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Fuzzy Preference Relations in Group Decision Making Problems Based on Ordered Weighted Averaging Operators

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Abstract

We study the problem of fuzzy preference relations in group decision making. Group decision problems need all experts express their preferences using the same preference representation format. However, in real practice, this is not always possible because each expert has their unique characteristics which regard to knowledge, skills, experience, and personality, which implies that different experts may express may express their evaluation by means of different preference representation formats. Therefore, we use the order weighted averaging (OWA) operator and order weighted geometric (OWG) operator in the aggregation of group decision making problems. The main contribution of this paper is the easy and extensible solution to group decision problems. Therefore, we studied literatures about OWA OWG and fuzzy preference relation. We results are created the proposed of solving a multi-criteria decision making problem using OWA and OWD operator. Finally, we give an empirical example where we can see the different results obtained by using the OWA and OWG operator in fuzzy preference relations in group decision making.

Keywords: Group decision making; Fuzzy preference; order weighted averaging (OWA) operators; order weighted geometric (OWG) operators

1. Introduction

Group decision making is currently an important part of decision science. Its theory and methods in engineering, economics, management, and many other fields have been widely used. Several authors have provided interesting results on group decision making or social choice theory and multi-criteria decision making with help of fuzzy sets theory [14, 15]. However, some certain decision-making methods such as Bayesian networks**,** it must suppose between the various properties independent of each other in probability calculation. The aggregation step is a necessary and very important task to carry out when we want to obtain a final solution of group multi-attribute decision making problems. It would be useful to use the ordered weighted averaging (OWA) operator 15]. The OWA operator has been used in a wide range of applications such as [4-6, [19]). The OWA operator is very useful technique for aggregating the information providing a parameterized family of aggregation operators that includes the maximum, the minimum andthe average, among others [1]. The use of the OWA operator in different types of distances measures has been studied in [9]. Recently, [4] have developed a geometric version of the OWA operator, the ordered weighted geometric (OWG) operators. OWG operator has been extensively analyzed by the different researcher [3]. It is based on the geometric mean and the OWA operator. It allows incorporating the concept of fuzzy majority in the decision process when the information is provided using a ratio scale.

In this paper, we analyzed the process using OWA operators in the aggregations as suggested by [15]. The purpose of this research is to see the different results obtained by using the OWA operator, and the OWG operators in fuzzy preference relations in group decision making. In order to do this, the paper is set out as follows. The concept of regular

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increasing monotone linguistic quality, OWA and OWG are introduced in Section 2. Section 3 is devoted to present the OWA and OWG operator to fuzzy preference relations in group decision making. An example of its used in decision making is given in Section 4. Finally, some conclusion remarks are pointed out in Section 5.

2. Preliminaries

2.1. The Regular Increasing Monotone (RIM) and Linguistic Quantifer

Q is RIM quantifier, we measure the overall success of the alternative $x = (a_1, a_2, ..., a_n)$ by $F_Q(a_1, a_2, ...a_n)$. Where F_Q is an OWA operator derived from Q. i.e. the weights associated with this quantifier guided aggregation are obtained as follows.

$$w_i = Q(\frac{i}{n}) - Q(\frac{i-1}{n}),\ \text{i} = 1, 2, \ldots, \text{n} \qquad (1)$$

The membership function of a non decreasing relative quantifier can be represented as

$$Q(r) = \begin{cases} 0 \quad if\ r < a \\ \dfrac{r-a}{b-a} \quad if\ a \le r \le b \\ 1 \quad if\ r > b \end{cases} \qquad (2)$$

with $a, b, r \in [0,1]$

Some examples of relative quantifiers are shown in Figure 1, where the parameters, (a, b) are (0.3, 0.8), (0, 0.5), (0.5,1)

