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INTERCULTURAL FEATURES AND THE THEME OF TRAVELLING IN BILINGUAL MATHEMATICS LESSONS

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Abstract

The present qualitative research is focused on bilingual mathematics education. The research presents findings of a case study of one bilingual Slovak and English mathematics 40-minute lesson within an after school elective bilingual mathematics course running weekly since October, 2015. The lesson took place in March, 2016, and was attended by nine learners aged 12-13, eight boys and one girl. The learners are cases of successive school additive bilingual education. The elective course as a whole is a case of immerse bilingual educational programme. In terms of sociolinguistic settings, the course lessons are cases of bilingual education with external second language. The researcher designed and realized the course lessons in terms of CLIL approach, i.e. Content and Language Integrated Learning. The main aim of the case study was to examine if bilingual mathematics instruction does or does not prevent learners from solving math word problems. Secondly, the analysis of transcription of the lesson audio-record served for identification of intercultural features which might hinder the learning process. The analysis of the transcribed audio-record indicates that the bilingual context did not prevent students from solving math word problems, although each of the students worked at their individual rate. On the other hand, some students were confused by the comma as a thousands-separator in multi-digit numbers, and this actually hindered their learning and problem solving process. This fact has been identified as an intercultural difference which had to be explicitly explained to the students. In order to lessen the possible negative influences of bilingual context on mathematics education, teachers need to predict students' responses to various intercultural differences which students are unfamiliar with.
Key words: *bilingual mathematics education, CLIL, thousands-separator, land area, per cents*.

Introduction

The present qualitative research analyses specific phenomena occurring in bilingual mathematics education at a lower secondary level in Slovakia based on a case study of one bilingual mathematics lesson within a long-term elective after-school course. Secondary comprehensive bilingual schools have had quite a long tradition in Slovakia. However, bilingual education at a lower secondary and primary level is just being developed in Slovakia, and naturally attracts much attention of parents, teachers, researchers, and also representatives of governmental institutions. This is assumably in concordance with nowadays common requirement on European citizens to be fluent speakers of several languages. In relation to this concerned bodies of the European Union (COM(95), 1995; COM(2003), 2003) as well as the Slovak Ministry of Education (Conception of foreign language teaching at primary and secondary schools, 2007) recommends CLIL, i.e. Content and Language Integrated Learning (Mehisto, Marsh & Frigols, 2008), as an appropriate approach to bilingual education at a primary and secondary level. Simultaneously, there is also the urgent requirement of mathematical literacy, as defined by OECD (OECD, 2016, Mathematics Framework, p. 65). Following the socioeconomic demands on individuals, it is just natural that the number of schools which provide bilingual education starting even at pre-primary level is constantly increasing. Hand in hand with this, there appear such phenomena in educational environments which catch and call for researchers' interest.

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In fact, bilingual or multilingual mathematics education has already been of scientific interest for several decades. Most researches into bilingual mathematics education have been conducted in such sociolinguistic settings where the main instructional language is the majority language in wider society and students are speakers of minority languages (Khisty, 2001; Dominguéz, 2011; Moschkovich, 2002, 2007; Norén, 2011). In terms of the role of the learner's second language (henceforth abbreviated to L2, first language L1) Siegel (2003) refers to such setting as a *dominant L2 setting*. Other researchers report findings about bilingual mathematics education in such sociolinguistic settings where "the main classroom language is an indigenous or imported language with wide range of official uses and students speak several local languages and inhabit highly multilingual environments" (Barwell, 2005, p. 146). In Siegel's (2003) framework it is the *institutional L2 setting,* and is discussed in (Setati, 2003; Bernardo & Calleja, 2005; Adler, 1995).

