



# Constructing an optimal portfolio using Sharpe's single index model

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## I. INTRODUCTION

The foundation of Modern Portfolio Theory was laid by Markowitz in 1951. He began with the simple premise that since almost all investors invest in multiple securities rather than one, there must be some benefit in investing in a portfolio of securities. He measured riskiness of a portfolio through variability of returns and showed that investment in several securities reduced this risk. His work won him the Nobel Prize for Economics in 1990. Markowitz's work was extended by Sharpe in 1964, Lintner in 1965 and Mossin in 1966. Sharpe shared the Nobel Prize for Economics in 1990 with Markowitz and Miller for his contribution to the Capital Asset Pricing Model (CAPM). This model breaks up the riskiness of each security into two components - the market related risk which cannot be diversified called systematic risk measured by the beta coefficient and another component which can be eliminated through diversification called unsystematic risk.

### ❖ Single Index Model:-

Markowitz's efficient portfolio combines securities with a correlation of negative one in order to reduce risk in the portfolio to gain optimum return. In order to study N-security portfolio using Markowitz model, the inputs required are:

- Expected Returns,
- Variances of Returns,

The Markowitz model is extremely demanding in its data needs for generating the desired efficient portfolio. It requires  $N(N+3)/2$  estimates ( $N$  expected returns +  $N$  variances of returns +  $N*(N-1)/2$  unique covariance of returns). Because of this limitation the single index model with less input data requirements has emerged. The Single index model requires  $3N+2$  estimates (estimates of alpha for each stock, estimates of beta for each stock, estimates of variance see 2 for each stock, estimate for expected return on market index and an estimate of the variance of returns on the market index sum 2) to use the Markowitz optimization framework.

The steps to be followed in constructing optimal portfolio are as follows:

1. Determination of objectives
2. Selection of securities based on the objectives
3. Choose a suitable approach for construction portfolio
4. Apply the approach and construct the portfolio
5. Assessment of risk and return

## II. OBJECTIVES OF THE RESEARCH

- To create optimal portfolio using BSE SENSEX Scripts with the help of Sharpe Portfolio Optimization Theory.
- To apply Theoretical concept of Portfolio optimization theory on real market Movement and scripts.
- To help investors in taking investment decisions cautiously after studying risk and return involved in the security with the help of Sharpe Portfolio Optimization Theory.
- To aware the technical analyst regarding the practical applicability of Sharpe Portfolio Optimization Theory

## III. RESEARCH METHODOLOGY

### Research Hypothesis:

There is significance difference in the risk - return profile of portfolio created by considering any of the following parameters.

1. Return of feasible security.
2. Risk of feasible security.
3. Sharpe Single Index Portfolio optimization theory.

### Statistical Hypothesis:

1. There is no significance difference in monthly return of portfolio created with the help of Sharpe single index model and market portfolio (Sensex) (Anova test)
2. There is no significance difference in monthly variance of portfolio created with the help of Sharpe single index model and market portfolio (Sensex). (F - Test)

### Analysis Of Variance (ANOVA):

Analysis of variance (abbreviated as ANOVA) is an extremely useful technique concerning researches in the fields of economics, biology, education, psychology, sociology, and business/industry and in researches of several other disciplines.

This technique is used when multiple sample cases are involved. As stated earlier, the significance of the difference between the means of two samples can be judged through either z-test or the t-test, but the difficulty arises when we happen to examine the significance of the difference amongst more than two sample means at the same time.

## IV. LITERATURE REVIEW

### 1. Financial Portfolio Optimization through a Robust Beta Analysis:

This section of the topic based literature review discusses the various techniques for selecting security for portfolios, specifically Harry M. Markowitz's full covariance model and William F. Sharpe's Single Index Model. The purpose of this section is to get a general understanding of the portfolio problem that investors face on a regular basis and the methods that can assist in determining its solution. Furthermore, the two models described below establish the context and motivation for the research done in this thesis study. (*Department of Mechanical and Industrial Engineering University of Toronto*)

### 2. Optimal portfolio selection with or without the procedure of short sales:

The main purpose of this paper is to construct an optimal portfolio with the procedure of short sales and without the procedure of short sales by applying Sharpe's single-index model. The present study is based on the secondary data. For the purpose of constructing an optimal portfolio, a sample of thirty stocks listed on Bombay Stock Exchange (BSE) was selected in this study. BSE Sensitive Index (Sensex) has been used as market index. Monthly closing prices of selected stocks as well as market index for the period of Jan 2012 to Dec 2014 were used in this study. A unique cut off point was computed. The results of the study constructed an optimal portfolio and also represented the optimal portfolio with the 15 percentage invested in each stock. The present study found that the eleven out of thirty stocks have expected return greater than risk free rate of return and these eleven stocks have been used for optimal portfolio construction. (*Research Scholar, Department of Commerce, Kurukshetra University, Kurukshetra, Haryana*)