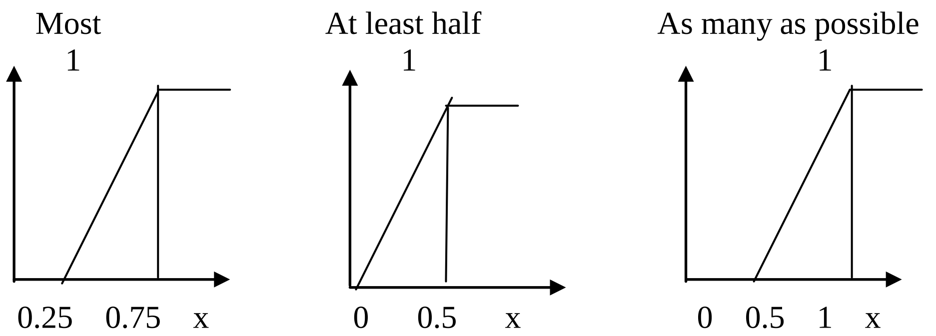

Figure 1. Relative Fuzzy Quantifiers

2.2. OWA Operators

OWA operator provides a parameterized family of aggregation operators which have been used in many applications [2, 5, 10, 17]. In the following, we provide a definition of the OWA operators.

Definition 1: An OWA operator of dimension n is mapped $F : R^n \to R$, that has an associated n vector $w = (w_1, w_2, ..., w_n)^T$ such as $w_i \in [0,1], 1 \le i \le n,$ and $\sum_{i=1}^{n} w_i = 1$ such that

$$OWA(a_1, a_2, ..., a_n) = w_1 b_1 + w_2 b_2 + ... + w_n b_n \quad (3)$$

where b_j is the j[th] largest value of the a_i

This operator OWA is another called descending ordered weighted averaging (DOWA)

Properties 1: The OWA operator satisfies the following properties.

1. It is an or-and operator, i.e., it remains between the minimum and the maximum of the arguments:
   $$\min(a_1, a_2, ..., a_n) \le OWA(a_1, a_2, ..., a_n) \le \max(a_1, a_2, ..., a_n)$$
2. It is commutative:
   $$OWA(a_1, a_2, ..., a_n) = OWA(a_{\pi(1)}, a_{\pi(2)}, ..., a_{\pi(n)})$$
3. It is idempotent:
   $OWA(a_1, a_2, ..., a_n) =$ a, if $a_i = a \ \forall i$
4. It is increasing monotonous:
   $OWA(a_1, a_2, ..., a_n) \ge OWA(d_1, d_2, ..., d_n)$, if $a_i \ge d_i \ \forall i$
5. It leads to maximum when $w = (1,0,...,0)^T$
6. It leads to minimum when $w = (0,0,...,1)^T$

The OWA operator is a mean or averaging operator. This is reflection of the fact that operator is commutative, monotonic, bounded and idempotent. Different families of OWA operators can be used by choosing a different manifestation of the weighting vector [7-9].

2.3. OWG Opersators

OWG operator provides a family of aggregation operators similar to the OWA operator. It consists in combining the OWA operator with the geometric mean.

Definition 2: An OWG operator of dimension n is mapped $F : R^{+n} \to R^{+}$, that has an associated n vector $w = (w_1, w_2, ..., w_n)^T$ such as $w_i \in [0,1], 1 \le i \le n,$ and $\sum_{i=1}^{n} w_i = 1$ such that

$$OWG(a_1, a_2, ..., a_n) = b_1^{w1} \times b_2^{w2} \times ... \times b_n^{wn} \quad (4)$$

where b_j is the jth largest value of the a_i and R^+ is the set of positive real number.

This operator OWG is another called descending ordered weighted averaging (DOWG). As it is seen in ([12], [12], [14]), the OWG operator has the following properties.

Properties 2: The OWG operator satisfies the following properties.

1. It is commutative: any permutation of the arguments has the same evaluation
2. It is increasing monotonous: if
   $$a_i \ge d_i \quad \forall i \Rightarrow OWG(a_1, a_2, ..., a_n) \ge OWG(d_1, d_2, ..., d_n)$$
3. It is bounded: $Min\{a_i) \le OWG(a_1, a_2, ..., a_n) \le Max\{a_i)$
4. It is idempotent: $OWG(a_1, a_2, ..., a_n) = a,\ if\ a_i = a, \forall i$

3. To Aggregate Fuzzy Perference Relation

3.1. Presentation on the Problem

[16] Introduced the Induced Ordered Weighted Averaging (IOWA) operator. [4]

provide some IOWA operators to aggregate fuzzy preference relations in group decision making problems. The OWA operator and OWG operator as well as Weighted Averaging (WA) operator are included in the more general class of IOWA operators.