The present research is focused on a less explored case of bilingual mathematics education, and that is in Siegel's (2003) framework the *external L2 sociolinguistic setting*, which means that "the main classroom language is a foreign or distant language" and "students are speakers of the dominant language" (Barwell, 2005, p. 146). The learners who attended the analysed lesson presented in this research are Slovak learners learning English as a second language in Slovakia, where English is popular and the most frequently taught foreign language at schools, although English is neither an official minority language in Slovakia, nor a dominant language in any of the neighbouring countries. Closely related research in bilingual mathematics education has been conducted in Slovakia by Lengyelfalusyová (2011, 2013), Kubeš (2012), Pokrivčáková (2013), Guffová (2014), Naštická (2014), Vitézová et al. (2016), and in the Czech Republic by Hoffmanová, Novotná & Hadj-Moussova (2003), Šteflíčková (2012), Tejkalova Prochazková (2013), Moraová & Novotná (2016). The present research is focused on examining if bilingual context does or does not prevent learners from solving math word problems, and on identifying such intercultural features which might hinder the learning process. This case study orderly complements worldwide research in bilingual mathematics education, especially such bilingual mathematics education which is performed in external L2 sociolinguistic setting (Siegel, 2003; Barwell, 2005).

Methodology of Research

General Background of Research

In this research the lesson and the attending students are analysed in view of the sociolinguistic perspective of bilingualism, which perceives bilinguals as members of social groups who use their languages for various functions in their everyday lives (Grosjean, 1994; Moschkovich, 2007; Valdés-Fallis, 1978). In the empirical research the researcher performed as the teacher in the lesson and the leader of the long-term elective mathematics course. The researcher designed the course, the analysed lesson and the applied worksheet for students in terms of CLIL approach (Mehisto, Marsh & Frigols, 2008).

Previous related empirical researches were aimed at younger students (Pokrivčáková, Menzlová & Farkašová, 2010; Kubeš, 2012) or, contrarily, on students at upper secondary educational level (Lengyelfalusyová, 2011, 2013; Guffová, 2014; Vitézová et al., 2016), and many of the researches dealt mostly with various language aspects rather than mathematics educational aspects (Pokrivčáková, 2013). Other related empirical researches were much aimed at how pre-service teachers perceive bilingual mathematics education and various teaching methods, such as CLIL (Hoffmanová, Novotná & Hadj-Moussova, 2003). The latter researches were supported by several theoretical researches (Moraová & Novotná, 2016).

Šteflíčková (2012) investigated difficulties of students with CLIL at lower secondary education in Czech and English bilingual education. Particularly, Šteflíčková used a diagnostic mathematics test (in English as L2) aimed at per cents. In Šteflíčková's quantitative research

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the researcher did not analyse difficulties related to intercultural differences in thousands-separators, nor was the researcher interested in use of decimal marker. In addition, Šteflíčková provided a complex and detailed analysis of two students' solutions to eight math word problems related to per cents proposed within bilingual context. Šteflíčková's two detailed analyses support Bialystok's (2001) inference suggesting that bilingualism does not have any negative effect on mathematical problem solving providing that the learners' language proficiency is at least adequate for understanding the problem. Having in mind this prerequisite, in the present research the researcher dared to use the math word problems in form of a worksheet, as explained below and shown in Figure 1.

Sample Selection

The 40-minute lesson was attended by nine students, eight boys and one girl, aged 12-13. The lesson took place in March, 2016, and was a part of an after school elective bilingual mathematics course running weekly since October, 2015, in Nitra, Slovakia. The students are cases of successive school additive bilingual education (Horecký, 2002; Horňáková, 2007; Štefánik, Palcútová & Lanstyák, 2004). The elective course as a whole is a case of immerse bilingual educational programme (Pokrivčáková, Menzlová & Farkašová, 2010). In the present research the researcher focused the investigation on the group of lower secondary education students, as it has been long neglected in current researches on bilingual mathematics education in L2 external setting, as explained in previous section. Due to tight limitations of teachers' possibilities to implement innovative educational approaches in Slovak school system, the research was conducted in a small group of students who attended the long-term elective course. As for the theme selection of the lesson, a geography-related theme was chosen, since the students were also taught geography in CLIL approach within an obligatory school course.

