Construction and Finding the Area of the New Geometric Figure, *n*-cardioid

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Abstract *– This paper is an original and basic research that will give the definition, the process of construction and the formula for finding the area of a new geometric figure formed on a mathematical problem of a goat tied in the corner of a shed. This new geometric figure was named as n-cardiod. The n-cardiois is the union of the arc* X_i *formed when a tension that measures half of the polygons perimeter is tied at any vertex of closed* P_n *and is rotated in a clockwise and counter clockwise direction until it touches the polygon, where* P_n *is a regular polygon with vertices* $x_1, x_2, ..., x_n$ *, n sides and length of sides s for all* $n \in \mathrm{N}$ *. The area of the n-cardioid will be solved using the formula* $A = \left(\frac{5n^2+4}{24}\right)s^2\pi$.

Keywords *– n-cardioid, polygons, arc, area.*

INTRODUCTION

Geometry as branch of mathematics deals with the deduction of the properties, measurement, and relationships of points, lines, angles, and figures in space. Its importance lies less in its results than in the systematic method Euclid used to develop and present them.

There are many geometric figures that can be seen in plant forms, in fabric designs, in art and in architecture.

One problem given to engineering students that challenge them in application of area of sectors caught the researcher's attention. The problem says that "*A goat is tied to a corner of a 4m by 5m shed by a 6m rope. What area of the ground can the goat graze?"* The problem is solvable by applications of areas of sectors. The area of the ground that the goat can graze is the total area of the sectors. Geometrically, what if the shed is a square and the smaller sector has a radius that is equal to the length of the side of square. In doing so, the larger sector has a radius that is twice the smaller sector. By inspection, the radius of the larger sector is equal to the half of the perimeter of square. The researcher wants to extend the problem about it. What if the shed is a closed regular polygon and the rope measures half of its perimeter?

This study contributes a body of knowledge particularly in geometry. Also, it could be a way for exploring a new geometric figure and its properties. This research work could also significant to mathematics instructor, for this may help them to solve *n*-cardioid problems and other problems related to this. Finally, to the researcher, it develops logical skills and reasoning, wakes up a wide interest in field of mathematics especially in geometry.

OBJECTIVES OF THE STUDY

This study introduces a geometric figure called *n*-cardioid. Specifically it aims to give a definition of *n*-cardioid; discuss how to construct an *n*-cardioid;. and generalize a formula for the area bounded by the *n*-cardiod.

MATERIALS AND METHODS

This is a basic research so the researcher collects and compiles data from mathematics books. It presents some properties of related geometric figures that are important in the introduction of a new geometric figure.

RESULTS AND DISCUSSION

Definition of *n*-cardiod

This section defines the *n*-cardiod and its parts.

Definition 1. Let P_n be a regular polygon with vertices $x_1, x_2, ..., x_n$, *n* sides and length of sides *s* for all $n \in \mathrm{N}$. Then there exist $a_1, a_2, ..., a_m$ such that

$m \le n$, then sector $a_i x_1 a_1$ with radius $\left(r = \frac{ns}{2}\right)$ said to be a *reflexive sector or sector* X_1 if

i. x_2 lies on the line segment $x_1 a_1$,

ii. x_n lies on the line segment $a_i x_1$ where $i = n-2$ if *n* even, otherwise $i = n$.

iii. The angle of the sector is a reflex angle that is consist of 2 exterior angle and an interior angle of the regular polygon.

Example 1. The below figure is the reflexive sector of the new geometric figure whose radius equal is 4.