In OWA operator, We suppose that we have a group of experts, $E = \{e_1, e_2, \ldots, e_m)$, which provide preferences about a set of alternatives, $X = \{x_1 x_2, \ldots, x_n\}$, by means of the fuzzy preference relations, $\{P^1, P^2, \cdots, P^m\}$, $P^k = [p_{ij}^k]$, $p_{ij}^k \in [0,1]$, which are additive reciprocal, i.e., $p_{ij}^k + p_{ji}^k = 1, \ \forall i, j, k.$

In OWG operator, We suppose that we have a group of experts, $E = \{e_1, e_2, \ldots, e_m)$, which provide preferences about a set of alternatives, $X = \{x_1 x_2, \ldots, x_n\}$, by means of the fuzzy preference relations, $\{P^1, P^2, \cdots, P^m\}$, $P^k = [p_{ij}^k]$, $p_{ij}^k \in [0,1]$. [20] suggests measuring p_{ij}^k using a ratio scale, and in particular the 1 to 9 scale: $p_{ij}^k = 1$ indicates indifference between x_i to x_j, p_{ij}^k = 9 indicates that x_i is unanimously preferred to x_j, and $p_{ij}^k \in$ {2,3,…,8} indicates intermediate evaluations. It is usual to assume the multiplicative reciprocity property $p_{ij}^k \,.\, p_{ij}^k = 1 \ \ \forall i, j$..

$a_i \geq d_i \quad \forall i \Rightarrow OWG\,(a_1, a_2, ..., a_n) \geq OWG\,(d_1, d_2, ..., d_n)$

3.2. Reciprocity of the Collective Multiplicative Perference Relation

Properties 3: In OWA operator, the collective preference relation, $P^c = (P_{ij}^c)$ obtained by using OWA operator $\phi_Q(P_{ij}^1, p_{ij}^2, ..., p_{ij}^m) = OWA(P_{ij}^1, p_{ij}^2, ..., p_{ij}^m)$ guided by a fuzzy linguistic quantifier Q is also reciprocal.

Proof:

If $\sigma : \{1,2,...,m\} \rightarrow \{1,2,...,m\}$ a permutation such that $p_{ij}^{\sigma(k)} \geq p_{ij}^{\sigma(k+1)}, \forall k = 1,2,...,n-1$

$$P_{ij}^c = \sum_{k=1}^{m} w_k \, p_{ij}^{\sigma(k)} = \sum_{k=1}^{m} w_k (1 - p_{ji}^{\sigma(k)}) = 1 - \sum_{k=1}^{m} w_k \, p_{ji}^{\sigma(k)} = 1 - p_{ji}^c$$

And thus P^c verifies the reciprocity property.

Properties 4: In OWG operator, the collective preference relation, $P^c = (P_{ij}^c)$ obtained by using OWG operator $OWG(P_{ij}^1, p_{ij}^2, ..., p_{ij}^m)$ guided by a fuzzy linguistic quantifier Q is also reciprocal. *i.e.*, $P_{ij}^c * P_{ji}^c = 1$

Proof:

As we assume that $P^k = [p_{ij}^k]$ reciprocal then $p_{ij}^k * p_{ji}^k = 1$, and therefore if $\{b_{ij}^1, ..., b_{ij}^m\}$ are ordered from largest to lowest. $\{b_{ji}^1, ..., b_{ji}^m\}$, being $b_{ji}^k = 1 / b_{ij}^k$, are ordered from lowest to large. We have:

$$p_{ij}^c \,.\, p_{ji}^c = \prod_{k=1}^{m} (b_{ij}^k)^{w_k} \,.\, \prod_{k=1}^{m} (b_{ji}^k)^{w_{m-k+1}}$$

$$= \prod_{k=1}^{m} (b_{ij}^{k})^{w_k} \cdot \prod_{k=1}^{m} (\frac{1}{b_{ij}^{k}})^{w_{m-k+1}}$$

$$= \prod_{k=1}^{m} (b_{ij}^{k})^{w_k - w_{m-k+1}} = \prod_{k=1}^{m} (b_{ij}^{k})^{\overline{w}_k}$$

Where

$$\overline{w}_k = [Q(\frac{k}{m}) - Q(\frac{k-1}{m})] - [Q(\frac{m-k+1}{m}) - Q(\frac{m-k}{m})]$$

If we set

$A(k) = Q(\frac{k}{m}) + Q(1 - \frac{k}{m})$, then

$\overline{w}_k = A(k) - A(k-1)$

Since Q is a linguistic quantifier with membership function verifying $Q(1-x) = 1 - Q(x)$, then $A(k) = 1, \ \forall k$ and in consequence $\overline{w}_k = 1, \forall k$.