## V. APPLICATION OF SHARPE SINGLE INDEX MODEL ON ANALYZED DATA

### 1. Calculate the excess return to $\beta$ Ratio for each of the given stock under review

Table-1

Security	Expected Return	Beta	Variance Error	$(R_i - R_f)/B$
Axis Bank	-0.5149	2.6019	362.3575	-0.4631
Bajaj Auto	1.0527	1.1133	34.9814	0.325838
Bharti Airtel	0.5207	1.1921	54.4589	-0.14198
BHEL	1.6251	2.6063	108.8339	0.358806
Cipla	2.1541	0.1331	58.0559	10.99666
Coal India	0.8216	1.6841	61.5894	0.078127
Dr Reddy's	2.5560	0.1799	50.5652	10.37443
GAIL	1.4578	1.0677	52.2330	0.719082
HDFC	1.7753	0.9570	22.4792	1.134036
HDFC Bank	1.8782	1.2563	11.6838	0.945751
Hero Moto Corp	2.5361	0.7923	33.3813	2.329989
Hindalco	1.9422	1.4483	101.5328	0.864599
HUL	2.2957	0.5963	43.6191	2.69254
ICICI Bank	-1.2543	3.8446	191.9576	-0.50573
Infosys	-0.0529	0.5523	210.5488	-1.34513
ITC	0.9162	0.3634	24.6689	0.622604
L&T	0.8670	2.5205	94.5885	0.070231
Mahi. & Mahi.	1.6963	1.0250	40.9478	0.981742
Maruti Suzuki	3.8845	2.2902	58.1752	1.394857
NTPC	0.0872	1.3307	80.6352	-0.45301
ONGC	0.3370	1.6340	27.7936	-0.21604
Reliance Industry	0.2461	1.2732	24.8076	-0.34865
SBI	-2.9922	1.7614	412.1288	-2.09043



Sesa Sterlite	1.6754	1.4224	241.3086	0.692761
Sun Pharma	1.5506	0.4094	156.9271	2.102034
Tata Motors	2.5436	1.0452	50.2395	1.773479
Tata power	-0.3377	1.9610	68.8283	-0.52408
Tata Steel	0.7764	1.7060	137.0327	0.05065
TCS	3.1093	-0.4970	55.3013	-4.8676
Wipro	1.7020	-0.6030	79.8972	-1.67836

**2. Ranking of security on the basis of excess return to beta ratio.**

Risk free return = 8.311% p.a.

**Table-2**

Rank	Security	Expected Return	Beta	Variance Error	(Ri – Rf)/B
1	Cipla	2.1541	0.1331	58.0559	10.996659
2	Dr Reddy's	2.5560	0.1799	50.5652	10.374429
3	HUL	2.2957	0.5963	43.6191	2.69254
4	Hero Moto Corp	2.5361	0.7923	33.3813	2.3299891
5	Sun Pharma	1.5506	0.4094	156.9271	2.1020345
6	Tata Motors	2.5436	1.0452	50.2395	1.7734788
7	Maruti Suzuki	3.8845	2.2902	58.1752	1.3948573
8	HDFC	1.7753	0.9570	22.4792	1.1340355
9	Mahi. & Mahi.	1.6963	1.0250	40.9478	0.9817418
10	HDFC Bank	1.8782	1.2563	11.6838	0.9457511
11	Hindalco	1.9422	1.4483	101.5328	0.8645985
12	GAIL	1.4578	1.0677	52.2330	0.7190818
13	Sesa Sterlite	1.6754	1.4224	241.3086	0.6927612
14	ITC	0.9162	0.3634	24.6689	0.6226044
15	BHEL	1.6251	2.6063	108.8339	0.3588064
16	Bajaj Auto	1.0527	1.1133	34.9814	0.3258383
17	Coal India	0.8216	1.6841	61.5894	0.0781268
18	L&T	0.8670	2.5205	94.5885	0.0702307
19	Tata Steel	0.7764	1.7060	137.0327	0.0506495
20	Bharti Airtel	0.5207	1.1921	54.4589	-0.141983
21	ONGC	0.3370	1.6340	27.7936	-0.216044
22	Reliance Industry	0.2461	1.2732	24.8076	-0.34865
23	NTPC	0.0872	1.3307	80.6352	-0.453007
24	Axis Bank	-0.5149	2.6019	362.3575	-0.463097
25	ICICI Bank	-1.2543	3.8446	191.9576	-0.505733
26	Tata power	-0.3377	1.9610	68.8283	-0.524085
27	Infosys	-0.0529	0.5523	210.5488	-1.34513
28	Wipro	1.7020	-0.6030	79.8972	-1.678358
29	SBI	-2.9922	1.7614	412.1288	-2.090427
30	TCS	3.1093	-0.4970	55.3013	-4.867598