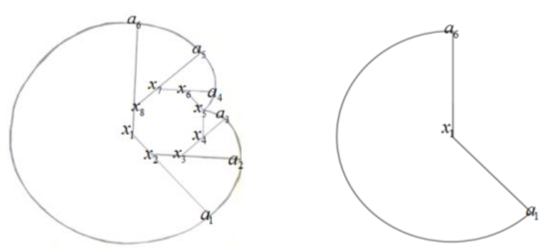

Figure 1

Definition 2. Let P_n be a regular polygon with vertices $x_1, x_2, ..., x_n$, *n* sides and length of sides *s* for all $n \in \mathrm{N}$. Then there exist $a_1, a_2, ..., a_m$ such that $m \le n$, the sector is said to be a *polysector* X_i if :

i. *n* is odd $X_i = \{a_{i-1} x_i a_i ; 2 \le i \le n$ where x_{i+1} lies on the line segment $x_i a_i$ if $i < \left\lceil \frac{n}{2} \right\rceil$ or x_{i-1} lies on line segment $x_i a_{i-1}$,

ii. *n* is even, then $X_i = \begin{cases} a_{i-1} x_i a_i ; 2 \le i < \frac{n}{2} \\ a_{i-1} x_i x_{i+1} ; i = \frac{n}{2} \\ x_{i-1} x_i a_{i-2} ; i = \frac{n}{2} + 2 \\ a_{i-2} x_i a_{i-3} ; \left(\frac{n}{2} + 2\right) < i \le n \end{cases}$ where x_{i+1} lies on the line segment $x_i a_i$ for $2 \le i < \frac{n}{2}$ or x_{i-1} lies on line segment $x_i a_{i-1}$ for $\left(\frac{n}{2} + 2\right) < i \le n$,

iii. the angle of the sector is an exterior angle of the P_n.

Remark 1 If *n* is even, there is no sector X_i formed when $i = \left(\frac{n}{2} + 1\right)$.

Definition 3. Let P_n be a regular with *n* sides and let X_i be the polysector, then the *sequential sector* is the set $\bigcup_{i=2}^{n} X_i$.

Example 2. The figure below shows the sequential sector of the new geometric figures with octagon as its reference polygon. The octagon is just a reference in this figure and it does not belong to the sequential sector.

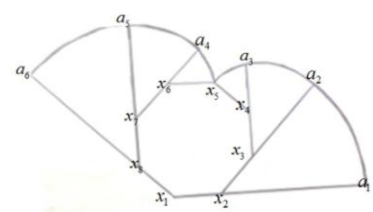

Figure 2

Definition 3.1.4. Let P_n be a regular with *n* sides. Let S_{X_i} denote the arc formed by the sector X_i, then $\bigcup_{i=1}^{n} S_{X_i}$ is called *n-cardioid.*

Example 3. The 8-cardioid forms when a 4-m tension is tied at any vertex of the closed regular octagon whose side's measures one meter each and then moves counter clockwise and clockwise direction until it touches the part of the polygon as shown in the Figure 3.

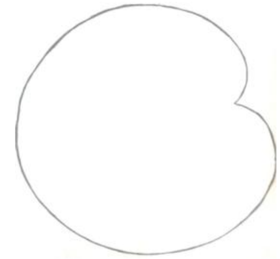

Figure 3

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This *n*-cardioid does not have sides, *n* is just a reference to show where this geometric figure came from and P_n does not belong to *n*-cardioid.

Construction of *n*-cardioid

Given a closed regular polygon P_n with *n* sides and length of sides *s* and a tension that measures half of the regular polygon's perimeter. The *n*-cardioid forms when a tension is tied at any vertex of the P_n. Then it moves counter clockwise and clockwise direction until it touches the part of the polygon.

Finding the Area of *n*-cardioid

The generalized formulas are being discussed in this section.

Lemma 1. Let P_n be a regular with *n* sides and length of each side *s*. Let X_1 be the reflexive sector, then area of the sector X_1 is given $A_{X_i} = \frac{n^2+2n}{8}s^2\pi$.

Illustration Find the area of the reflexive sector or sector X_1 from a hexagon with 4 meter sides.

Solution

$$A_{X_r} = \frac{n^2+2n}{8}s^2\pi$$
$$= \frac{6^2+2(6)}{8}4^2\pi$$
$$A = 96\pi$$

The area of the X_1 is 96π square meters.