This implied that $p_{ij}^{c} \cdot p_{ji}^{c} = \prod_{k=1}^{m} (b_{ij}^{k})^{\overline{w}_k} = \prod_{k=1}^{m} (b_{ij}^{k})^{0} = 1$.

3.3. Consistency Property

The ordinal consistency of fuzzy judgment matrix is an important issue. The lack of consistency in decision making can lead to inconsistent conclusions. It is difficult to ensure a consistent pair-wise comparison. A judgment method for ordinal consistency of fuzzy judgment matrix was proposed according to the transitivity of binary relation [19].

Properties 5: In OWA operator, if the set of fuzzy preference relations are additive consistent (Herrera-Viedma *et al.*, 2004), *i.e.*, $P_{ij}^{k} + P_{jl}^{k} + P_{li}^{k} = 3/2$

The collective preference relation, $P^{c} = (P_{ij}^{c})$ obtained by using OWA operator $OWA(P_{ij}^{1}, p_{ij}^{2}, ..., p_{ij}^{m})$ guided by a fuzzy linguistic quantifier Q is also additive consistent. i.e. $P_{ij}^{c} + P_{jl}^{c} + P_{li}^{c} = 3/2$

Proof:

$$P_{ij}^{k} + P_{jl}^{k} + P_{li}^{k} = \sum_{k=1}^{m} w_k * p_{ij}^{\sigma(k)} + \sum_{k=1}^{m} w_k * p_{jl}^{\sigma(k)} + \sum_{k=1}^{m} w_k * p_{li}^{\sigma(k)}$$

$$= \sum_{k=1}^{m} w_k * (p_{ij}^{\sigma(k)} + p_{jl}^{\sigma(k)} + p_{li}^{\sigma(k)})$$

$$= \sum_{k=1}^{m} w_k * (\frac{3}{2}) = \frac{3}{2}$$

This proves the additive consistency of $P^{c} = (P_{ij}^{c})$.

Properties 6: In OWG operator, if the set of fuzzy preference relations are multiplicative consistent, i.e. $P_{ij}^{k} * P_{jl}^{k} = P_{li}^{k}$

The collective preference relation, $P^{c} = (P_{ij}^{c})$ obtained by using OWG operator

$OWA(P_{ij}^{1}, p_{ij}^{2},..., p_{ij}^{m})$ guided by a fuzzy linguistic quantifier Q is also multiplicative consistent. i.e. $P_{ij}^{c} * P_{jl}^{c} = P_{li}^{c}$

Proof:

$$P_{ij}^{k} * P_{jl}^{k} = \sum_{k=1}^{m} w_k p_{ij}^{\sigma(k)} * \sum_{k=1}^{m} w_k p_{jl}^{\sigma(k)} = \sum_{k=1}^{m} w_k * (p_{ij}^{\sigma(k)}, p_{jl}^{\sigma(k)})$$

$$= \sum_{k=1}^{m} w_k p_{il}^{\sigma(k)} = p_{il}^{k}$$

This proves the multiplicative consistency of $P^c = (P_{ij}^c)$

4. OWG Operations and OWA Operations to Aggregate Fuzzy Perference

4.1. Solving a Multi-Criteria Decision Making Problem Using S the Owg Operator

If $\tilde{A}^k$ is the fuzzy judgment matrix of evaluator k, $\tilde{a}_{ij}$ the fuzzy assessments between criterion j and criterion j of evaluator k, $\tilde{A}^k = [\tilde{a}_{ij}]^k$, where $\tilde{a}_{ij} = (l_{ij}, m_{ij}, r_{ij})$. The linguistic scale and corresponding triangular fuzzy numbers showed as Table 1.