Instrument and Procedures

In the lesson students were asked to complete a fill-in-the-gaps exercise, i.e. their task was to fill in the gaps in texts which were part of the worksheet used in the lesson (Figure 1). Students worked in a form of a whole-class discussion. The researcher and teacher in one person organized their work and decided who would be the next to read the text and fill in the gap if necessary, which included solving a specific task in each of the cases. The lesson theme was Iceland, which enabled the teacher/researcher to interlink learning objectives for several subjects. Two primary objectives, in terms of CLIL, were focused on mathematics and the English language. The interlinking theme covered secondary learning objectives of the lesson focused on acquiring new geographic knowledge. Besides, the instruction and the main text, the worksheet included a simple glossary in form of a footnote. In this case the glossary did not contain any mathematical terms.

Based on the direct participant observation and on the analysis of the transcribed audio-record of the lesson the present research analyses how students solved the proposed mathematical tasks. Primarily, the research investigates in what manner students were affected by the bilingual context while solving math word problems comprised in the worksheet. The main focus of the research is to examine whether the intercultural differences between Slovak (L1) and English (L2) mathematical notations – the difference between the English comma as a thousands-separator in multi-digit numbers and the Slovak decimal comma functioning as a decimal marker – in any way hinder the learning and mathematical problem solving process of the students.

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TASK 1

Look at the map and fill in the gaps in the texts below.

Capital and Area

The capital[1] of Iceland is _ _ _ _ _ _ _ _ _ . It lies in the _ _ _ _ _-western part of the country. The size of Iceland is 103,000 square kilometres. Since the total area of Slovakia is 49,036 km², we can say that Iceland is around _ _ _ _ _ bigger than Slovakia, or that Slovak area is approximately[2] one _ _ _ _ of Icelandic area.

Ice and Fire

Iceland attracts[3] tourists with its wonderful contrasts, especially its many active volcanoes[4] covered with ice of tremendous[5] glaciers[6]. Although the glaciers in Iceland belong among the largest ones left in Europe, they cover only about 10% of the Icelandic land, which is _ _ _ _ _ km². However, in Greenland[7], another amazing island[8], much larger and much closer to the North Pole than Iceland, ice covers around 80 % of the total 2,166,000 km², which is _ _ _ _ _ _ _ km². The largest Icelandic glacier is called _ _ _ _ _ _ _ _ _ _ _, including[9] the highest Icelandic peak[10], Hvannadalshnúkur, _ _ _ _ metres high.

[1] hlavné mesto; [2] približne; [3] priťahuje; [4] sopky; [5] obrovské; [6] ľadovce; [7] Grónsko; [8] ostrov; [9] vrátane; [10] vrch, štít

Figure 1: Texts read and completed by students in the analysed lesson.

Data Analysis

Based on the analysis of the transcribed communication which emerged and was audio-recorded in the lesson, the occurred mathematics-related parts of conversation were divided into two main categories. The first category covers such utterances and conversations in which students struggle with reading certain mathematical texts and proposing answers and results in both instructional languages. More specifically, the text covered in this category includes reading numbers, especially consisting of four and more digits (*multi-digit numbers*), and reading the international abbreviation of square kilometres. Reading multi-digit numbers requires the knowledge of the intercultural difference between decimal comma in Slovak mathematical discourse (while in English it is the decimal point, which was not used in the task which students were asked to solve in the lesson) and the comma between hundreds and thousands, and between millions and hundred thousands etc. in English mathematical discourse (*comma as a thousands-separator*). Acquiring this knowledge was one of the lesson learning objectives, therefore frequent occurrence of students' misunderstanding of the comma had been expected. The second category covers such utterances and conversations in which students negotiate their understanding of the math word problem, including their solutions, in both languages of instruction. In the analysed lesson this category of utterances is elicited by two types of tasks. The first type is related to multiples and parts of a whole. The second type is related to per cents (see Figure 1). In the analysis the relationship between the type of the math word problem and the students' manner of solving them is not investigated. The research focuses on the influences of the bilingual context on the students' quality of understanding and solving the math word problems.

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Results of Research

Nine students (S1 – S9) were reading the text, sentence by sentence, as they were asked by the teacher (T), and if necessary, they had to find missing information in a simple map of Iceland (four gaps), estimate (two gaps) or calculate (two gaps) numerical information. The four cases in which students had to estimate or calculate the missing numerical information were, actually, simple math word problems presented in a non-traditional form of a fill-in-the-gap exercise. While reading the text in L2, students struggled with reading numbers, as illustrated in Episodes 1 and 2. Both students and the teacher occasionally mixed the two instructional languages, which is marked by italics in the transcription, and Slovak utterances are translated into English in slash brackets.