**1. Calculations for determining cut off rate:**

**Table-3**

Security	B2/E2	(Ri – Rf) *B/E2	Cum. (Ri – Rf)*B/E2	Cum.B2/E2	Ci
Cipla	0.000305	0.003358	0.003358	0.000305	0.048424
Dr Reddy's	0.00064	0.006638	0.009996	0.000945	0.142832
HUL	0.008153	0.021952	0.031948	0.009098	0.408878
Hero Moto Corp	0.018807	0.04382	0.075767	0.027905	0.781576
Sun Pharma	0.001068	0.002245	0.078013	0.028973	0.795966
Tata Motors	0.021744	0.038562	0.116575	0.050717	0.973454
<b>Maruti Suzuki</b>	<b>0.09016</b>	<b>0.12576</b>	<b>0.242335</b>	<b>0.140877</b>	<b>1.15445</b>
HDFC	0.040745	0.046206	0.288541	0.181622	1.151132
Mahi & Mahi.	0.025658	0.02519	0.313731	0.20728	1.135403
HDFC Bank	0.135091	0.127763	0.441493	0.342371	1.073128
Hindalco	0.02066	0.017863	0.459356	0.363031	1.063157
GAIL	0.021826	0.015695	0.475051	0.384857	1.046612
Sesa Sterlite	0.008384	0.005808	0.480859	0.393241	1.040194
ITC	0.005353	0.003333	0.484192	0.398594	1.035414
BHEL	0.062413	0.022394	0.506586	0.461007	0.955743
Bajaj Auto	0.035428	0.011544	0.51813	0.496435	0.916278



Coal India	0.046048	0.003598	0.521727	0.542484	0.853164
L&T	0.067165	0.004717	0.526444	0.609649	0.775682
Tata Steel	0.021238	0.001076	0.52752	0.630886	0.753683
Bharti Airtel	0.026095	-0.0037	0.523815	0.656981	0.72149
ONGC	0.096059	-0.02075	0.503062	0.75304	0.611941
Reliance Industry	0.065341	-0.02278	0.480281	0.81838	0.541212
NTPC	0.021962	-0.00995	0.470332	0.840342	0.517201
Axis Bank	0.018683	-0.00865	0.46168	0.859025	0.497467
ICICI Bank	0.077001	-0.03894	0.422738	0.936026	0.420609
Tata power	0.05587	-0.02928	0.393458	0.991896	0.37086
Infosys	0.001449	-0.00195	0.391509	0.993344	0.36852
Wipro	0.004551	-0.00764	0.383871	0.997895	0.35979
SBI	0.007528	-0.01574	0.368133	1.005424	0.342622
TCS	0.004467	-0.02174	0.34639	1.009891	0.32105

**2. Formation of the optimal portfolio by calculating the percentage invested in each security (as found above in step by the following formula)**

As per this model first 2 securities should be there in optimal portfolio. Proportion for these securities can be found out with the help of below mention formula.

$$X_i = \frac{Z_i}{\sum_{j=1}^n Z_j}$$

Where,

$$z_i = \frac{\beta_i}{\sigma_{e_i}^2} \cdot \left( \frac{R_i - R_f}{\beta_i} - C^{\text{a}} \right)$$

**Table-4**

Security	zi	Proportion
Cipla	0.022571873	0.17482121
Dr Reddy's	0.032797088	0.254016427
HUL	0.02102808	0.162864395
Hero Moto Corp	0.027902491	0.216107329
Sun Pharma	0.002472093	0.019146589
Tata Motors	0.012878204	0.099742858
Maruti Suzuki	0.009464213	0.073301192
	0.129114043	1

❖ **Analysis of Risk (Variance) of Portfolio (F- Test)**

**1. Portfolio V/s Sensex Risk Comparison**

Ho = There is no significance difference in the Variance of return of sensex and portfolio as per Sharpe single index model.

Ha = There is significance difference in the Variance of return of sensex and portfolio selected stock of Sharpe single index model.

**Table-5**

F-Test Two-Sample for Variances		
	Portfolio	Sensex
Mean	2.349932459	1.485489876
Variance	20.51850518	14.48533239
Observations	23	23
df	22	22
F	1.416502199	
P(F<=f) one-tail	0.210307775	
F Critical one-tail	2.047770309	



**Interpretation:**

Calculated value of F test is 1.416502199 and critical value is 2.047770309 than accepted region as it is lower than table value, which shows that there is enough evidence to reject null hypotheses. It indicates that there is significance difference in the Variance of return of Sensex and portfolio selected stock with the help of Sharpe single index model.

