

## Modelos de fechamento de difusão turbulenta e os ciclos de varreduras e ejeções em camadas limite convectivas

Gradient-diffusion closure and the ejection-sweep cycle in convective boundary layers

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### Abstract

*The inadequacy of conventional gradient-diffusion closure in modeling turbulent heat flux  $H = \overline{w'\theta'}$  within the convective atmospheric boundary-layer is often alleviated by accounting for nonlocal transport. Such nonlocal effects are a manifestation of the inherent asymmetry in vertical transport in the convective boundary layer, which is in turn associated with third-order moments (skewness and fluxes of fluxes). In this work, the role of these third-order moments in second-order turbulence closure of the sensible heat flux is examined with the goal of reconciling the models to various closure assumptions. Surface layer similarity theory and mixed-layer parametrizations are used here, complemented by LES results when needed. The turbulent heat flux with various closure assumptions of the flux transport term ( $T = \overline{w'w'\theta'}$ ) is solved, including both local and nonlocal approaches. We connect  $T$  to ejection-sweep cycles in the flow field using the Gram-Charlier cumulant expansion of the joint probability distribution of vertical velocity and potential temperature. In this nonlocal closure, the transport asymmetry models that include the vertical velocity skewness as a correction term to  $H$  originate from ejection-sweep events. Vertical inhomogeneity results in a modified-skewness correction to the nonlocal contribution to the heat flux associated with the relative intensity of ejections and sweeps.*

**Keywords:** Convective boundary layer, second-order closure, ejection-sweep cycle

### Resumo

*As limitações do modelo de gradiente-difusão para o fechamento do fluxo turbulento de calor sensível  $H = \overline{w'\theta'}$  na camada limite atmosférica convectiva é muitas vezes compensada introduzindo-se termos de transporte não-local. Esse transporte é resultante da assimetria inerente ao transporte vertical na camada limite convectiva, que por sua vez está associada aos momentos de terceira ordem (coeficiente de assimetria e fluxos dos fluxos). Nesse trabalho, o papel desses momentos de terceira ordem é examinado com o objetivo de reconciliar os modelos acima citados com diversas hipóteses de fechamento. São utilizadas teoria de similaridade da camada superficial e parametrizações da camada misturada, complementadas por resultados de LES. O valor de  $H$  é obtido utilizando hipóteses de fechamento para o termo de transporte de fluxo ( $T = \overline{w'w'\theta'}$ ), incluindo abordagens locais e não-locais.  $T$  é relacionado a ciclos de ejeção-varredura do escoamento através da expansão de Gram-Charlier para a probabilidade conjunta das distribuições de velocidade vertical e temperatura potencial. Nesse fechamento não-local, eventos de ejeção-varredura influenciam modelos de assimetria do transporte que utilizam o coeficiente de assimetria da velocidade vertical para corrigir  $H$ . A não-homogeneidade vertical resulta em uma modificação da correção à contribuição não-local ao fluxo de calor, associada à intensidade das ejeções e varreduras.*

**Palavras-chave:** Camada-limite convectiva, modelo de fechamento de segunda ordem, ciclos ejeção-varredura

### 1 Introduction

Heat fluxes introduced at the bottom of the convective atmospheric boundary layer are diffused by turbulence against the vertical gradients in mean temperature. Efforts to explain the vertical profile of this heat flux are centered at models which modify the gradient-diffusion theory (K-theory) by a nonlocal correction term (e.g. Deardorff (1966)), transport asymmetry models (e.g. Moeng and Wyngaard (1989)), or mass-flux approaches (e.g. Siebesma *et al.* (2007)). Despite the differences between these models, it is agreed upon that this additional contribution to the heat flux originates from large-scale turbulent eddies (nonlocal effects) that is tied to strong updrafts/downdrafts, skewness in vertical velocity and significance of third-order mixed moments, and/or bottom-up and top-down diffusion. Here we address the differences between these models with the aim of reconciling them to the fact that third-order moments, namely the turbulent flux transport term in the heat flux budget, is responsible for this nonlocal effect and reproduce the various aforementioned assumptions. The starting point is the heat flux budget in the convective boundary layer, which under horizontally homogeneous conditions, high Reynolds number, negligible Coriolis effect and with the Boussinesq approximation reads

$$\frac{\partial \overline{w'\theta'}}{\partial t} = -\overline{w'^2} \frac{\partial \bar{\theta}}{\partial z} - \frac{\partial \overline{w'w'\theta'}}{\partial z} - \frac{1}{\rho_0} \overline{\theta' \frac{\partial p'}{\partial z}} + \frac{g}{\theta_0} \overline{\theta'^2}$$