/Episode 1, time 17:06/
S5: The size of Iceland is one hundred three thousand-
T: -one hundred and three thousand-
S5: -one hundred and three thousand square kilometres.

/Episode 2, time 17:30/
S7: Since the total area of Slovakia is
štyridsaťdeväť celá nula tridsaťšesť kilometrov štvorcových.
/forty-nine point zero thirty-six square kilometres./

Student S5 correctly read number 103,000 in L2, English, and did not mistake the comma between thousands and hundreds for Slovak decimal comma (Episode 1). He managed to read the number correctly before the teacher explained the intercultural difference. It cannot be definitely stated if he had already known the intercultural difference, or he simply realized that he was reading quite a big number since it referred to the area of Iceland. There is also the possibility that he read the number correctly because he was not misled by digits after the comma, as they were three zeros, unlike student S7 a few seconds later. Student S7 read number 49,036 incorrectly in L1, Slovak, and did mistake the comma between thousands and hundreds for Slovak decimal comma (Episode 2). She might have been misled by the digits after the comma, as they were not only zeros. We can, however, definitely say she neither realized nor had she known the intercultural difference. In addition, she was not fully concentrated on the fact she was reading about the area of Slovakia which cannot be counted in tens of square kilometres.

Around five minutes later the teacher explicitly explained the difference between comma as a thousands-separator and decimal comma used in Slovak mathematical discourses. The teacher did so because she was aware that students were just about to solve a math word problem about per cents and that she needed to avoid their let-down in case they would be told their results are wrong despite correct procedures. After student S2 switched into Slovak, the conversation continued in Slovak without language switching, therefore Episode 3 is presented below in its translated form only. As it is shown in Episode 3 (the last line), student S2 already started solving the task correctly. However, the teacher ignored him and decided to explicitly explain the difference between the English and the Slovak meaning of the comma because of the above mentioned reasons.

/Episode 3, time 21:05/
S2: Although …the glaciers in island...Iceland belong among the largest
ones left in Europe, they cover only about ... ten per cent of the Icelandic land, which is... *Neviem, porozmýšľam.*
/I don't know, I'll think about it./

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T: Uhm, so now we want to find out---
S2: ---how many square kilometres it is.
T: ---how many square kilometres the glaciers in Iceland form. We know that it is ten per cent of the total area.
S3: So, one per cent is ten point three---
T: Watch out, this is one hundred and three…thousand.
S1: Aha.
S3: I thought it is the decimal comma.
T: The comma---
S2: One thousand and thirty is one per cent.

Before the lesson proceeded to Episode 3 and the explicit explanation of the intercultural difference between the commas and subsequently to the tasks about per cents, students first easily managed to solve the tasks about a part of a whole and multiples (Figure 1, comparing the total areas). As shown in Episode 4, four students, namely S8, S3, S4 and S6 (*whereas student S7 was still wondering about the unexplained comma) had no difficulties when estimating the ratio of the total areas of Iceland and Slovakia, i.e. their mathematical thinking was not hindered by the bilingual context. They only were not absolutely sure how to propose their solutions in L2. Unsurprisingly, one of the students (**S6) blended the two languages in his answer.

/Episode 4, time 18:28/
S7: We can say that Iceland is around...eee-
S8: - two times bigger.
T: *No, čiže máme tam rozlohu Islandu,*
/Well, we know the Icelandic area…/
the size or the area, the area of Iceland is one hundred and three thousand, yes? The area of Slovakia is forty-nine thousand and thirty-six. *Takže áno, niekto to už povedal.*
/So, yes, someone has already said it./
*(Then, addressing student S7 in L1.)
My musíme zistiť, koľkokrát je územie… je Island väčší ako Slovensko.
/We need to find out how many times the area is… Iceland is larger than Slovakia./
S7: Aha.
T: *Približne.* */Approximately/*
S3: *Dvakrát?* */Two times?/*
S4: *Dvakrát.* */Two times./*
T: Yes. *A jedným slovom-* */In one word-/*
S8: Twice.
T: Twice. It's twice bigger.
**S6: *Ja som napísal, že dva* time. */I have written two time./*
T: Two times is correct.
S3: *Čiže máme iba doplniť hento, že* is around twice?
/So, we should write that…/
T: Yes, is around twice bigger than Slovakia. (Addressing S8 by his name.)
S8: -or that Slovak area is approximately one half… one half of Icelandic.

