On the Sum of Corresponding Factorials and Triangular Numbers: Runsums, Trapezoids and Politeness

Asia Pacific Journal of Multidisciplinary Research
Vol. 3 No. 4, 95-101
November 2015 Part II
P-ISSN 2350-7756
E-ISSN 2350-8442
www.apjmr.com

Romer C. Castillo (M.Sc.)
Batangas State University, Batangas City, Philippines
romercastillo@rocketmail.com

Date Received: August 3, 2015; Date Revised: September 18, 2015

Abstract – *When corresponding numbers in the sequence of factorials and sequence of triangular numbers are added, a new sequence of natural numbers is formed. In this study, these positive integers are called factoriangular numbers. Closely related to these new numbers are the runsums, trapezoidal and polite numbers. Some theorems on runsum representations of factoriangular numbers are proven here, as well as, a theorem on factoriangular number being represented as difference of two triangular numbers. Unambiguous definitions of trapezoidal number and number of trapezoidal arrangements are also given, including how these differ from runsum, polite number, number of runsums and politeness.*

Keywords – *factoriangular number, polite number, runsum, trapezoidal arrangement, trapezoidal number*

INTRODUCTION

The sequence {2, 5, 12, 34, 135, 741, 5068, 40356, 362925,…} is a result of the addition of corresponding factorials and triangular numbers. This is sequence A101292 in [1]. In this study, these integers are named *factoriangular* numbers. Related to these numbers are the runsums, trapezoidal and polite numbers.

Knott [2] defines a runsum as a sum of a sequence of consecutive whole numbers. Runsums can be either trivial or nontrivial. A trivial runsum is a runsum consisting of a single number and thus, every whole number is a runsum. Triangular numbers are the special runsums that begin at 1.

Because the series sum can be shown as a trapezium, runsum is also called trapezoid; a runsum or trapezoid is a difference of two triangular numbers [3]. Similarly, a polite number is defined as a positive integer that can be written as the sum of two or more consecutive positive integers and if all numbers in the sum are strictly greater than 1, the polite number is also called trapezoidal number because it represents pattern of points arranged in a trapezoid [4]. It turns out that almost all natural numbers are polite and the only impolite are the powers of 2 [5].

A natural number n, greater than 1 and not a power of 2, has more than one runsum representation. The number of runsums of n depends on the prime factorization of n [3]. This is similar to the number of trapezoidal arrangements of n, which is the number of ways n can be written as the difference of two triangular numbers [6]. Related to this is the politeness of a positive number which is defined as the number of ways the number can be expressed as the sum of consecutive positive integers [4].

The main objective of this study is to explore the runsum representations of factoriangular numbers. Secondary to this is to redefine trapezoidal number and number of trapezoidal arrangements to have a clearer distinction from runsum, polite number, number of runsums and politeness.

METHODS

This study is a basic research in number theory. It employs mathematical exploration, exposition and experimentation with repeated trials and inventiveness to arrive at a proof. The methods of investigation here hold on the scientific approach as it uses experimental mathematics. In finding the runsum representations of factoriangular number, the discussions on runsums of length n given by Knott in [3] serve as the groundwork for the computations.

RESULTS AND DISCUSSION

The term *factoriangular number* is used here to name a sum of a factorial and its corresponding triangular numbers. The following notations are also used: n for natural numbers, $n!$ for factorial of a natural number, T_n for triangular number, and Ft_n for factoriangular number. The formal definition is as follows:

Definition 1. The nth factoriangular number is given by the formula $Ft_n = n! + T_n$, *where* $n! = 1 \cdot 2 \cdot 3 \cdots n$ *and* $T_n = 1 + 2 + 3 + ... + n = n(n+1)/2$.

