t_m reached a maximum value to 25 °C. For each point bar minimum and maximum values of final moisture was 10 % and 14 %.



Scheme 5. Change the average moisture content W (curve 1) temperature t_c on the thermometer dry (curve 2) and temperature t_m on the thermometer wet (curve 3) spruce wood in time.

Conclusions

1. We formulated a mathematical model of elastic-viscous-plastic deformation of wood during drying process, which takes into consideration plastic deformations, caused by mechanical-sorption creep and anisotropy of mechanical characteristics of the material and gives the opportunity to determine two-dimensional stress-strain state in the conditions of non-isothermal humidity transfer.

2. On the basis of the created mathematical models and methods of analysis application software is developed for the modeling of elastic-viscous-plastic state of capillary-porous materials during the convective drying, which consists of the above classes and gives the oppor-

tunity to automatize finite element analysis stress-strain state of wood during convective drying.

3. As a result of calculable experiments, conducted with the use of developed applied programmatic facilities regularity of anisotropy influence of thermo-physical and mechanical descriptions of wood were set, it initial humidity on changing of two-dimensional temperature-humidity and elastic-viscous-plastic state of wood during the convective drying.

4. Based on the mathematical model study of the stress-strain state and temperature and moisture patterns formulated optimization problem of drying capillary-porous materials. Using genetic algorithms solve the optimization problem that can improve the quality of drying capillary-porous materials, including wood.

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