Later on, despite the initial obstacles that students faced in recognizing the comma as a thousands-separator (Episode 3), they understood the mathematical problem hidden in the text and were able to find the solution to the tasks about per cents without the teacher's help, though each student worked at their individual rate (Episode 7). After the teacher's explicit explanation, students individually kept on calculating the result. Several students used their pocket calculators or cell phones, which, naturally, led into the following dialogue (originally in Slovak only):

/Episode 5, time 22:28/
S3: Slovak pocket calculators are all made in England?
T: No, but it is rather international.

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The question of students S3 (Episode 5), as well as the remark of student S6 five minutes later (Episode 6, originally in Slovak only), represents students' understanding and acceptance of the intercultural difference.

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/Episode 6, time 27:05/
S6: My pocket calculator also displays the commas.
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After the teacher explained the intercultural difference between the commas, students continued with calculating ten per cent of the Icelandic area. As shown in Episode 7 (originally in Slovak only), some students figured out the correct results very quickly (S2), some were struggling with it and rather guessing (S3). Finally, after student S2 performed the calculation at the blackboard, student S8 was asked to read the result aloud, and several students read it aloud simultaneously to show, proudly, that they managed to calculate it too.

```
/Episode 7, time 22:52/
T: Ten per cent of one hundred and three thousand. Well, come here
and try
   it at the blackboard.
S2: It is ten thousand three hundred.
S3: One million three thousand. One million thirty thousand.
```

Next, after student S3 read the sentence about glaciers in Greenland, the teacher asked student S9 to calculate eighty per cent of the total Greenlandic area at the blackboard. Meanwhile, other students were working individually at their desks. The teacher checked the student at the blackboard and helped him use the comma as a thousands-separator correctly. Suddenly, one of the students working at their desks (S2) proposed his result aloud in L1, and was correct, which the teacher affirmed by reading the number in L2. As other students also finished their calculations, they looked at the blackboard and S2 asked if that was the correct result. The teacher affirmed the result again and asked students whether their calculators displayed the same result. Other three students joyfully affirmed they arrived at the same number.

Another intercultural difference which students faced in the analysed lesson was reading the international abbreviation of square kilometres. The official Slovak wording of *square kilometres* is in the inverse order of the word classes, i.e. noun (*kilometre*) comes before adjective (*square*). Reading the international abbreviation km^2 in Slovak is extraordinary even within broader Slovak context, since in the Slovak language it is more usual to denote adjectives before nouns, except few cases, mostly technical terms. This intercultural difference revealed its consequences in the way students read their texts. Some of them read the abbreviation as *kilometres square*, one of the students translated it incorrectly as *kilometre second*. What seems to be positive is that students were willing to experiment with the foreign language. In addition, students were not resistant against the teacher's help when she read the abbreviation correctly, and they immediately accepted it and started using the correct linguistic form.

The crucial fact, however, is that neither struggling with reading numbers nor the inverse reading of abbreviation km^2 prevented students from solving the math word problems comprised in the text of fill-in-the-gaps type. At first, they easily figured out the proportion of the land areas in both directions. Next, students managed to solve the two tasks on per cents, yet, each of them at their individual rate.

Discussion

In nowadays multicultural world it is just natural that bilingual and multilingual education becomes more and more frequent. The scarcity of literature and scientific papers addressing the issue of bilingual mathematics education in *L2 external setting* (Siegel, 2003; Barwell,

2005) just intensifies the need to investigate various aspects of such education. For current investigations of bilingual mathematics education in L2 external setting we can only proceed from researches on bilingual and multilingual mathematics education in general, regardless the mutual position of the target language and the mother tongue in given sociocultural setting, regardless the learners' age, and regardless types of mathematical tasks used in the researches.