Let $R(Ft_n)$ be the number of runsums of a factoriangular number or the number of ways a factoriangular number can be represented as a runsum of different lengths or number of terms. This is formally defined as follows:

Definition 2. The number of runsums of Ft_n that is not a power of 2 is given by

$$R(Ft_n) = (e_1+1)(e_2+1)(e_3+1)\cdots$$

where e_1, e_2, e_3,... are the exponents of the prime numbers, p_i, greater than 2 in the prime factorization $Ft_n = 2^c \cdot p_1^{e_1} \cdot p_2^{e_2} \cdot p_3^{e_3} \cdots$, *integer $c \geq 0$. A power of 2 has only one runsum representation.*

$R(Ft_n)$ is also equal to the number of odd divisors of Ft_n. The $R(Ft_n)$ for the first 20 Ft_n, as well as, the number of trapezoidal arrangements and politeness, are presented in Table 1.

Table 1. Number of Runsums [$R(Ft_n)$], Trapezoidal Arrangements [$Tr(Ft_n)$] and Politeness of Ft_n

n	Ft_n	$R(Ft_n)$ or $Tr(Ft_n)$ including trivial	$Tr(Ft_n)$ excluding trivial or Politeness
1	2	1	0
2	5	2	1
3	12	2	1
4	34	2	1
5	135	8	7
6	741	8	7
7	5068	4	3
8	40356	12	11
9	362925	18	17
10	3628855	8	7
11	39916866	8	7
12	479001678	4	3
13	6227020891	12	11
14	87178291305	32	31
15	1307674368120	8	7
16	20922789888136	4	3
17	355687428096153	24	23
18	6402373705728171	12	11
19	121645100408832190	16	15
20	2432902008176640210	128	127

Notice that $Ft_1 = 2$ has only one runsum and that is the trivial runsum of length 1 whose only term is the number 2 itself. Further, each of $Ft_2 = 5$, $Ft_3 = 12$ and $Ft_4 = 34$ has only two runsums. Aside from the trivial runsum, their respective runsum representations are as follows: 5 = 2 + 3, 12 = 3 + 4 + 5, and 34 = 7 + 8 + 9 + 10.

There is a way to efficiently determine all runsum representations of a particular number. The first step is to determine the length or number of terms of a runsum. The following guidelines in determining the length of a runsum were adapted from Knott [3] with some modifications and enhanced discussions:

1. Every natural number has a runsum of length 1, the runsum whose only term is the number itself and which is called a trivial runsum.
2. Every odd number has a runsum of length 2. A runsum of length 2 is of the form $j + (j + 1)$, j is a positive integer; and which is the same as $2j + 1$, an odd number.
3. Every multiple of 3, except 3 itself, has a runsum of length 3. A runsum of length 3 is of the form $j + (j + 1) + (j + 2)$, j is a positive integer; or $3j + 3$, which is a multiple of 3. Notice that $3j + 3 = 3j + T_2$ and $2j + 1 = 2j + T_1$, where T_n is the nth triangular number. The generalization is given next.
4. Every natural number of the form $ij + T_{i-1}$, $i,j \geq 1$, has a runsum of length i. ($T_0 = 0$ is included here.)

Table 2. Runsum Representations of $Ft_5 = 135$

Length: i	First Term: j	Runsum Representation
1	135	135
2	67	67 + 68
3	44	44 + 45 + 46
4	129/4	-
5	25	25 + 26 + 27 + 28 + 29
6	20	20 + 21 + 22 + 23 + 24 + 25
7	114/7	-
8	107/8	-
9	11	11 + 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19
10	9	9 + 10 + 11 + 12 + 13 + 14 + 15 + 16 + 17 + 18
11	80/11	-
12	69/12	-
13	57/13	-
14	44/14	-
15	2	2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 + 16
16	15/16	-

P-ISSN 2350-7756 | E-ISSN 2350-8442 | www.apjmr.com
Asia Pacific Journal of Multidisciplinary Research, Vol. 3, No. 4, November 2015

Hence, to find a runsum representation of a factoriangular number, there is a need to determine a positive integer *j* such that $Ft_n = ij + T_{i-1}$ or

$$j = \frac{Ft_n - T_{i-1}}{i},$$

where *i* is the length of the runsum and *j* is the first term. For instance, consider finding the runsums of Ft_5 = 135, the results of which are given in Table 2.