Definition 3:

If $\tilde{a}_{ij} = (l_{ij}, m_{ij}, r_{ij})$, the expected value if defined as

$$E(\tilde{a}_{ij}) = \begin{cases} \frac{1}{4}(l_{ij} + 2m_{ij} + r_{ij}) & if\ \tilde{a}_{ij} = \tilde{x} \in \{\tilde{1},...,\tilde{9}\} \\ \frac{1}{4}(l_{ij}.2m_{ij}.r_{ij}) & if\ a_{ij} = \frac{1}{\tilde{x}} \end{cases} \quad (5)$$

Table 1. The Linguistic Scale and Corresponding Triangular Fuzzy Numbers

Fuzzy number	Linguistic scales	Scale of fuzzy number
$\tilde{1}$	Equally important	(1,1,1)
$\tilde{3}$	Weakly important	(2,3,4)
$\tilde{5}$	Essentially important	(4,5,6)
$\tilde{7}$	Very strongly important	(6,7,8)
$\tilde{9}$	Absolutely important	(7,8,9)
$\tilde{2}, \tilde{4}, \tilde{6}, \tilde{8}$	Intermediates value ($\tilde{x}$)	(x-1,x,x+1)
$1/\tilde{x}$	Between two adjacent judgments	(1/(x+1), 1/x, 1/(x-1))

Steps of multi-criteria decision making problem using the OWG operator

Step 1: The experts provide the following multiplicative Fuzzy preference relations on a set of alternatives. Which satisfy the consistency and reciprocal properties (Properties 4, Properties 6)

Step 2: Calculation of the equivalence expected judgment matrix of $(p_{ij}^{k})_{n\times n}$ (by definition 3).

Step 3: Using OWG operator and aggregating the equivalence expected judgment matrix as group judgment matrix $G=(u_{ij})_{n\times n}$

Where $u_{ij}=\prod_{k=1}^{m}(q^{k})^{w_k}$, where q^{k} is the k[th] largest value of the $E(p_{ij}^{k})$

w_k is obtained by the Regular Increasing Monotone (RIM) and linguistic quantifier

Step 4: Using OWG operator to calculate the alternatives index

$r_i^{'}=\prod_{j=1}^{n}(v_j)^{w_j^{'}}$, where v_j is the jth largest value of the $\{u_{i1},u_{j2},...,u_{jn}\}$

$w_k^{'}$ is obtained by the Regular Increasing Monotone (RIM) and linguistic quantifier

Step 5: Calculate the normalized vector

$$r_i=\frac{r_i^{'}}{\sum_{i=1}^{n}r_i^{'}} \qquad (6)$$

Step 6: Rank the alternatives

4.2. Empirical Study in OWG Operator

Suppose a set of three experts provide the following multiplicative Fuzzy preference relations on a set of three alternatives (p^1,p^2,p^3). which satisfy the consistency and reciprocal properties (Properties 4, Properties 6)

$$p^1=\begin{bmatrix}\tilde{1} & \tilde{3} & \tilde{6}\\ \frac{1}{\tilde{3}} & \tilde{1} & \tilde{2}\\ \frac{1}{\tilde{6}} & \frac{1}{\tilde{2}} & \tilde{1}\end{bmatrix}=\begin{bmatrix}(1,1,1) & (2,3,4) & (5,6,7)\\ (\frac{1}{4},\frac{1}{3},\frac{1}{2}) & (1,1,1) & (1,2,3)\\ (\frac{1}{7},\frac{1}{6},\frac{1}{5}) & (\frac{1}{3},\frac{1}{2},1) & (1,1,1)\end{bmatrix}$$

$$p^2=\begin{bmatrix}\tilde{1} & \tilde{3} & \tilde{9}\\ \frac{1}{\tilde{3}} & \tilde{1} & \tilde{3}\\ \frac{1}{\tilde{9}} & \frac{1}{\tilde{3}} & \tilde{1}\end{bmatrix}=\begin{bmatrix}(1,1,1) & (2,3,4) & (8,9,10)\\ (\frac{1}{4},\frac{1}{3},\frac{1}{2}) & (1,1,1) & (2,3,4)\\ (\frac{1}{10},\frac{1}{9},\frac{1}{8}) & (\frac{1}{4},\frac{1}{3},\frac{1}{2}) & (1,1,1)\end{bmatrix}$$

$$p^1 = \begin{bmatrix} \tilde{1} & \tilde{2} & \tilde{4} \\ \frac{1}{\tilde{2}} & \tilde{1} & \tilde{2} \\ \frac{1}{\tilde{4}} & \frac{1}{\tilde{2}} & \tilde{1} \end{bmatrix} = \begin{bmatrix} (1,1,1) & (1,2,3) & (3,4,5) \\ (\frac{1}{3},\frac{1}{2},1) & (1,1,1) & (1,2,3) \\ (\frac{1}{5},\frac{1}{4},\frac{1}{3}) & (\frac{1}{3},\frac{1}{2},1) & (1,1,1) \end{bmatrix}$$