Earlier researches focused highly on code switching and language switching within bilingual education and discussed if language switching is as negative as it seems at first glance. Although in this research we do not deal with the question of code- and language-switching, we consider it natural behaviour of learners in bilingual mathematics classroom (Moschkovich, 2002, 2007; Grosjean, 1994). Yet, a very important aspect of language-switching for mathematics education is the influence of bilingual context on the bilinguals' solution time and error rate when solving mathematical tasks, since switching between languages naturally slows down the solution process and may increase error rates. This issue was addressed by many researchers (Marsh & Maki, 1976; McLain & Huang, 1982), whose investigations were, however, conducted mostly among adult bilinguals, in experimental environment (i.e. neglecting natural classroom practices, such as interaction between learners and the teacher) and concerning only simple arithmetic operations, not math word problems. Still, their results are of considerable importance for any type of bilingual mathematics education, since they proved that bilingual context may slow down learners' arithmetic computations, but the error rates are not statistically significantly dependent on the bilingual nature of the educational context. Their findings support the idea that learners should be allowed to use their preferred language (including the *mental, voiceless* language use) for arithmetic operations, which we did during the analysed lesson, and that is also in accordance with CLIL approach.

The results of the present research are limited by the sample size which was only nine learners. On the other hand, the detailed analysis of the audio recordings elicits new ideas about how learners might be approached and assisted in bilingual mathematics education in order to reduce the negative influences and strengthen the positive potentials of bilingual or multilingual mathematics education. Besides individual approach, also the efficient timing of explicit explanation of intercultural differences might be inevitable in some cases, as indicated by the present research. This issue requires further investigations. Another important aspect which needs to be taken into consideration in bilingual and multilingual mathematics education is the adequate choice of interdisciplinary themes and themes related to everyday life to be comprised in lessons and mathematical texts in order to catch and hold learners' interest in mathematics and its role in human lives. The choice of context for mathematical tasks and word problems is partially addressed by Libbrecht & Goosen (2016) who suggest that learning is much more efficient when conceptualized within learners' context. Similarly, Dominguez (2011) argues that mathematics education for bilingual learners should capitalize on their experiences as cognitive resources. CLIL proponents (Mehisto, Marsh & Frigols, 2008) also underline the importance of interdisciplinary focus in language education. Implementation of context-based tasks in mathematics education follows from the broad applicability of mathematics in everyday life and numberless sciences.

Quite a rare case study was performed by Qi (1998) whose research subject was an adult bilingual solving math word problems. The bilingual, as stated by the researcher, switched her languages during arithmetic operations quickly and automatically, and it facilitated the math word problem solving rather than inhibited. Bialystok (2001) suggests that bilingualism does not have any negative effect on mathematical problem solving if the learners' language proficiency is at least adequate for understanding the problem. Considering Bialystok's inference, it has to be added that there is a significant difference between language proficiency and familiarity with specific intercultural features within mathematical context. As shown in this research, most learners in the class had no difficulty to understand the mathematical task comprised in the worksheet, though they would not have arrived at the correct solution – despite correct procedures – if the teacher had not explicitly explained the comma as a thousands-separator.

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The analysis of the situations which occurred in the audio-taped lesson does not serve for investigation of how learners acquire *new* pure mathematical knowledge. The present research focuses on how learners deal with solving math word problems related to per cents within bilingual context including intercultural difference in mathematical notation of thousands-separator. In the analysed lesson learners did not learn any *unfamiliar* (for them) mathematical knowledge, nor were they exposed to or asked to use any *new* computational procedure. They were *practicing* the procedure of counting given part of land area or glacier area in per cents, i.e. they were asked to apply their *previously acquired knowledge* in order to solve math word problems. The research question was if the intercultural difference in mathematical notation of thousands-separator would prevent them from arriving at the correct solution. The conducted research, however, investigates neither if, nor how bilingual context and the related intercultural difference influences learning mathematical knowledge which would be *new* for the learners. The situation investigated in this research is, however, a situation which can with high probability often occur in bilingual mathematics class with L2 external setting.