Length *i* > 16 was not included anymore since T_{16} = 136 is greater than Ft_5 = 135 that will definitely result to a negative *j*. It was given in Table 1 that R(135) = 8 and in Table 2, these 8 representations are those runsums of length 1, 2, 3, 5, 6, 9, 10 and 15. As indicated by the non-integer value of *j*, 135 has no runsum representation of length 4, 7, 8, 11, 12, 13, 14 and 16.

Of special interest here is that when *i* = *n* of Ft_n. Notice that Ft_5 = 135 has a runsum of length 5. The list of runsum representations of length *n* of the first 10 Ft_n is presented in Table 3.

Table 3. Runsum Representations of Length *n* of the First 10 Factoriangular Numbers

n	Ft_n	Runsum Representation of Length *n*
1	2	2
2	5	2 + 3
3	12	3 + 4 + 5
4	34	7 + 8 + 9 + 10
5	135	25 + 26 + 27 + 28 + 29
6	741	121 + 122 + 123 + 124 + 125 + 126
7	5068	721 + 722 + 723 + 724 + 725 + 726 + 727
8	40356	5041 + 5042 + 5043 + 5044 + 5045 + 5046 + 5047 + 5048
9	362925	40321 + 40322 + 40323 + 40324 + 40325 + 40326 + 40327 + 40328 + 40329
10	3628855	362881 + 362882 + 362883 + 362884 + 362885 + 362886 + 362887 + 362888 + 362889 + 362890

A theorem regarding this runsum representation of a factoriangular number is hereby established:

Theorem 1. For n ≥ 1, Ft_n has a runsum representation of length n.

Proof:

For *n* = 1, Ft_1 = 2 has the trivial runsum of length 1.

For *n* ≥ 2, consider the formula for determining a positive integer $j = (Ft_n - T_{i-1})/i$, where *i* is the length and *j* is the first term of the runsum, as previously discussed. But here, let *i* = *n*, and to prove the theorem, show that *j* is a positive integer. Hence,

$$j = \frac{Ft_n - T_{n-1}}{n}$$

$$\Leftrightarrow \quad j = \frac{n! + \frac{n(n+1)}{2} - \frac{(n-1)n}{2}}{n}$$

$$\Leftrightarrow \quad j = \frac{n(n-1)! + \frac{n^2 + n - n^2 + n}{2}}{n}$$

$$\Leftrightarrow \quad j = \frac{n(n-1)! + n}{n}$$

$$\Leftrightarrow \quad j = (n-1)! + 1,$$

which implies that *j* is a positive integer and the theorem was proven.

It has to be noted that the first part of the proof (that is, for *n* = 1) can be incorporated to the second part of the proof (that is, for *n* ≥ 2 to become for *n* ≥ 1) if *n* = 1 → T_0 = 0 is to be considered.

With the above proof, the following corollary had been established as well:

Corollary 1.1. The first term of the runsum representation of length n of the Ft_n, for n ≥ 1, is (n – 1)! + 1.

Since *n* is the length of the runsum considered in Theorem 1 and Corollary 1.1, the proof of the next corollary is trivial:

Corollary 1.2. The last term of the runsum representation of length n of the Ft_n, for n ≥ 1, is (n – 1)! + n.

Nevertheless, using the fundamental formula for factoriangular number, the terms of its runsum of length *n* can also be shown as follows:

$$Ft_n = n! + T_n \quad \Leftrightarrow$$

$$Ft_n = n(n-1)! + (1 + 2 + 3 + ... + n) \quad \Leftrightarrow$$

$$Ft_n = [(n-1)! + 1] + [(n-1)! + 2] + [(n-1)! + 3] + ... + [(n-1)! + n].$$

Notice that *n*(*n* – 1)! Is the addition of (*n* – 1)! *n* times. Again, the first and last term, respectively, of the runsum representation of length *n* of any Ft_n are (*n* – 1)! + 1 and (*n* – 1)! + *n*.

