

# Portfolio Dynamics. A Macroeconomic Model

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**Abstract** *This article approaches the issue-oriented analysis of three markets: the output market, the money market and the labor market. The model we take into account regards the market demand and preserves a dynamic structure, mainly based on the money market component. The complexity of this analysis is supposed to use an extensive macroeconomic modeling and the results are obtained in order to provide a wide panel of indicators and measures to help the users to distinguish correctly between different investment opportunities. The aim is to ensure the investor a convincing profitability, through the configuration of his own portfolio, with the associated risk that he is willing to assume.*

**Key words** Macroeconomic model, Gross Domestic Product, portfolio, bonds, market

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## 1. Introduction

The analysis of aggregate economic behavior has been the main concern for the macroeconomic theory in the latest years and it is based on the main key variables which include: the rate of employment, the real income, consumption and investment which are settled as real variables, the money supply, the interest rate and the return rate on capital as the financial variables and nominal variables such as: the prices and the inflation rate. The analysis is increasingly focusing increasingly on international variables (trade balance and balance of payments), which are considered to have an important influence on the dynamic on the market.

## 2. Literature review

Anghelache (2016) develops on the instrument and methods of econometrics, applicable in economic researches. Anghelache *et al.* (2015) cover several key aspects regarding economic modeling. Bo *et al.* (2015) approach a particular situation of dynamic portfolio selection. Brokas *et al.* (2016) study the skewness in dynamic portfolio choice. Faria and Correia-da-Silva (2012) research the relevance of stochastic volatility in the scope of dynamic portfolio choice. Legendret and Togola (2015) offer some explicit solution to dynamic portfolio choice issues. Princ studies the case of Central European countries in the regard of dynamic portfolio formations. Anghelache (2008) and Anghelache *et al.* (2013) provide solid reference for studies based on macroeconomic indicators.

## 3. Types of markets

Invariably these notions are abstract in the sense that we cannot refer to a specific price level, or at a certain interest rate, but most of all there are many values of interest rate and variety of goods and services, each with individual price. So it therefore requires a high level of aggregation in order to formulate an economic model.

Conventionally economic analysis divides the level of aggregation into four types of market, as follows:

- The output market;
- Money market;
- Bond market;
- Labor market.

Regarding the four main market types listed above, only three of them might be considered to be independent and therefore one of them can be eliminated, as long as the equilibrium of the balance is made by the other three markets. The bond market is conventionally eliminated and the analysis focuses on the three other markets.

To have a complete and correct assessment of the current macroeconomic theory is important to take into account when analyzing, the evolution over time and the default elements of the data series in order to develop a model of complex technical methods (Legendret and Togola, 2015). The base model is built in the following, primarily starting with the assessment of the market demand, described by the results/outputs obtained in the economy and monetary market. The other side is represented by the market supply on the labor market, where both demand and supply sides consider a dynamic macroeconomic model with the correct specification of monetary policy.

The starting point for building a macroeconomic model considers taking into account the national income accounts (Faria and Correia-da-Silva, 2012). Thus, from this point of view the gross domestic product (GDP) can be calculated on the basis of expenditure in the final output value. In the case of a closed economy compositing relation to GDP is as follows:

$$\text{GDP} = \text{C} + \text{I} + \text{G} \quad (1)$$

Where,

C = consumption expenditures in private sector;

I = private investments;

G = government expenditure on products and services.

Therefore the result can be consumed either by the private sector or the private sector invested or acquired by the Government. We further agree that the invested goods have an invested amount that do not suffer a devaluation, so we will not distinguish between gross and net worth of investments (Anghelache *et al.*, 2015).

In an open economy, national income accounts transactions have to consider the influence of the foreign residents (Anghelache, 2008) the above relationship can be written as:

$$\text{GDP} = \text{C} + \text{I} + \text{G} + \text{X} - \text{M} \quad (2)$$

Where,

X = exports;

M = imports.

Besides the two relationships of composing the GDP above, there are still at least two other methods of decomposing of GDP, namely:

$$\text{GDP} = \text{C} + \text{S} + \text{T} \quad (3)$$

Where,

S = total savings;

T = net taxes.

In the above equation, the revenue can be divided in terms of consumption, savings or tax payments (Bo *et al.*, 2015). On the other hand, the gross domestic product can be expressed in terms of revenue generated by the production process; such income may be realized as a form of salary or the profit in the private sector, income from rents, dividends, interest income, etc. All forms of expression are important to their income and are selected as a part of a specific analysis in time. Generally, the two forms listed above are most commonly used in conventional macroeconomic theory and in a particular way the income component is very important (Princ, 2013).