As already stated, one of the tightest limitations of the present research is the L2 external setting (Siegel, 2003; Barwell, 2005). Only little research has been done in L2 external setting of bilingual mathematics education, none in the issue of covering the intercultural difference in mathematical notation of thousands-separator. On the other hand, regardless the position of the target language with respect to the learners' mother tongue a few researches were conducted in the issue of using information and communication technologies (ICT) in multilingual mathematics teaching and learning in general. These are in some way related to the presented case study. As shown in Episodes 5 and 6, learners realized the intercultural difference of mathematical notation for thousands-separator and decimal mark when they used their pocket calculators. This issue is also addressed in Hofstede (1991), Marcus (2008) and Libbrecht & Goosen (2016). In relation to ICT tools and *their* language, Libbrecht & Goosen (2016) suggest that usually learners need to be self- informed of such typographical aspects and differences. We argue that in some cases self-informing of the learners may take too long and thus is rather ungainly in school practice. Nevertheless, we suppose that with regards to the need of individual approach of learners it is only the teacher who can make the best decision in this question.

In terms of the sociolinguistic view of bilingualism (Grosjean, 1994; Moschkovich, 2007, Valdés-Fallis, 1978) students and the teacher attending the lesson acted as bilinguals, as they actively used both of the instructional languages for various purposes. Yet, their bilingualism in relation to their mathematical skills has proven to function individually. While some students needed more time to solve the word problems, i.e. the bilingual context did actually hindered their learning process, other students were very quick at solving the tasks. Among those who solved the task correctly were students who proposed their answers in L2 and also such students who proposed their solutions in L1. Unsurprisingly, some students "blended" the two instructional languages when proposing their solutions. The present empirical research suggests that the intercultural difference between the English and the Slovak meaning of a comma in mathematical notation of multi-digit numbers may hinder the students' process of solving math word problems. The amount of the negative influence of the bilingual context depends on various factors. One of the factors is the students' background knowledge and personal intercultural experience. Another factor is the timing of the teacher's intervention in the students' process of grasping the meaning from the math word problem wording. Teachers have to decide if they wait and let students struggle with negotiation of the meaning and discover it by themselves, or they provide students with explicit explanation of the intercultural difference in question. When deciding for the explicit explanation, teachers have to make another important decision – when it is most appropriate to explain the difference, either *early before* students encounter such situation when they need to be familiar with it, or *right at that moment*, or even *certain amount of time after* encountering such situation. The influence of the timing of the teacher's intervention on the quality of students' problem solving performance needs to be subjected to further research. Also, it would be useful to investigate the influence of the form in which math

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word problems are proposed to students on the quality of their problem solving performance within bilingual instruction. Similar intercultural mathematics-related difference is in large extent represented in metric and imperial units of length, velocity, weight, temperature and the like. This issue is of high importance in Slovak and English bilingual education – similarly in Czech and English bilingual mathematics education, as suggested by Moraová & Novotná (2016) – since in Slovakia metric units are used, whereas in English speaking countries imperial units are preferred. Studying how students cope with such intercultural mathematics-related differences would significantly enrich current research on bilingual mathematics education in sociolinguistic setting with external L2, and in general.

Conclusions

Bilingual mathematics education is a very complex process covering both language and mathematics education. Regarding the limitations of the presented research, the chosen research design allows analysing in detail how students negotiate the meanings of specific mathematical notations within bilingual context in L2 external sociolinguistic setting, which has concrete applications in school practice. Undeniably, the issue of thousands-separator is very frequent not only in bilingual mathematics education, but also in mathematics education where various ICT tools are employed and most probably use the international mathematical notation. The detailed analysis of audio-records obtained in a lesson, which was embedded within a long-term elective after-school course, provides a deeper look in school practices and students' and teachers' communication and challenges in bilingual mathematics school-set education. The findings indicate that students cope with the intercultural difference in thousands-separators in individual manner and at individual rate. Nevertheless, if students' language proficiency is sufficient for understanding the math word problem context, they are able to solve the proposed math word problem. Yet, students need specific assistance of the teachers in order to understand and accept intercultural differences covered in multilingual mathematics discourse.

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