The first terms of the runsums of length *n* of Ft_n for *n* ≥ 1 form an interesting sequence, {2, 2, 3, 7, 25, 121, 721, 5041, 40321, 362881, …}, which is the same as sequence A038507 in [1]. The last terms of the runsums of length *n* of Ft_n for *n* ≥ 1 also form an interesting sequence, {2, 3, 5, 10, 29, 126, 727, 5048, 40329, 362890, …}, which is the same as sequence A213169 in [1]. Note that the first term, in both sequences, is the single term of the trivial runsum of Ft_1 = 2.

When the first and last terms of the runsums of length n of Ft_n, for $n \geq 2$, are added, that is,
$[(n-1)!+1]+[(n-1)!+n]=2(n-1)!+n+1$,
the sums form a new sequence, {5, 8, 17, 54, 247, 1448, 10089, 80650, 725771, …}. Further, the following theorem was also established:

Theorem 2. For $n \geq 2$, the sum of the first term and the last term of the runsum of length n of Ft_n is twice the Ft_n divided by n.

Proof:

Since the sum of the first and last terms of the runsum of length n of Ft_n is $2(n-1)! + n + 1$, there is a need to show that $2Ft_n / n$ is equal to that. Hence,

$$\frac{2Ft_n}{n} = \frac{2\left[n!+\frac{n(n+1)}{2}\right]}{n}$$

$$\Leftrightarrow \quad \frac{2Ft_n}{n} = \frac{2n(n-1)!+n(n+1)}{n}$$

$$\Leftrightarrow \quad \frac{2Ft_n}{n} = 2(n-1)!+n+1,$$

and the proof is completed.

Since a triangular number is a sum of integers from 1 to n, it is also a runsum – a special runsum that begins at 1 and whose longest representation is of length n. What makes triangular numbers as runsums more special is that any other runsum can be expressed as a difference of two triangular numbers. For example:

$$25+26+27+28+29=(1+2+3+...+29)-(1+2+3+...+24)$$
$$\Leftrightarrow \quad 135=435-300$$
$$\Leftrightarrow \quad Ft_5=T_{29}-T_{24}.$$

The eight different expressions of $Ft_5 = 135$ as difference of two triangular numbers were determined and presented in Table 4.

Table 4. Expressions of $Ft_5 = 135$ as Difference of Two Triangular Numbers

Length	Runsum Representation of Ft_5	Difference of Two Triangular Numbers
1	135	$T_{135} - T_{134}$
2	67 + 68	$T_{68} - T_{66}$
3	44 + 45 + 46	$T_{46} - T_{43}$
5	25 + 26 + 27 + 28 + 29	$T_{29} - T_{24}$
6	20 + 21 + 22 + 23 + 24 + 25	$T_{25} - T_{19}$
9	11 + 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19	$T_{19} - T_{10}$
10	9 + 10 + 11 + 12 + 13 + 14 + 15 + 16 + 17 + 18	$T_{18} - T_8$
15	2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 + 16	$T_{16} - T_1$

The simple conversion from a runsum representation to a difference of two triangular numbers is easy to see. Notice that it is just the difference of the triangular number of the last term and the triangular number of the first term minus 1. Further, the length of the runsum is equal to the difference between the subscripts of T.