where overbars indicate ensemble-averaging and primes denote temporal fluctuations around the mean value. The variables  $w$  and  $\theta$  are the vertical velocity and potential temperature respectively,  $z$  is the vertical coordinate,  $g$  is gravitational acceleration, and the subscripts “0” are for reference-state density and potential temperature respectively, and  $p$  is pressure. The pressure-temperature covariance term can be represented by the modified Rotta closure that includes the effects of buoyancy after Moeng and Wyngaard (1986)

$$-\frac{1}{\rho_0} \overline{\theta' \frac{\partial p'}{\partial z}} = -\frac{\overline{w'\theta'}}{\tau} - \frac{1}{2} \frac{g}{\theta_0} \overline{\theta'^2}$$

where  $\tau$  is a relaxation timescale. The resultant heat flux budget then takes the form

$$\overline{w'\theta'} = -\sigma_w^2 \tau \frac{d\bar{\theta}}{dz} - \tau \frac{d\overline{w'w'\theta'}}{dz} + \frac{\tau}{2} \frac{g}{\theta_0} \overline{\theta'^2}$$

which provides the general framework for the subsequent analysis, particularly the gradient in the flux transport term  $\overline{w'w'\theta'}$ , which we associate with local (K-theory) and nonlocal (ejection-sweep) closures.

### 2 Methodology

The vertical profiles used here span the surface layer similarity theory (Monin-Obukhov similarity theory (MOST)) and mixed layer parametrization under convective conditions from Kaimal and Finnigan (1994) and Stull (1988). Whenever needed, the large eddy simulation (LES) of Moeng and Wyngaard (1989) are used to complement the missing profiles, particularly the third-order moments. Three convective boundary layer cases are included to examine the effects of the surface layer mean-gradients on the closure assumptions. These cases are summarized in Table 1 and vertical profiles are shown in Figure 1.

Table 1. Parameters for the three simulation cases

Parameter	$-L$ (m)	$u_*$ (m s <sup>-1</sup> )	$w_*$ (m s <sup>-1</sup> )	$u_*/w_*$	$\overline{w'\theta'}_s$
Case 1	12	0.25	1.49	0.17	0.1
Case 2	49	0.4	1.49	0.27	0.1
Case 3	100	0.57	1.66	0.34	0.14

With these vertical profiles, both local and nonlocal closures for the term  $\overline{w'w'\theta'}$  are considered. The first yields a second-order ordinary differential equation for the heat flux

$$\frac{d^2 \overline{w'\theta'}}{dz^2} = \frac{1}{\tau \sigma_w^2} \frac{d(\tau \sigma_w^2)}{dz} \frac{d\overline{w'\theta'}}{dz} + \frac{\overline{w'\theta'}}{\tau^2 \sigma_w^2} + \frac{1}{\tau} \frac{d\bar{\theta}}{dz} - \frac{g}{2c\tau \sigma_w^2 \theta_0} \overline{\theta'^2} = 0$$