Before proceeding to the analysis of the model, we make the distinction between the notions of GDP in nominal terms compared to the expression in real terms (Brocas *et al.*, 2016). National income accounts are conventionally measured in local currency (USD, RON etc.), in nominal terms. However, nominal GDP growth between two different time periods may be due both to an increase in prices and an increase in real output or in both ways. In order to measure the work output in real terms it is necessary to adjust the nominal GDP deflated by an index of the price level. However, because aggregation is one level up from general, the issue with the choice of index deflating nominal level to the real level disappears.

#### 4. Market output

To simplify the model, we work with the highest degree of aggregation and assume that the economy produces a single good that can be used either for consumption or to be invested (Bo *et al.*, 2015). In the latter case, when it is used for consumption, it is accumulated as a capital good and combined with another factor -labor- producing additional revenue. For a closed economy, goods market equilibrium is given by equation:

$$Y = C+I+G \tag{4}$$

Where,

- Y= real revenue or national revenue;
- C = real private consumption demand;
- I = real private investments demand;
- G = real government expenditures.

The equation above was written in real terms. Assuming that we have a single income, this situation applies both in real and in nominal terms. If there are multiple outputs, conversion of nominal income to real income involves choosing a product expressed in the national currency for the definition of real income. The reason we consider the real value and not the nominal value takes into account the idea that individuals express their request in terms of real demand (real consumption, real investment etc.). Up to what level this is valid, it remains an empirical question, but according to Tudi, at least in terms of consumption, individuals are influenced by "money illusion". Another specification is represented by the distinction between consumer demand and investment demand, on one hand and the government spending and private consumption, on the other hand. The reason for this kind of contrast is that government spending decisions are related to different considerations versus private spending decisions. The same thing is available also for demand. We have to consider however that government spending can only take the form of consumption and investment. If we know what makes the three components C (consumption), I (investments), G (government), we may have a theory for Y (revenue). Therefore, the first step is to develop relationships behavior for these three aggregate demands. The simplest theory on this subject considers consumption function:

$$C=C(Y) \quad 0 < C' < 1 \tag{5}$$

Where,

$$C_u = \bar{I}, \quad G = \bar{G}.$$

Meaning that consumption depends on income, while investments and government spending are exogenous variables; by substituting these relations we get (Anghelache, 2016):

$$Y=C(Y)+ \bar{I} + \bar{G} \tag{6}$$

Meaning that Y can be solved uniquely by endogenous variables: the value of investments and government spending.

The next phase of analysis involves the use of a closer relationship between consumption in general and investments in particular. Starting with a first analysis, consumption can be expressed as a function of real disposable income, namely:

$$C = C(Y^D) \quad 0 < C' < 1 \quad (7)$$

Where,  $Y^D = Y - T$

and

$Y^D$  = real disposable income,  $T$  = real taxes.

Taxes are usually endogenous variables, with varying changes in income levels, but for now we assume the value of  $T$  as constant for applying our model.

In addition, more complex studies on the consumption function were performed from considerations concerning to maximize the utility. The basic idea refers to the fact that rather than being limited to current income, consumption is influenced by a continuous measurement of income that actually reflects the saving expectation of the economic agent and this idea that can be formulated in many ways.

One of the approaches, the one of Friedman (1957) is to express the life revenue stream to a permanent stable income extent.

Another approach, associated rather with "Life Cycle Theory" of Modigliani refers to the fact that an individual's life savings are based on its current health and forecasting future economies are based on current savings.

The consequence of these contributions is that consumption depends hypothetically to some form of health, interest rates and disposable income.

This, the equation  $C = C(Y^D)$  can be written as:

$$C = C(Y^D, r, A), \quad 0 < C_1 < 1; C_3 > 0 \quad (8)$$

Where,

$R$  = interest rate;

$A$  = real disposal from the private sector.

Without deviating from the topic, a lot of questions regarding this model need an answer. The first one is related to the definition of real disposable income, an idea widely approached in 1977 by Turnovsky, referring to the fact that expressing the model in real terms, the disposable income adequate definition is:

$$Y^D = Y - T + X \quad (9)$$

Where,

$B$  = the nominal stock of governmental bonds;

$P_b$  = the price of governmental bonds;

$P$  = output price level;

$X$  = real capital gain or loss rate.

The definition of disposable income is adjusted to include the interest of governmental bonds with gains or losses of real revenues. The expression of effective variable  $X$  (rate of real capital gain or loss) depends on the form in which assets are denominated (real or nominal value) and the manner in which the prices are expected to change.

We showed in the previous equation that expressing real disposable income,  $Y^D$  ensures expression of savings and accumulation availability in real terms and even if the theoretically definition is the appropriate one, we can still express the position of the consumer in equivalent terms ( $Y^D$ ) specification which implies that the marginal propensity to consume of interest income and capital gains is zero. It remains an empirical question whether the above assumption is valid or not, as an assumption that the marginal propensity to consume of any nature is the same.