However, the process can also be reversed, that is, expressing first a factoriangular number as a difference of two triangular numbers before converting it into a runsum representation. As in the previous discussion, there is a need to find a positive integer j such that $j=(Ft_n-T_{i-1})/i$, where i is the length and j is the first term of the runsum. After finding j, then

$$Ft_n = T_{j+i-1} - T_{j-1}$$

recalling that the subscript of the first T is the last term of the runsum, hence $j + i - 1$, and the subscript of the second T is the first term minus 1 or $j - 1$. However, there is still a need to show that this $Ft_n = T_{j+i-1} - T_{j-1}$ is the same as the Ft_n in $j=(Ft_n-T_{i-1})/i$ or $Ft_n = ij + T_{i-1}$. Hence,

$$Ft_n = T_{j+i-1} - T_{j-1}$$

$$\Leftrightarrow \quad Ft_n = \frac{(j+i-1)(j+i)}{2} - \frac{(j-1)j}{2}$$

$$\Leftrightarrow \quad Ft_n = \frac{(j^2-2ij+i^2-j-i)-(j^2-j)}{2}$$

$$\Leftrightarrow \quad Ft_n = \frac{2ij+i^2-i}{2}$$

$$\Leftrightarrow \quad Ft_n = ij + \frac{(i-1)i}{2}$$

$$\Leftrightarrow \quad Ft_n = ij + T_{i-1}.$$

Of another special interest is when the length of the runsum is equal to n of Ft_n. That is, for $Ft_5 = 135$, consider length 5 and examine the representation of Ft_5 as difference of two triangular numbers, in particular, $Ft_5 = T_{29} - T_{24}$. Accordingly, using the runsums of length n of the first 10 Ftn in Table 3, they are written as differences of two triangular numbers and presented in Table 5.

Examining the subscripts of T, notice the familiar sequences {2, 3, 5, 10, 29, 126, 727, 5048, 40329, 362890, …} which is $(n - 1)! + n$, being the last terms in the runsum representations, and {1, 1, 2, 6, 24, 120, 720, 5040, 40320, 362880, …} which is a sequence of factorials or more particular in this case, as one less than the first terms of the runsum representations, that is, $[(n - 1)! + 1] - 1 = (n - 1)!$.

P-ISSN 2350-7756 | E-ISSN 2350-8442 | www.apjmr.com
Asia Pacific Journal of Multidisciplinary Research, Vol. 3, No. 4, November 2015

Table 5. Runsum Representations of Length *n* of the First 10 Factoriangular Numbers Expressed as Difference of Two Triangular Numbers

N	Ft_n	Difference of Two Triangular Numbers
1	2	$T_2 - T_1$
2	5	$T_3 - T_1$
3	12	$T_5 - T_2$
4	34	$T_{10} - T_6$
5	135	$T_{29} - T_{24}$
6	741	$T_{126} - T_{120}$
7	5068	$T_{727} - T_{720}$
8	40356	$T_{5048} - T_{5040}$
9	362925	$T_{40329} - T_{40320}$
10	3628855	$T_{362890} - T_{362880}$

Another identity involving factoriangular numbers, triangular numbers and factorials can now be established as given in the following theorem:

Theorem 3. For n ≥ 1, T_n is the nth triangular number and Ft_n is the nth factoriangular number, $Ft_n = T_{(n-1)!+n} - T_{(n-1)!}$.

Proof:

Using the definition of triangular number,

$$T_{(n-1)!+n} - T_{(n-1)!} = \frac{[(n-1)!+n][(n-1)!+n+1]}{2} - \frac{(n-1)![(n-1)!+1]}{2} \Leftrightarrow$$

$$T_{(n-1)!+n} - T_{(n-1)!} = \frac{[((n-1)!)^2 + 2n(n-1)! + (n-1)! + n^2 + n] - [((n-1)!)^2 + (n-1)!]}{2}$$

$$\Leftrightarrow \qquad T_{(n-1)!+n} - T_{(n-1)!} = \frac{2n(n-1)! + n^2 + n}{2}$$

$$\Leftrightarrow \qquad T_{(n-1)!+n} - T_{(n-1)!} = n(n-1)! + \frac{n(n+1)}{2}$$

$$\Leftrightarrow \qquad T_{(n-1)!+n} - T_{(n-1)!} = n! + T_n$$

$$\Leftrightarrow \qquad T_{(n-1)!+n} - T_{(n-1)!} = Ft_n,$$

which finally arrived at the definition of factoriangular number that completes the proof.