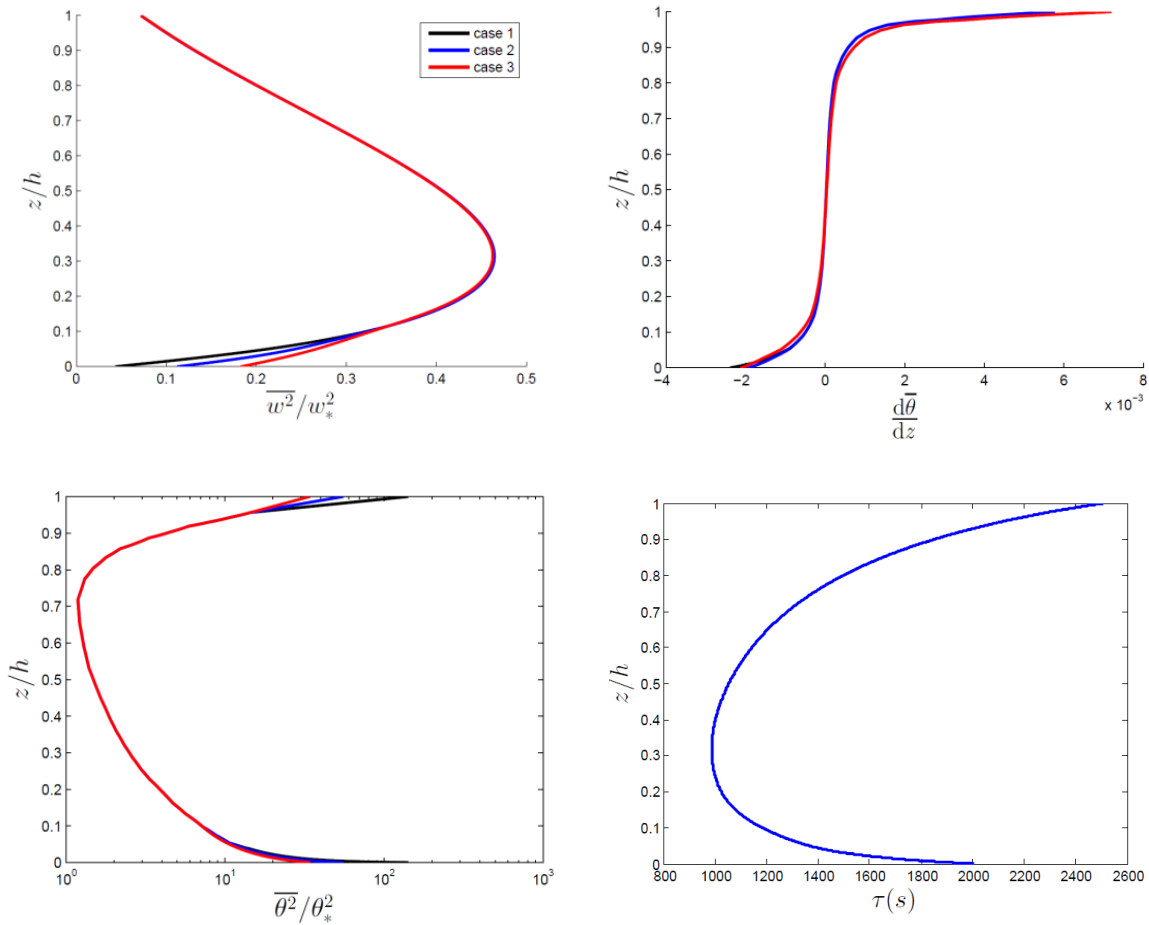


Figure 1. Vertical profiles of vertical velocity variance (top left), mean potential temperature gradient (top right), potential temperature variance (bottom left), relaxation timescale (bottom right).

The second relates the triple-moment to the ejections and sweeps in the flow field, where  $\overline{w'w'\theta'}$  is modeled via quadrant analysis of the joint distribution of vertical velocity and temperature (Raupach, 1981). This results in

$$\frac{\partial \overline{w'w'\theta'}}{\partial z} = \alpha \frac{\partial [\hat{S}_w \sigma_w \overline{w'\theta'}]}{\partial z}$$

where

$$\hat{S}_w = M_{03} - \frac{6\sqrt{2\pi}}{1 - A_1} \Delta S_0$$

is a modified vertical velocity skewness ( $M_{03}$  is vertical velocity skewness),  $\Delta S_0$  is the relative importance of ejections and sweeps estimated from the joint distribution of  $w'$  and  $\theta'$ , and  $A_1$  is a closure constant.

### 3. Results

The preliminary results of the study in Figure 2 show the heat flux profile resulting from the local closure approach (the second-order differential equation above), with the timescale  $\tau = h/\sigma_w$ . While gradient-diffusion is considered inadequate in simulating the flux due to lack of mean-gradients in the mixing layer, this approach seems to remain reasonable in relating third-order to second-order moments, especially by relating a turbulent quantity to the gradient of a turbulence quantity rather than a mean value. The nonlocal closure requires quadrant analysis of the  $(w', \theta')$  distribution along with other third-order moments (such as temperature and velocity skewness and other mixed moments).

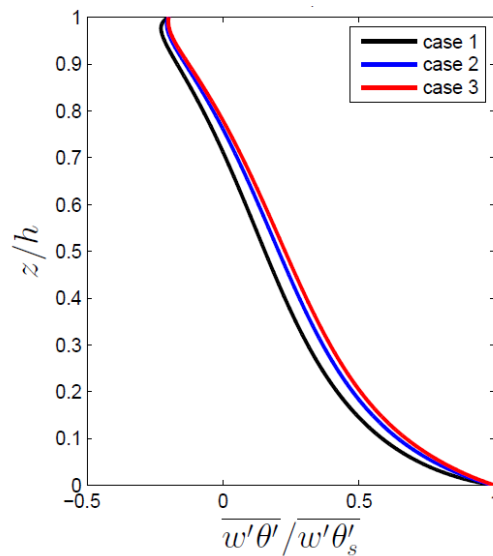


Figure 2. Heat flux profile (normalized by surface flux). The relaxation timescale used here is inversely proportional to the vertical velocity standard deviation.

## Final Considerations

On the basis of the sweep-ejection cycle in the flow field, which can be considered a nonlocal effect in the sense that it relates to large-eddy structures in the boundary layer, we can highlight the importance of third-order moments in contributing to this transport asymmetry and therefore to better representation of the heat flux under convective conditions. This approach is to be complemented by LES experiments and compared to countergradient correction and mass-flux models.

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