Step 1: Calculation of the equivalence expected judgment matrix of $(p_{ij}^k)_{n\times n}$

$$E(P^k) = (E(p_{ij}^k))_{n\times n}$$

$$E(p^1) = \begin{bmatrix} 1 & 3 & 6 \\ \frac{1}{3} & 1 & 2 \\ \frac{1}{6} & \frac{1}{2} & 1 \end{bmatrix} \quad E(p^2) = \begin{bmatrix} 1 & 3 & 9 \\ \frac{1}{3} & 1 & 3 \\ \frac{1}{9} & \frac{1}{3} & 1 \end{bmatrix} \quad E(p^3) = \begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix}$$

Step 2: Using OWG operator and aggregating the equivalence expected judgment matrix as group judgment matrix $G = (u_{ij})_{n\times n}$

$u_{ij} = \prod_{k=1}^{m} (q^k)^{w_k}$, where q^k is the k[th] largest value of the $E(p_{ij}^k)$

Using the linguistic quantifier "most" with the pair value (0.25, 0.75) and the corresponding OWG operators with weight $w = (\frac{1}{6}, \frac{2}{3}, \frac{1}{6})$

We obtain G

$$G = \begin{bmatrix} 1 & 2.1668 & 3.8335 \\ 0.4615 & 1 & 1.7692 \\ 0.2608 & 0.5652 & 1 \end{bmatrix}$$

Step 3: Using OWG operator to calculate the alternatives index

$r_i^{'} = \prod_{j=1}^{n} (v_j)^{w_j^{'}}$, where v_j is the j[th] largest value of the $\{u_{i1}, u_{j2}, ..., u_{jn}\}$

Using the linguistic quantifier "most" with the pair value (0.25, 0.75) and the corresponding OWG operators with weight $w^{'} = (\frac{1}{6}, \frac{2}{3}, \frac{1}{6})$

$$r_1^{'} = (3.8335)^{\frac{1}{6}} (2.1668)^{\frac{2}{3}} (1)^{\frac{1}{6}} = 4.12021$$

$$r_2^{'} = (1.7692)^{\frac{1}{6}} (1)^{\frac{2}{3}} (0.4615)^{\frac{1}{6}} = 0.97498$$

$$r_3^{'} = (1.)^{\frac{1}{6}} (0.5652)^{\frac{2}{3}} (0.2608)^{\frac{1}{6}} = 1.39980$$

Step 4: calculate the normalized vector

$$r_i = \frac{r_i^{'}}{\sum_{i=1}^{n} r_i^{'}} \qquad (7)$$

We obtain

$r_1 = 0.6343$, $r_1 = 0.1501$, $r_1 = 0.2104$

Step 5: Rank the alternatives

$$x_1 \succ x_3 \succ x_2$$

4.3. Solving a Multi-Vriteria Decision Making Problem Using the OWA Operator

Step 1: The experts provide the following multiplicative Fuzzy preference relations on a set of alternatives. Which satisfy the consistency and reciprocal properties (Properties 4, Properties 6)

Step 2: Using OWA operator and aggregating the fuzzy preference relations judgment matrix as group judgment matrix $G = (u_{ij})_{n \times n}$

Where $u_{ij} = \sum_{k=1}^{m} w_k q^k$, where q^k is the k^th^ largest value of the p_{ij}^k

w_k is obtained by the Regular Increasing Monotone (RIM) and linguistic quantifier

Step 3: Using OWA operator to calculate the alternatives index

$r_i^{'} = \sum_{j=1}^{n} (v_j) w_j^{'}$, where v_j is the j^th^ largest value of the $\{u_{i1}, u_{j2}, ..., u_{jn}\}$

$w_j^{'}$ is obtained by the Regular Increasing Monotone (RIM) and linguistic quantifier