A corollary to this theorem was also established in the above proof. This corollary does not involve factoriangular number but is interesting enough to be mentioned as follows:

Corollary 3.1. For n ≥ 1 and T_n is the nth triangular number, $n! = T_{(n-1)!+n} - T_{(n-1)!} - T_n$.

Runsums are analogous to trapezoidal arrangements. Numbers that can be expressed as difference of two triangular numbers are also called trapezoids or trapezoidal numbers. If trivial runsums are included then all numbers are trapezoidal numbers and the number of trapezoidal arrangements is equal to the number of runsums. If trivial runsums are excluded then the number of trapezoidal arrangements is one less than the number of runsums. The number of trapezoidal arrangements for the first 20 factoriangular numbers is also given in Table 1.

Further, if trivial runsums are excluded, then the powers of 2 are non-trapezoidal numbers. They do not have trapezoidal arrangements; what they have are rectangular arrangements. Furthermore, if in the difference of two triangular numbers T_0 are not included, for example $T_5 - T_0$ is not considered as a difference of two triangular numbers, then the number of trapezoidal arrangements of triangular numbers is two less than the number of runsums.

Relative to the above discussions, examples of geometrical arrangements are given in Fig. 1.

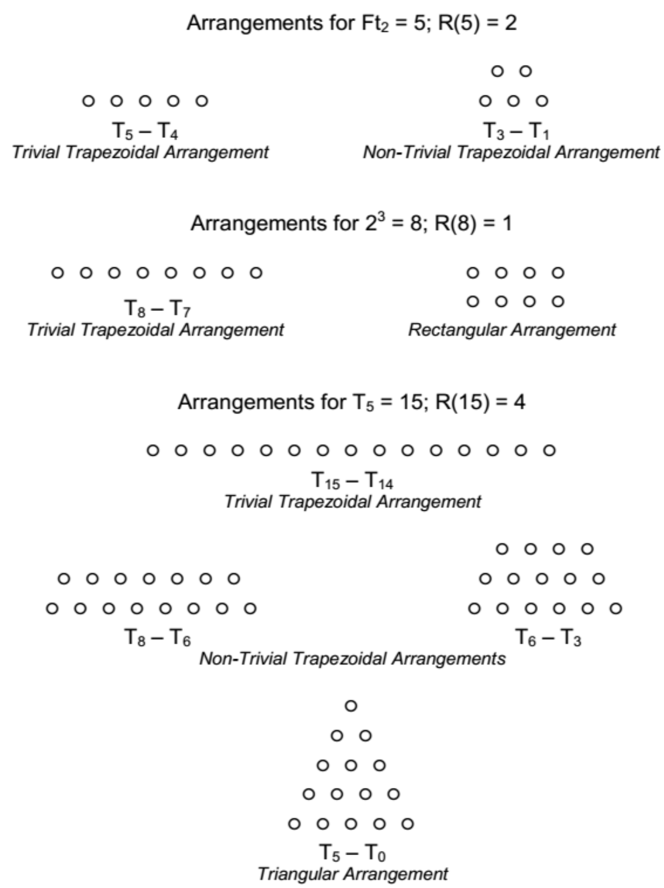

Fig. 1. Examples of Geometrical Arrangements

To exclude triangular arrangements and trivial trapezoidal arrangements or one-row arrangements, new definitions of trapezoidal number and trapezoidal arrangement, which may be different from previous definitions given in the literature, are stated herein. Note that a trapezoidal number can be arranged in a trapezoidal array of points, which may be done in one or more ways.

Definition 3. A trapezoidal number is a number that can be written as $T_m - T_n$, where T_i is the i^{th} triangular number and m and n are positive integers such that m is greater than n by at least two.