The second question is related to the form taken by the interest rate, considering that previous theories related to the consumption function do not take into account the influence of interest rate, considering that it is not significant in statistically terms. Weber's studies (1970) suggest that this is due to the lack of specification of how the interest rate was included in the composition of consumption function. Recent theories include interest rate as a linear influence, considering it insignificant. However, consumer utility of maximizing the consumption function, specified in terms of real and future consumption indicates that the actual interest rate gives its own influence, expressed by  $r-\pi$ , where  $\pi$  is the expected inflation rate.

Finally there is a question about the availability of adequate definition of real private revenues and defining a way for a macroeconomic model at a low level is given by:

$$A = \frac{M + P_b B + P_k K}{P} \quad (10)$$

Where,

M = nominal stock of money liabilities of the central bank;

K = actual stock of physical capital;

P<sub>k</sub> = price of capital goods.

In the upper equation includes two questions: the first one regards the nature of government bonds and the economic literature adopts two assumptions:

B are very short-term bonds with a fixed nominal price, so  $P_b = 1$ , they are permanent bonds with a fixed coupon payment per time unit, the price being in an inverse relation with the nominal interest rate,  $P_b = 1/r$ .

Starting from the assumption of an efficient market bonds, rational expectations and neutral risk, we arrive at the following relationship between nominal interest rate and the bond coupon price and unit value equal to  $P_b$  and T, meaning the time the bond matures:

$$r(t) = \frac{1}{P_b(t)} + \frac{\dot{P}_b(t)}{P_b(t)} \quad t \leq T; \quad P_b(T) = \dot{P}_b \quad (11)$$

The above equation assumes that prior to the maturity date of the bond; risk arbitrage neutral ensures that short-term interest rate equals the bond coupon rate of gain  $1/P_b$  and capital gain rate. On maturity, the bond is redeemed at face value  $\dot{P}_b$  and this relation has a significant importance in predicting future price bonds.

The second question that rises when defining the real private liabilities is to what extent government bonds constitute real liability. One can argue that future tax liabilities on interest earned from government bonds should be compensated, but current theories take into account government bonds as part of the actual availability and do not undergo other changes.

Considering private investment, the simplest form of the investment is:

$$I = I(r-\pi) \quad I' < 0 \quad (12)$$

That relationship may be explained by reduced current value criteria; a company that seeks to maximize profit has the opportunity to invest in those projects that have a positive return on the reduced investment amount. As interest rates increase, the number of projects on this asset decrease, therefore the value of the investments made by the company decreases, reaching a negative relationship between  $r$  and  $\pi$ . In the opposite case, if you expect the price to rise, the relevant interest rate is the actual rate,  $\pi - r$  (for example).

A lot of economic models propose that function as investment relationship:

$$I = I(r-\pi, Y, K) \quad I_1 < 0, I_2 > 0, I_3 < 0 \quad (13)$$

where investments depend directly on the level of income and have an inverse relationship with the actual rate of interest and the level of existing capital stock.

Suppose the desired capital stock is determined by  $K^*$

$$K^* = F(r-\pi, Y) \quad F_1 < 0, \quad F_2 > 0 \quad (14)$$

and that the current capital stock is adjusted with the help of  $K^*$  with a proportional rate with the difference  $(K^* - K)$  (the conventional relationship stock adjustment) so that the relationship:

$$I = K = \lambda (K^* - K) \quad (15)$$

Therefore the properties of the investment function derive as in the previous equation expression of  $K^*$ .

The basic macroeconomic results suppose that the outputs can be transformed into investment without cost. Latest developments of this theory of investment introduced the cost adjustment in this process. Substituting the consumption function and the function of investment products in the market balance, we reach the IS curve:

$$Y = C(Y-T, r-\pi, \frac{M+B+P_kK}{P}) + I(r-\pi) + G \quad (16)$$

Where we used the simplified measurement of disposable income and assumed that  $P_b=1$ . Taking into account the output price  $P$ , the capital assets price  $P_k$ , expected inflation rate  $\pi$  and the stock of assets  $M, B, K$ , the IS curve emerges from this combination of  $Y$  and  $r$  that will keep the market balance products. Deriving both sides of the equation depending on  $Y$ , we obtain:

$$\frac{(dr)}{(dY)IS} = \frac{1-C1}{I'+C2} \quad (17)$$

and  $I'+C2 < 0$  involves a graph with a curve with a downward slope, the economic explanation is that if income grows, consumption grows, but with a lower value. The only method by which additional income can be spent is through additional investments and for this to happen, the interest rate must decrease.

## 5. Conclusions

IS curve also depends on the position the function of investment; consumption and government spending function have. Therefore, an increase in government spending or consumption or investment functions will move to another curve IS curve positioned above the chart. This shift may be due to either:

- (I) an increase in inflation rate  $\pi$ ;
- (II) a decrease of the goods prices (outputs);
- (III) an increase in the stock of assets;
- (IV) an increase in the price of capital;
- (V) to lower taxes.

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