Lemma 2 Let P_n be a regular with *n* sides and length of each side *s*. Let X_i be a polysector then the radius X_i is given by $r = \left(\frac{ns}{2} - (i-1)s\right)$ for all $2 \le i \le \left\lceil \frac{n}{2} \right\rceil$.

Theorem 1. Let P_n be a regular with *n* sides and length of each side *s*. Let X_i be a polysector then, the area of each X_i is given by $A = \frac{\pi}{n}\left(\frac{ns}{2} - (i-1)s\right)^2$ for all $2 \le i \le \left\lceil \frac{n}{2} \right\rceil$.

Remark 2. Let P_n be a regular with *n* sides and let X_i be the polysector with an angle that is equal to the exterior angle of the P_n then, $X_2 = X_n, X_3 = X_{n-1}, \ldots, X_{\left\lceil \frac{n}{2} \right\rceil} = X_{\left\lfloor \frac{n}{2} \right\rfloor + 2}$ such that $n \in \mathrm{N}$.

Remark 3. Let P_n be a regular with *n* sides and length of each side *s*. Let A_{X_i} be the area of X_i. Since $\bigcup_{i=2}^{n} X_i$ .be the sequential sector then, area of the sequential sector is the $\sum_{i=2}^{n} A_{X_i}$.

Lemma 3. Let P_n be a regular with *n* sides and length of each side *s*. Let $\bigcup_{i=2}^{n} X_i$ .be the sequential sector then, area of the sequential sector is given $A = \frac{(n-1)(n-2)}{12}s^2\pi$.

Illustration Find the area of the sequential sector of the 8-cardioid whose sides of the reference polygon is 2 cm.

Solution:

$$A = \left(\frac{n^2-3n+2}{12}\right)s^2\pi$$
$$= \left(\frac{8^2-3(8)+2}{12}\right)2^2\pi$$
$$A = 14\pi$$

The area of the sequential sector is 14π sq. cm.

Remark 4. Let P_n be a regular with *n* sides and length of each side *s*. Let A_{X_i} be the area of X_i. If $\bigcup_{i=1}^{n} X_i$ are the sectors formed by *n*-cardioid, then area of the *n*-cardioid is the $\sum_{i=1}^{n} A_{X_i}$.

Theorem 2. Let P_n be a regular polygon with *n* sides and let *s* be the length of each sides. Let $\bigcup_{i=1}^{n} X_i$ are the sectors formed by *n*-cardioid, then the area of the *n*-cardioid is given by $A = \frac{5n^2+4}{24} s^2 \pi$.

Illustration Find the area of the 6-cardioid whose reference sides is 3 feet long.

Solution

$$A = \frac{5n^2+4}{24} s^2 \pi$$
$$= \frac{5(6)^2}{24} 3^2 \pi$$
$$= \frac{21}{2} \pi ft^2$$

The area of the 6-cardioid is $\frac{21}{2} \pi ft^2$.

Theorem 3. The area of the *n*-cardioid diverges.

CONCLUSION AND RECOMMENDATION

Through Mathematics we can simplify complicated problems. Instead of finding the area and arc length of each sectors formed, you can easily on finding the total area of all the sector forms when an object is tied to a corner of a closed regular polygon by a tension that is equal to the half of polygon’s perimeter. This object is rotate on the closed regular polygon. By observation while analyzing the problems and computing for the answers, you can find out a pattern that leads to come up with the generalize formula. Generalized formula on finding the area and the distance of the objects can graze is formed to simplify the computations.

The *n*-cardioid is still open for the other properties. Future researchers that may be done by other Mathematics practitioners may form a sequence from the area and circumlength of the *n*-cardioid. They may also find the longest line on the on the *n*-cardioid, then generate a formula. Finding the formula of the distance of the line that divides the *n*-cardioid on two equal parts given only the length of the sides and number of sides of the reference polygon.

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