Definition 4. The number of trapezoidal arrangements of any positive integer k, except triangular numbers and powers of 2, is given by

$$Tr(k) = [(e_1+1)(e_2+1)(e_3+1)\cdots]-1$$

and the number of trapezoidal arrangements of triangular numbers, T_i, except $T_1 = 1$, or for $i \geq 2$, is given by

$$Tr(T_i) = [(e_1+1)(e_2+1)(e_3+1)\cdots]-2\,,$$

where e_1, e_2, e_3, ... are the exponents of the prime numbers, p_i, greater than 2 in the prime factorization $k = 2^c \cdot p_1^{e_1} \cdot p_2^{e_2} \cdot p_3^{e_3} \cdots$, integer $c \geq 0$. The number 1 and numbers equal to a power of 2 have no trapezoidal arrangement.

These definitions also imply that a triangular number that is equal to odd prime times a power of 2 is not a trapezoidal number and has no trapezoidal arrangement. It has only the one-row arrangement and the triangular arrangement of points.

With the above definitions, consider now the following theorem and its corollary:

Theorem 4. Except $Ft_1 = 2$, all factoriangular numbers are trapezoidal numbers.

Proof:

As provided in Theorem 3, Ft_n for $n \geq 1$ can always be expressed as $T_{(n-1)!+n} - T_{(n-1)!}$. Clearly, $(n-1)! + n$ is greater than $(n-1)!$ by n. Thus, when $n = 1$, Ft_1 is not a trapezoidal number but when $n \geq 2$, Ft_n is a trapezoidal number since $(n-1)! + n$ exceeds $(n-1)!$ by at least two.

Corollary 4.1. Except $Ft_1 = 2$, no factoriangular number is a power of 2.

Proof:

A factoriangular number, except 2, is a trapezoidal number and hence, it has at least one trapezoidal arrangement and is therefore, not a power of 2.

Finally, for this section, consider the concept of the politeness of a number. This concept is very closely related to runsums and trapezoidal arrangements. The definitions of polite number and politeness are as follows:

Definition 5. A polite number is a positive integer that can be written as the sum of two or more consecutive positive integers.

Definition 6. The politeness of a positive integer is the number of ways the integer can be expressed as the sum of consecutive positive integers.

Thus, all numbers, except the powers of 2, are polite numbers and the politeness is the same as the number of runsum representations excluding the trivial runsum. The politeness of the first 20 factoriangular numbers are also given in Table 1.

It may be further deduced that all trapezoidal numbers are polite numbers but the converse is not true. By Definition 5, all triangular numbers are also polite numbers. Hence, triangular numbers that are equal to odd prime times a power of 2 are the polite numbers that are not trapezoidal.

The following theorem, similar to Theorem 4, had been established also:

Theorem 5. Except $Ft_1 = 2$, all factoriangular numbers are polite numbers.

Proof:

By Theorem 4, all factoriangular numbers are trapezoidal and by Definition 3 and Definition 5, all trapezoidal numbers are polite. Hence, all factoriangular numbers are polite numbers.

CONCLUSIONS

Factoriangular numbers resulted from the addition of corresponding factorials and triangular numbers. These new numbers have interesting runsum representations and trapezoidal arrangements.

The *n*th factoriangular number has a runsum representation of length *n*, the first term of which is $(n-1)! + 1$ and the last term is $(n-1)! + n$. For $n \geq 2$, the sum of the first and last terms of this runsum is twice the *n*th factoriangular number divided by *n*. Further, another relation between factoriangular number and triangular number is given by $Ft_n = T_{(n-1)!+n} - T_{(n-1)!}$ while another relation between factorial and triangular number is given by $n! = T_{(n-1)!+n} - T_{(n-1)!} - T_n$.

The trapezoidal number and the number of trapezoidal arrangements can be redefined to give clear distinction from runsum, polite number, number of runsums and politeness. With the new definitions, it can be concluded that, except for 2, all factoriangular numbers are trapezoidal and polite numbers and no factoriangular number greater than 2 is a power of 2. Also, all trapezoidal numbers are polite but not all polite numbers are trapezoidal.

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