Step 4: Calculate the normalized vector

$$r_i = \frac{r_i^{'}}{\sum_{i=1}^{n} r_i^{'}} \qquad (8)$$

Step 5: Rank the alternatives

4.4. Empirical Study in OWA Operator

Suppose a set of four experts $E = \{e_1, e_2, e_3, e_4\}$ provide the following additive Fuzzy preference relations on a set of four alternatives $X = \{x_1, x_2, x_3, x_{4\}}$ is (p^1, p^2, p^3, p^4). which satisfy the consistency and reciprocal properties (Properties 3, Properties 5)

$$p^1 = \begin{bmatrix} 0.5 & 0.3 & 0.4 & 0.1 \\ 0.7 & 0.5 & 0.6 & 0.3 \\ 0.6 & 0.4 & 0.5 & 0.2 \\ 0.9 & 0.7 & 0.8 & 0.5 \end{bmatrix} \quad p^2 = \begin{bmatrix} 0.5 & 0.4 & 0.6 & 0.2 \\ 0.6 & 0.5 & 0.7 & 0.3 \\ 0.4 & 0.3 & 0.5 & 0.1 \\ 0.8 & 0.7 & 0.9 & 0.5 \end{bmatrix}$$

$$p^3 = \begin{bmatrix} 0.5 & 0.5 & 0.8 & 0.5 \\ 0.5 & 0.5 & 0.8 & 0.5 \\ 0.2 & 0.2 & 0.5 & 0.2 \\ 0.5 & 0.5 & 0.8 & 0.5 \end{bmatrix} \quad p^4 = \begin{bmatrix} 0.5 & 5/12 & 1/3 & 0 \\ 7/12 & 0.5 & 5/12 & 1/12 \\ 2/3 & 7/12 & 0.5 & 1/6 \\ 1 & 11/12 & 5/6 & 0.5 \end{bmatrix}$$

Step 1: Using OWA operator and aggregating the fuzzy preference relations judgment matrix as group judgment matrix $G=(u_{ij})_{n\times n}$

$u_{ij}=\sum_{k=1}^{m} w_k q^k$, where q^k is the k[th] largest value of the p_{ij}^k

Using the linguistic quantifier "most" with the pair value (0.25, 0.75) and the corresponding OWA operators with weight $w=(0,0.5,0.5,0)$

$$G=\begin{bmatrix} 0.5 & 0408 & 0.5 & 0.15 \\ 0.592 & 0.5 & 0.65 & 0.3 \\ 0.5 & 0.35 & 0.5 & 0.15 \\ 0.85 & 0.7 & 0.85 & 0.5 \end{bmatrix}$$

Step 2: Using OWA operator to calculate the alternatives index

$r_i^{'}=\sum_{j=1}^{n}(v_j)w_j^{'}$, where v_j is the j[th] largest value of the $\{u_{i1},u_{j2},...,u_{jn}\}$

Using the linguistic quantifier "most" with the pair value (0.25, 0.75) and the corresponding OWG operators with weight $w^{'}=(0,0.5,0.5,0)$

$r_1^{'}=0.454$, $r_2^{'}=0.0.546$, $r_3^{'}=0.425$, $r_4^{'}=$ 0.775

Step 3: calculate the normalized vector

$$r_i=\frac{r_i^{'}}{\sum_{i=1}^{n} r_i^{'}} \quad (9)$$

We obtain

$r_1=0.206$, $r_2=0.248$, $r_3=0.194$ $r_4=0.352$

Step 4: Rank the alternatives

$$x_4 \succ x_2 \succ x_1 \succ x_3$$

5. Conclusion

In this paper, we have studied the use of the OWA operators and OWG operators in the aggregation of fuzzy preference relation in group decision making. We have shown how this operator is adequate for the synthesis of ratio judgments in multi-criteria decision making problems with additive preference relations (OWA operator) and multiplicative preference relations (OWG operator), *i.e.*, Analytic Hierarchical Process (AHP), where it's need to aggregate multiplicative (or additive) reciprocal preference relation which satisfies the consistency property. We have illustrated its use in multi-criteria decision making problems with OWA operator and OWG operator.

In the future, we will research the use of the OWA operator and OWG operator for designing AHP process for improvement of consistency problem.

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