**2. Five measures for the security on a different portfolio:**

**Table-6**

Measure	Sharpe	Treynor	Jensen	Fama	MM
Sensex	0.209012	0.79549	0	0	0
Ranking	7	9	9	7	7
Cipla	0.191735	11.00029	1.358259	-0.13193	-0.06576
Ranking	8	1	3	8	8
Dr Reddy's	0.261211	10.37266	1.722932	0.372904	0.19867
Ranking	4	2	1	4	4
HUL	0.229921	2.692728	1.131323	0.146023	0.079581
Ranking	5	4	6	6	5
Hero Moto Corp	0.283267	2.330097	1.215869	0.483944	0.282613
Ranking	2	5	4	3	2
Sun Pharma	0.068171	2.102021	0.534894	-1.77791	-0.53603
Ranking	9	6	8	9	9
Tata Motors	0.228053	1.773442	1.022156	0.154769	0.072472
Ranking	6	7	7	5	6
Maruti Suzuki	0.275808	1.394864	1.372687	0.773664	0.254225
Ranking	3	8	2	1	3
Portfolio	0.366445	2.849125	1.196448	0.713133	0.599186
Ranking	1	3	5	2	1

**VI. INTERPRETATION**

- As Sharpe measure portfolio performance get 1st rank among all security and sensex.
- As Treynor measure portfolio performance get 3rd rank among all security and sensex.
- As Jensen measure portfolio performance get 5th rank among all security and sensex.
- As Fama measure portfolio performance get 2nd rank among all security and sensex.
- As MM measure portfolio performance get 1st rank among all security and sensex.

**VII. CONCLUSION**

This research attempts to construct an optimal portfolio by applying Sharpe's Single Index Model of Capital Asset Pricing. Taking BSE 30 as market index and considering daily indices for the Jan 2013 to Dec 2014 period, the proposed method formulates a unique cut off point (Cut off rate of return) and selects stocks having excess of their expected return over risk-free rate of return surpassing this cut-off point. Percentage of investment in each of the selected stocks is then decided on the basis of respective weights assigned to each stock depending on respective b value, stock movement variance representing unsystematic risk, return on stock and risk free return vis-a-vis the cut off rate.

Sharpe single Index Model is basically develop for creating a portfolio from list of securities in such a way which provide best return at lowest risk. In this research work we had try to do the same thing by selecting sensex 30 security as sample as on date 12th March and then apply Shape Single Index Model on this securities for creating portfolio. That inputs required like  $\beta$ (beta) of security,  $\sigma$ (standard deviation) of security and index average monthly closing return of security and index error than standard deviation of error than etc...has been calculated by using closing price of sensex 30 security of last 2 years monthly basis.

After applying Sharpe Single Index Model we come to know that as per this model only 7 securities are satisfying criteria for Sharpe Single Index Model.



## REFERENCES

1. Donald E. Fischer, Ronaki J. Jordan “Security Analysis and Portfolio Management”
2. Edwin J. Elton and Martin J. Gruber “Modern Portfolio Theory and Investment Analysis”
3. M. Obaidullah “Indian Stock Market - Theories and Evidence”
4. Prasanna Chandra “The Investment Game - How to win”
5. Samir K. Barua, J .R. Varma and V. Raghunathan “Portfolio Management”
6. Zvi Bodie, Alex Kane and Alan J. Marcus “Investments”
7. Financial Portfolio Optimization Through a Robust Beta Analysis (Department of Mechanical and Industrial Engineering University of Toronto)
8. Bayesian Portfolio Analysis (Doron Avramov Finance Department, the Hebrew University of Jerusalem, Mt. Scopus Jerusalem 91905, Israel)
9. Modern portfolio theory, 1950 to date (Edwin J. Elton a, Martin J. Gruber b, USA b Department of Finance, Stern School of Business, New York University,)
10. Estimating the Distribution of Sharpe Ratios (Nadeem Aftab, Ingo Jungwirth, Tomás Sedliacik, and Nadir Virk June 30, 2008...)
11. Optimal portfolio selection with or without the procedure of short sales(Research Scholar, Department of Commerce, Kurukshetra University, Kurukshetra, Haryana,)
12. [www.rbi.com](http://www.rbi.com)
13. [www.bse.com](http://www.bse.com)
14. [www.bseindia.com](http://www.bseindia.com)
15. [www.moneycontrol.com](http://www.moneycontrol.com)
16. Research Methodology by C.R. Kothari.